

AD-A206 116

RADC-TR-88-212
Final Technical Report
October 1988



I&W APPLICATIONS OF CATASTROPHE THEORY

Synectics Corporation

A.E.R. Woodcock, L. Cobb, M.E. Familant, J. Markey

DTIC
ELECTE
MAR 27 1989
S D
C&D

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

This effort was funded totally by the Laboratory Director's fund.

ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, NY 13441-5700

89 3 24 01

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b. RESTRICTIVE MARKINGS N/A		
2a. SECURITY CLASSIFICATION AUTHORITY N/A			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A			5. MONITORING ORGANIZATION REPORT NUMBER(S) RADC-TR-88-212		
4. PERFORMING ORGANIZATION REPORT NUMBER(S) WH-86-LQ-00			7a. NAME OF MONITORING ORGANIZATION Rome Air Development Center (IRDW)		
6a. NAME OF PERFORMING ORGANIZATION Synectics Corporation		6b. OFFICE SYMBOL (if applicable)	7b. ADDRESS (City, State, and ZIP Code) Griffiss AFB NY 13441-5700		
6c. ADDRESS (City, State, and ZIP Code) 111 East Chestnut Street Rome NY 13440		8a. NAME OF FUNDING/SPONSORING ORGANIZATION Rome Air Development Center			
8b. OFFICE SYMBOL (if applicable) IRDW		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER F30602-87-C-0054			
8c. ADDRESS (City, State, and ZIP Code) Griffiss AFB NY 13441-5700		10. SOURCE OF FUNDING NUMBERS			
		PROGRAM ELEMENT NO. 61101F	PROJECT NO. LDFP	TASK NO. 02	WORK UNIT ACCESSION NO. C7
11. TITLE (Include Security Classification) I&W APPLICATIONS OF CATASTROPHE THEORY <i>Unintentional Manover Groups</i>					
12. PERSONAL AUTHOR(S) A.E.R. Woodcock, L. Cobb, M.E. Familant, J. Markey					
13a. TYPE OF REPORT Final		13b. TIME COVERED FROM May 87 TO May 88		14. DATE OF REPORT (Year, Month, Day) October 1988	
15. PAGE COUNT 214					
16. SUPPLEMENTARY NOTATION This effort was funded totally by the Laboratory Director's fund.					
17. CUSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Catastrophe Theory, Indications and Warning, Non-linear Mathematics		
12	04				
19. ABSTRACT (Continue on reverse if necessary and identify by block number) Mathematical and statistical tools based on catastrophe theory have been developed and implemented as a fully functional software system which permits the capturing of data derived from analyst's perceptions of CMG-related data elements and their analysis with a computer program based on statistical catastrophe theory. This system permits individuals with no mathematical background to undertake a rigorous analysis on non-linear I&W-related and other phenomena. These tools have been used in experiments with intelligence analysts involving the simulated detection of CMGs, one of the most difficult problems of tactical analysis. A set of unclassified notional indicators predicting the development of an CMG was developed and ten specific settings of these indicators were presented to intelligence analysts who were asked to assess the probability of CMG development. The resulting assessments were captured and analyzed and this revealed the existence of perceptual ambiguity as well as the potential for sudden and gradual perceptual changes, perceptual hysteresis, and perceptual trapping. (Continued)					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a. NAME OF RESPONSIBLE INDIVIDUAL Patricia M. Langendorf			22b. TELEPHONE (Include Area Code) (315) 330-3126		22c. OFFICE SYMBOL RADC (IRDW)

DD Form 1473, JUN 86

Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

UNCLASSIFIED

Block 19 (Continued)

The experienced intelligence analysts who served as subjects had no mathematical background, yet were enthusiastic about the technology. They were particularly interested in a perceived capability to analyze themselves and to identify and correct inconsistent and ambiguous responses. This research has provided evidence that I&W indicators are neither linear nor uncorrelated in the mind of experienced intelligence analysts, a finding which is itself of value. The technology is directly applicable to communicating analyst understanding to battlefield commanders, and to capturing ambiguities in battlefield commander's perception of the combat environment. The technology also appears to be of value as an aid to intelligence analysts and decision-makers since it provides facilities for:

1. Alerting individuals to conditions where small changes in indicator input can give rise to either gradual or sudden changes of perception in the same situation under different conditions.
2. Making available an analytic capability that can give rational interpretations of non-linear and apparently counter-intuitive behavior and for clarifying the causes and effects of ambiguous perceptions.
3. Identifying and characterizing the different types of responses of intelligence analysts and others to features of I&W-related data sets and providing methods that can be used to support the training of such analysts and the interpretation of their assessments of particular sets of indicators.

This technology provides a method for developing a new understanding of the non-linear aspects of the military analytic process. This understanding can be extended from the intelligence analysis of OMGs to the command and control (C²) arena by providing new methods for identifying and selecting particular responses from a list of response options available to the commander. Such a facility is of obvious importance to C², and is being investigated.

UNCLASSIFIED

ACKNOWLEDGEMENTS

Dr. Woodcock would like to thank Ms. Patricia Langendorf, RADC Intelligence and Reconnaissance Directorate, Rome Air Development Center, Griffiss Air Force Base, IWCAT COTR, for her continuing interest in and support of the IWCAT project.

Dr. Woodcock is very grateful to Dr. Frederick Diamond, Chief Scientist, RADC, for his support of and interest in the IWCAT effort.

Dr. Woodcock would like to acknowledge the participation in the IWCAT project of Dr. Loren Cobb, Medical University of South Carolina, Charleston, South Carolina, particularly for his work in providing a new version of the Cusp Surface Analysis program. Dr. Woodcock would also like to acknowledge the participation of Dr. M. Elliott Familant, Mr. John Markey, Mr. Wayne Worthington, Mr. Tony DePace, and Mr. Greg Howard (Synectics Corporation) in the IWCAT project.

Dr. Woodcock would like to thank Ms. Dorothea Martin for the proof reading and production of this document.

Accession For	
NTIS CRA&I	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
A-1	



EXECUTIVE SUMMARY

This is the final technical report for the "I&W Applications of Catastrophe Theory (IWCAT)" project performed by Synectics Corporation (Dr. A. E. R. Woodcock, Project Director and Chief Scientist) for The Rome Air Development Center (RADC) (Ms. P. Langendorf, COTR), Contract Number: F30602-87-C-0054. The IWCAT effort has provided a successful demonstration of the use of catastrophe theory to analyze indications and warning (I&W)-related data and has provided new insights to the process of the analysis and understanding of military indicators, particularly in the area of Operational Maneuver Groups (OMGs).

Mathematical and statistical tools based on catastrophe theory have been developed and implemented as a fully functional software system which permits the capturing of data derived from analyst's perceptions of OMG-related data elements and their analysis with a computer program based on statistical catastrophe theory. This system permits individuals with no mathematical background to undertake a rigorous analysis of non-linear I&W-related and other phenomena. These tools have been used in experiments with intelligence analysts involving the simulated detection of OMGs, one of the most difficult problems of tactical analysis. A set of unclassified notional indicators predicting the development of an OMG was developed and ten specific settings of these indicators were presented to intelligence analysts who were asked to assess the probability of OMG development. The resulting assessments were captured and analyzed and this revealed the existence of perceptual ambiguity as well as the potential for sudden and gradual perceptual changes, perceptual hysteresis, and perceptual trapping.

The experienced intelligence analysts who served as subjects had no mathematical background, yet were enthusiastic about the technology. They were particularly interested in a perceived capability to analyze themselves and to identify and correct inconsistent and ambiguous responses. This research has provided evidence that I&W indicators are neither linear nor uncorrelated in the mind of experienced intelligence analysts, a finding which is itself of value. The technology is directly applicable to communicating analyst understanding to battlefield commanders, and to capturing ambiguities in battlefield commander's perception of the combat environment. The technology also appears to be of value as an aid to intelligence analysts and decision-makers since it provides facilities for:

1. Alerting individuals to conditions where small changes in indicator input can give rise to either gradual or sudden changes of perception in the same situation under different conditions.
2. Making available an analytic capability that can give rational interpretations of non-linear and apparently counter-intuitive behavior and for clarifying the causes and effects of ambiguous perceptions.
3. Identifying and characterizing the different types of responses of intelligence analysts and others to features of I&W-related data sets and providing methods that can be used to support the training of such analysts and the interpretation of their assessments of particular sets of indicators.

This technology provides a method for developing a new understanding of the non-linear aspects of the military analytic process. This understanding can be extended from the intelligence analysis of OMGs to the command and control (C2) arena by providing new methods for identifying and selecting particular responses from a list of response options available to the commander. Such a facility is of obvious importance to C2, and is being investigated.

TABLE OF CONTENTS

SECTION 1. IWCAT PROJECT REVIEW	1-1
1.1 The IWCAT Effort in Context	1-1
1.2 Catastrophe Theory can Provide New Tools for the I&W Analyst	1-2
1.3 Operational Maneuver Group Characteristics	1-5
1.4 IWCAT Concept of Operations	1-6
1.4.1 Mapping I&W Problems to Catastrophe Theory Surfaces	1-6
1.4.2 A Catastrophe Theory-Based Knowledge Development Environment	1-8
1.4.2.1 Test Data Sets	1-10
1.4.2.2 The Collection of Test Assessments	1-11
1.4.2.3 The Processing of OMG Assessment Data	1-11
1.5 Cusp Surface Analysis	1-15
1.5.1 Estimating Parameters	1-15
1.5.2 Making Predictions	1-15
1.6 OMG Threat Assessment Analyses	1-20
1.6.1 Mapping Data to the Catastrophe Model Surface	1-20
1.6.2 Specific Analyst Assessments	1-24
1.6.2.1 The Perceptions of "Analyst A"	1-33
SECTION 2. THE IWCAT EFFORT IN CONTEXT	2-1
2.1 New Facilities are Needed to Support the I&W Analyst	2-1
2.2 Activities of I&W Intelligence Analysts	2-2
2.3 Operational Maneuver Group Characteristics	2-4
2.3.1 OMGs and Other Military Units	2-5
2.3.2 Identifying OMGs	2-5
2.3.2.1 Time and Location of OMG Insertion	2-5
2.3.2.2 Concomitant Activity of Other Military Units	2-5
2.3.2.3 Changes to Military Units Becoming OMGs	2-6
2.4 Catastrophe Theory can Provide New Tools for the I&W Analyst	2-7
2.5 Models of Perception can be Based on Catastrophe Theory	2-8
2.5.1 Sudden Changes in Perception	2-11
2.5.2 Divergence	2-11
2.5.3 Bimodality	2-11
2.5.4 Hysteresis	2-11

TABLE OF CONTENTS (Continued)

SECTION 3. IWCAT CONCEPT OF OPERATIONS -----	3-1
3.1 Overview -----	3-1
3.2 Mapping I&W Problems to Catastrophe Theory Surfaces -----	3-3
3.2.1 Single Valued and Multi-Valued Functions -----	3-4
3.2.2 Singly Stable and Bi-Stable Perceptions -----	3-4
3.3 A Catastrophe Theory-Based Knowledge Development Environment --	3-9
3.3.1 The OMG Indications and Warnings Problem -----	3-10
3.3.2 The Source Evaluation System -----	3-10
3.3.3 The Test Data Sets -----	3-11
3.3.3.1 Number of Active Indicators -----	3-13
3.3.3.2 Level of Confidence -----	3-15
3.3.3.3 Scenario Presentation -----	3-15
3.3.3.4 Sample Sequences -----	3-19
3.3.4 The Collection of Test Assessments -----	3-24
3.3.5 The Processing of OMG Assessment Data -----	3-24
SECTION 4. CUSP SURFACE ANALYSIS -----	4-1
4.1 Preliminary Notes on Terminology -----	4-1
4.2 Introduction -----	4-1
4.3 Estimating the Parameters -----	4-6
4.4 Testing the Model -----	4-7
4.5 Making Predictions -----	4-12
4.6 A Sample Output -----	4-14
SECTION 5. USING THE IWCAT SYSTEM -----	5-1
5.1 Statistical Analysis and Catastrophe Theory -----	5-1
5.1.1 Key Variables -----	5-1
5.2 System Requirements -----	5-2
5.2.1 Hardware -----	5-2
5.2.2 Software -----	5-2
5.2.3 Getting Started -----	5-2
5.3 The IWCAT System -----	5-3
5.3.1 Menu Display Overview -----	5-3
5.3.2 Read About the Program -----	5-3
5.3.3 Enter Information About Yourself -----	5-6
5.3.4 Make a Test File -----	5-6
5.3.5 Practice Session -----	5-10
5.3.5.1 Practice Data Presentation -----	5-10
5.3.5.2 Practice OMG Threat Assessment -----	5-15

TABLE OF CONTENTS (Continued)

5.3.6	Take the Test -----	5-15
5.3.6.1	Test Data Presentation -----	5-15
5.3.6.2	OMG Threat Assessment -----	5-19
5.3.7	Make a File for Analysis -----	5-19
5.3.8	Perform Cusp Analysis -----	5-23
5.3.8.1	Testing the Model -----	5-28
5.3.8.2	Making Predictions -----	5-35
5.3.9	Termination of Analytic Session -----	5-35
SECTION 6.	OMG THREAT ASSESSMENT ANALYSIS -----	6-1
6.1	Statistical Analysis of the OMG Threat Assessment Data Base -----	6-1
6.1.1	Mapping Data to the Catastrophe Model Surface -----	6-3
6.1.2	Sudden and Gradual Perceptual Changes -----	6-6
6.1.3	Divergence -----	6-6
6.1.4	Ambiguity -----	6-6
6.1.5	"Slicing" the Cusp Surface -----	6-6
6.1.6	Perceptual Hysteresis -----	6-11
6.1.7	Perceptual "Trapping" -----	6-11
6.1.8	Counter-Intuitive or Paradoxical Behavior -----	6-15
6.2	Specific Analyst Assessments -----	6-15
6.2.1	"Analyst A" -----	6-17
6.2.2	"Analyst B" -----	6-24
6.2.3	"Analyst C" -----	6-30
6.2.4	"Analyst D" -----	6-35
6.2.5	"Analyst E" -----	6-37
APPENDIX A:	AN OVERVIEW OF CATASTROPHE THEORY -----	A-1
A.1	Catastrophe Theory – A Proven Framework -----	A-1
A.2	The Properties of the Cusp Catastrophe -----	A-3
A.3	The Properties of the Butterfly Catastrophe -----	A-6
A.4	Catastrophe Theory-Based Models of Military Behavior -----	A-6
A.4.1	A Cusp-Catastrophe-Based Model -----	A-6
A.4.2	A Butterfly-Catastrophe-Based Model -----	A-8
APPENDIX B:	BIBLIOGRAPHY -----	B-1

LIST OF EXHIBITS

1-1	Catastrophe Theory-Based I&W Assessment Activities -----	1-4
1-2	Overview of the IWCAT Concept of Operations -----	1-7
1-3	Components of the IWCAT Analyst Computer Display -----	1-9
1-4	Analyst OMG Threat Assessment and Data Base Formation Activities -----	1-12
1-5	Relationship of the OMG Threat Assessment Data Base to the Cusp Catastrophe Manifold Control Plane -----	1-13
1-6	Fitting OMG Threat Assessment Data Base to the Cusp Manifold Surface ----	1-14
1-7	A Conventional or a Catastrophe Theory-Based Approach? -----	1-17
1-8	Criteria for Acceptance of the Cusp Catastrophe-Based Model -----	1-18
1-9	Cusp Surface Analysis: Making Predictions -----	1-19
1-10	IWCAT System Activities -----	1-21
1-11	I&W Analyst-Derived Data Plotted on the Control Plane of the Cusp Model--	1-22
1-12	Mapping Data to the Cusp Model Surface -----	1-23
1-13	Cusp Model of Sudden and Gradual Changes in Analyst Perceptions -----	1-25
1-14	Cusp Model of Divergent Perceptions -----	1-26
1-15	Cusp Model Can Provide a New Understanding of the Causes of Perceptual Ambiguity -----	1-27
1-16	"Slicing" the Cusp Surface -----	1-28
1-17	Perceptual Hysteresis -----	1-29
1-18	Partial Perceptual "Trapping" -----	1-30
1-19	Complete Perceptual "Trapping" -----	1-31
1-20	Counter-Intuitive or Paradoxical Behavior -----	1-32
1-21	Analysis of OMG Threat Assessment Data -----	1-34
1-22	Analysis of OMG Threat Assessment Data -----	1-35
1-23	Analysis of OMG Threat Assessment Data -----	1-37
1-24	Analysis of OMG Threat Assessment Data -----	1-38
1-25	Analysis of OMG Threat Assessment Data -----	1-39
2-1	IWCAT Environment -----	2-3
2-2	Input Characteristics -----	2-9
2-3	Increasing Detail -----	2-10
2-4	Sudden Changes -----	2-12
2-5	The Property of Divergence -----	2-13
2-6	The Property of Bimodality -----	2-14
2-7	Perceptions Can Exhibit Hysteresis -----	2-15

LIST OF EXHIBITS (Continued)

3-1	Overview of the IWCAT Concept of Operations -----	3-2
3-2	Functional Relationships -----	3-5
3-3	Singly-Stable and Bi-Stable Perceptions -----	3-6
3-4	Catastrophe Theory-Based I&W Assessment Activities -----	3-8
3-5	The "Source Evaluation System" -----	3-12
3-6	Selected I&W-Related Indicators -----	3-14
3-7	Indicator Activity Patterns as a Function of Number of Active Indicators ---	3-16
3-8	Components of the IWCAT Analyst Computer Display -----	3-17
3-9	Analyst OMG Threat Assessment and Data Base Formation Activities -----	3-20
3-10	Relationship of the OMG Threat Assessment Data Base to the Cusp Catastrophe Control Plane -----	3-26
3-11	Fitting the OMG Threat Assessment Data Base to the Cusp Catastrophe Manifold -----	3-27
4-1	The Linear Model Used by Multiple Regression -----	4-2
4-2	The Cusp Catastrophe Model -----	4-4
4-3	Sections Through the Cusp Catastrophe Model -----	4-5
4-4	The Type N_3 PDF for the Cusp Catastrophe Model -----	4-8
4-5	A Conventional or a Catastrophe Theory-Based Approach? -----	4-9
4-6	Criteria for Acceptance of the Cusp Catastrophe-Based Model -----	4-11
4-7	Cusp Surface Analysis: Making Predictions -----	4-13
4-8	The Effect of Linear Factors -----	4-15
4-9	Sample Output from the Cusp Surface Analysis Program -----	4-16
5-1	IWCAT System Overview Menu Display -----	5-4
5-2	IWCAT System Overview Menu Display, Option 1: Read About the Program -----	5-5
5-3	IWCAT System Overview Menu Display, Option 2: Enter Information About Yourself -----	5-7
5-4	IWCAT System Overview Menu Display, Option 3: Make a Test File -----	5-8
5-5	Making a Test File -----	5-9
5-6	IWCAT System Overview Menu Display, Option 4: Practice Session -----	5-11
5-7	Entering OMG Assessment Practice Data -----	5-12
5-8	Practice Session Activities -----	5-13
5-9	Practice Data Display -----	5-14

LIST OF EXHIBITS (Continued)

5-10	IWCAT System Overview Menu Display, Option 5: Take the Test -----	5-16
5-11	Enter OMG Test Data Assessments -----	5-17
5-12	OMG Test Data Presentation -----	5-18
5-13	OMG Threat Assessment Activities -----	5-20
5-14	IWCAT System Overview Menu Display, Option 6: Make a File for Analysis -----	5-21
5-15	Making a File for Statistical Analysis -----	5-22
5-16	IWCAT System Overview Menu Display, Option 7: Perform Cusp Analysis -----	5-24
5-17	Cusp Analysis Activities -----	5-25
5-18	Sample Output from the Cusp Surface Analysis Program -----	5-27
5-19	Sample Output from the Cusp Surface Analysis Program -----	5-29
5-20	Sample Output from the Cusp Surface Analysis Program -----	5-30
5-21	Sample Output from the Cusp Surface Analysis Program -----	5-31
5-22	Sample Output from the Cusp Surface Analysis Program -----	5-32
5-23	Sample Output from the Cusp Surface Analysis Program -----	5-33
5-24	Sample Output from the Cusp Surface Analysis Program -----	5-34
6-1	IWCAT System Activities -----	6-2
6-2	I&W Analyst-Derived Data Plotted on the Control Plane of the Cusp Model -	6-4
6-3	Mapping Data to the Cusp Model Surface -----	6-5
6-4	Cusp Model of Sudden and Gradual Changes in Analyst Perceptions -----	6-7
6-5	Cusp Model of Divergent Perceptions -----	6-8
6-6	Cusp Model Can Provide a New Understanding of the Causes of Perceptual Ambiguity -----	6-9
6-7	"Slicing" the Cusp Surface -----	6-10
6-8	Perceptual Hysteresis -----	6-12
6-9	Partial Perceptual "Trapping" -----	6-13
6-10	Complete Perceptual "Trapping" -----	6-14
6-11	Counter-Intuitive or Paradoxical Behavior -----	6-16
6-12	Analysis of OMG Threat Assessment Data -----	6-18
6-13	Analysis of OMG Threat Assessment Data -----	6-20
6-14	Analysis of OMG Threat Assessment Data -----	6-22
6-15	Analysis of OMG Threat Assessment Data -----	6-23

LIST OF EXHIBITS (Continued)

6-16	Analysis of OMG Threat Assessment Data -----	6-25
6-17	Analysis of OMG Threat Assessment Data -----	6-27
6-18	Analysis of OMG Threat Assessment Data -----	6-29
6-19	Analysis of OMG Threat Assessment Data -----	6-31
6-20	Analysis of OMG Threat Assessment Data -----	6-33
6-21	Analysis of OMG Threat Assessment Data -----	6-34
6-22	Analysis of OMG Threat Assessment Data -----	6-36
A-1	The Elementary Catastrophes -----	A-2
A-2	The Cusp Catastrophe Function $V_{CC}(x)$ -----	A-4
A-3	The Cusp Catastrophe Manifold and Control Plane -----	A-5
A-4	The Butterfly Catastrophe Potential Function $V_{BC}(x)$ and Manifold -----	A-7
A-5	A Military Analysis and Problem-Solving Landscape -----	A-9
A-6	Application of Catastrophe Theory to Military Analysis -----	A-10
A-7	A Military Analysis and Problem-Solving Landscape -----	A-11

SECTION 1. IWCAT PROJECT REVIEW

This is the final technical report for the "I&W Applications of Catastrophe Theory (IWCAT)" project performed by Synectics Corporation (Dr. A. E. R. Woodcock, Project Director and Chief Scientist) for The Rome Air Development Center (RADC) (Ms. P. Langendorf, COTR).

The IWCAT effort has provided a successful demonstration of the use of catastrophe theory to analyze indications and warning (I&W)-related data and has provided new insights to the process of the analysis and understanding of military indicators, particularly in the area of Operational Maneuver Groups. Methods have been made available which can serve to identify conditions under which sudden and gradual changes, divergence, ambiguities, paradoxical reversals, hysteresis, and perceptual "trapping" can take place and can aid in assessing their impact on the interpretation of the data.

A fully functional IWCAT software system has been developed which permits the capturing of data derived from analyst's perceptions of OMG-related data elements and their analysis with a computer program based on statistical catastrophe theory. This system could serve as the basis of a fully operational I&W facility which is sensitive to nonlinear changes in perception, a capability which cannot be provided by systems which use linear regression techniques for analysis.

Several members of Synectics staff who have been involved in various forms of I&W and intelligence analysis activity participated in testing the IWCAT system as it neared completion. Information generated by this process was then subjected to analysis using the cusp analysis program. It is a suggestive finding of this statistical analyses performed during this investigation that the nature of the response of the different analysts to the OMG threat test data appeared to depend upon their background and experience. The suggestions of analyst background- and experience response-specificity is a tentative finding due to the small sample size of analysts that were used in the experiment. However, such a suggestion can have profound implications on the way that I&W and other forms of intelligence analyses are performed. These possibilities should be the subject of further analytic activities and investigations with the aid of the IWCAT system which can form the basis of a test-bed for such a study.

1.1 THE IWCAT EFFORT IN CONTEXT

The I&W Applications of Catastrophe Theory (IWCAT) effort has determined that it is feasible to use catastrophe theory and related mathematical techniques to provide new facilities to support the activities of I&W analysts in areas of importance to the Air Force. Investigations have concentrated on the use of indicators related to the formation of Operational Maneuver Groups (OMGs) as a test of the system. This effort has involved the development of a new flexible and adaptable problem-solving and decision-making environment that is able to capture and use small, and apparently insignificant, changes in information that are the precursors of dramatic changes in overall system behavior.

This final technical report identifies OMG detection as the specific I&W-related problem which is amenable to analysis with the techniques of applied catastrophe theory. In general, suitable I&W problems for such analysis will be those in which several key influences determine system behavior and which exhibit some or all of the following properties:

1. Gradual and sudden changes, divergence, bimodality, and hysteresis that are characteristic of the behavior exhibited by the elementary catastrophes.
2. Small changes in the information (provided to an I&W analyst, for example) can give rise to either small or large changes in perception under the same conditions.
3. Small biases in information can give rise to dramatically different analytic results.

After extensive discussions with the government, the IWCAT team selected an I&W problem which involved the recognition of an OMG, one of the most difficult problems in tactical analyses, as a test problem. A set of ten indicators predicting the development of a Soviet OMG was developed. Settings of these indicators were presented to military analysts who were asked to assess the probability of OMG development. These assessments were captured and analyzed with the aid of a statistical program based on catastrophe theory.

1.2 CATASTROPHE THEORY CAN PROVIDE NEW TOOLS FOR THE I&W ANALYST

Recent advances in mathematics in such areas as catastrophe theory have provided a new understanding of the nature of highly complicated and inherently nonlinear systems. These advances have paved the way for the application of new mathematical techniques to such problems as those associated with I&W. These applications can be supported through the development and use of new analytic "tools" based upon catastrophe theory. However, in order to avoid prohibitively long training periods, such tools should be made available to I&W analysts and decision-makers in such a way that these individuals are not required to understand their mathematical details. The IWCAT system has achieved such "mathematical transparency" through the use of menus, other forms of man-machine interface techniques, and self-documentation by means of appropriate text files.

Military analysts and decision-makers in the indications and warning (I&W) area are faced with the need to analyze and understand large amounts of often conflicting and contradictory data derived from sensors, communications systems, and other sources. These tasks often have to be performed under severe time-pressure and if used in the field, at some actual physical risk to the analysts and decision-makers themselves. Faced with the problem of information overload in critical periods of combat, such individuals will have to resort to the use of analytic methods that capture the essence of system behavior and "friendly" graphics devices that can facilitate the understanding, reasoning, and decision-making activities of analysts in ways that develop and reinforce their perceptions. The IWCAT effort has produced a prototype computer-based system that can support the I&W analyst by providing a new technology for capturing I&W analysts' perceptions of situations of interest and communicating an understanding of these perceptions to battlefield commanders, for example.

The IWCAT technology can also be of value of I&W analysts and decision-makers by:

1. Alerting individuals to conditions where small changes in indicator input can give rise to either gradual or sudden changes of perception in the same situation under different conditions.
2. Clarifying the causes and effects of different perceptions of the same situation.
3. Providing an analytic capability that can give rational interpretations of nonlinear and apparently counter-intuitive behavior.

- 4 Identifying and characterizing the different types of responses of I&W analyses and others to features of I&W-related data sets.
5. Providing methods that can be used to support the training of such analysts and the interpretation of their assessments of particular sets of indicators.

The IWCAT system provides a synthetic environment in which different indicators, representing the different key factors used by I&W analysts to make assessments and provide warning of an OMG are combined in a mathematically rigorous manner to provide an overall perception of the situation of interest. These key indicators determine position on a geometric structure (technically known as the catastrophe manifold, and referred to as the cusp or catastrophe surface in this report) which consists of regions which can be described as flat plains and cliffs. The flat plains represent regions in which the perception of the analyst is unambiguous. The cliffs, by contrast, represent conditions (represented by a particular set of indicator values) under which sudden, perceptual changes can take place (Exhibit 1-1). Furthermore, these cliffs mark the boundaries of regions where the analyst's perceptions are ambiguous and where incorrect, misleading, and ambiguous assessments can be made.

During the IWCAT project, a series of unclassified notional indicators, considered by the IWCAT project team to reflect the activities and characteristics of a Soviet military formation known as an Operational Maneuver Group (OMG), were identified and sets of these indicators were presented to test analysts in order to determine their assessment of OMG threat. Data generated by this process was analyzed with the aid of nonlinear statistical procedures based on catastrophe theory.

Use of these mathematical techniques in the IWCAT effort has made possible the development of a wide range of new I&W analytic tools that can be used to support the activities of military analysts and decision-makers. Systems that exhibit some or all of the properties of gradual and sudden changes in behavior, divergence, bimodality, and hysteresis have an underlying nature to which the catastrophe theory-based analysis can be applied. Such properties are associated with the general phenomena of perception and also with the specific activities of I&W analysts.

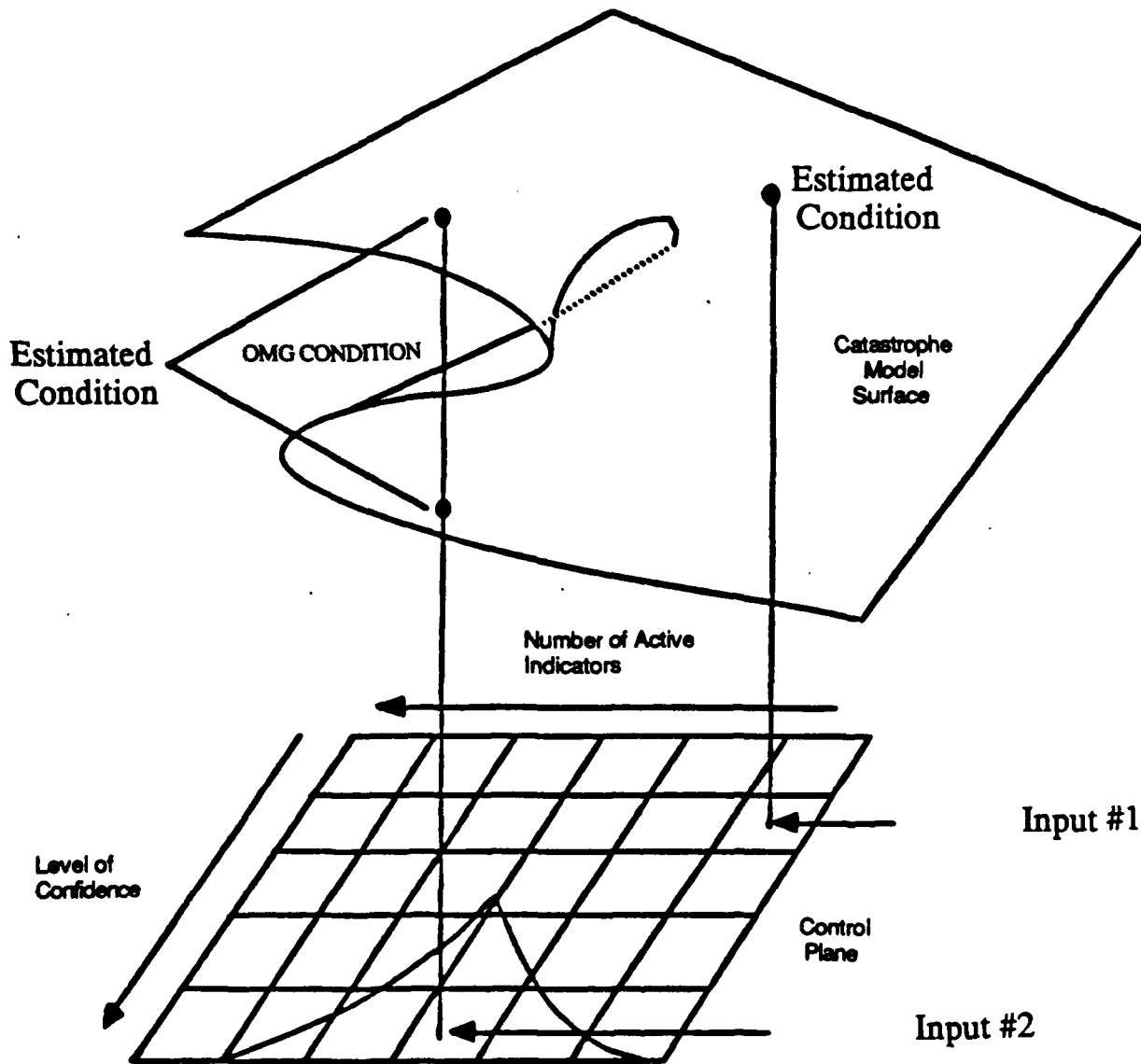
In the application of the theory to modeling perception carried out during the IWCAT project, two major control, or input, factors whose actions determine the nature of the perception of the I&W-related situation by an observer have been identified as the control factors of a model of perception based on catastrophe theory:

1. The number of active indicators in the sets of OMG-related indicators presented to the I&W analyst.
2. The level of confidence that a particular set of indicators represent actual military activities based on an assumed knowledge of the capabilities of intelligence collection and processing capabilities, for example.

The action of these factors will determine the perception of the object or scene by an observer and this perception will be described as the behavior or output variable of the system in the catastrophe-theoretic model of perception described below.

Exhibit 1-1

Catastrophe Theory-Based I&W Assessment Activities



1.3 OPERATIONAL MANEUVER GROUP CHARACTERISTICS

An OMG is a highly mobile military unit which evolved from the Soviet Mobile Group, a highly mobile tank formation used extensively during World War II and is designed to operate behind NATO lines. Its mission is to attack or raid valuable targets, destroy or limit the nuclear capability of the west, disrupt reinforcement supply lines, and maintain a close proximity with NATO troops thereby making the introduction of tactical nuclear weapons difficult. An OMG is designed to facilitate a quick win by destroying NATO defensive capacity and opening a second front during the offensive.

The introduction of an OMG appears to be designed to deter the west from deploying tactical nuclear weapons once hostilities start. NATO uses tactical nuclear weapons to partially offset the imbalance of conventional forces that exists between its forces and those of the Warsaw Pact. However, for NATO to use tactical nuclear weapons, Warsaw Pact forces must be well separated from NATO forces. If the two forces are in close proximity to each other, there is a risk that friendly forces will be destroyed if tactical nuclear weapons are used.

OMG's do not operate in isolation. They depend upon other military units for support. For the OMG to be most effective, it must arrive behind NATO lines intact. Because of this, an OMG will attempt to penetrate an opponent's defenses only after they have been weakened or diverted by first echelon forces. An OMG completes the breakthrough started by the first echelon forces. While attempting to break through NATO lines, the OMG will be supported by heavy artillery preparation and a barrage of covering fire. Artillery and air support are considered decisive elements in modern combat. The two major artillery units supporting the OMG are the Division Artillery Group (DAG) and the Regimental Artillery Group (RAG). Both groups are usually reinforced with nondivisional artillery battalions. Air defense for the OMG is provided by integrated systems of antiaircraft artillery, surface-to-air missiles (SAMs) and interceptor aircraft of frontal aviation. They provide air coverage at all altitudes.

There are several classes of criteria which can be used to identify an OMG. They are the time at which OMGs will be inserted into battle, the location at which the OMG will be inserted, the activities of other units that will be done in support of the OMG, and the changes that occur to a military unit prior to its operation as an OMG.

1. Time and Location of OMG Insertion: The places where the OMG is inserted will be weak points in the NATO defense. It is expected that OMGs will be inserted at locations characterized as having low combat power, lack of defense in depth, and low force density.
2. Concomitant Activity of Other Military Units: There are a number of activities in which Soviet forces will engage in support of the OMG's penetration of NATO defensive lines. Among them are the introduction of jammers to disrupt NATO air and fire support nets and command and control in the sector at which the break will occur; ground based air defense in support of a breakthrough operation; heavy artillery preparation and covering fire barrage immediately before penetration.
3. Changes to Military Units Becoming OMGs: For a military unit to operate as an OMG, it must increase its mobility and self-sufficiency. It will make the following kinds of attachments: self-propelled artillery; combat engineers; lift capacity; signal troops to provide long range communication; and increased amounts of fuel and ammunitions.

SECTION 1.4 IWCAT CONCEPT OF OPERATIONS

United States Air Force I&W analysts have the mission of providing a timely recognition and reporting of changes in military events that are of interest to the United States. Activities performed under the IWCAT contract have reviewed typical I&W activities and have led to the identification of those classes of problems which are amenable to analytic procedures based on catastrophe theory. Syntectics' IWCAT project staff has determined, in collaboration with the government, that the conditions under which an Operational Maneuver Group (OMG) is formed from an otherwise "normal" pattern of soviet military advance are of sufficient interest to the government to warrant its selection as the appropriate "I&W situation" as specified in the IWCAT statement of work.

The overall concept of operations for the IWCAT project is illustrated in Exhibit 1-2. The operation of the IWCAT knowledge development environment involves several major phases of activity, including the following:

1. The introduction to the knowledge development environment facility provides the user of the IWCAT facility with a practice use of this facility and a series of "help" and other text files that aid in its use.
2. The generation of OMG-related test data sets involves a dedicated scenario generator that produces groups of indicators whose properties have been chosen to reflect those of an OMG.
3. The presentation of test data sets to I&W analysts permits intelligence analysts to undertake OMG-related threat assessment activities and the construction of an OMG Threat Assessment Data Base.
4. The performance of a statistical analysis of the OMG Threat Assessment Data Base. Creation of this data base as outlined above sets the scene for its analysis with the aid of techniques based on statistical catastrophe theory.
5. The review of the results of this statistical analysis provides a new level of insight into the processes of perception and threat assessment undertaken by I&W analysts and can set the scene for the development of new types of operational facilities for OMG threat analysis, for example.

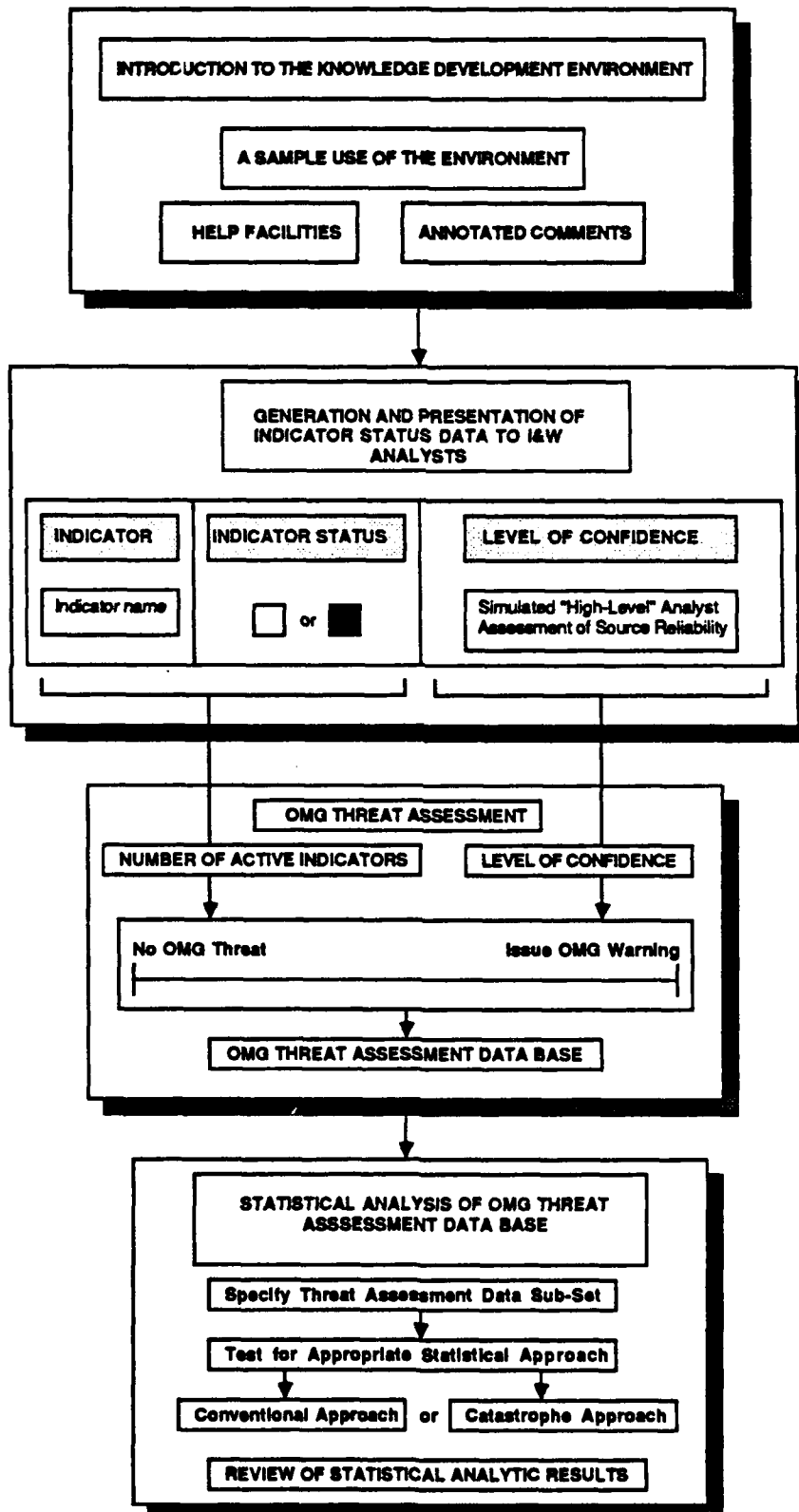
1.4.1 MAPPING I&W PROBLEMS TO CATASTROPHE THEORY SURFACES

Catastrophe theory describes a series of structures called catastrophe manifolds which resemble stylized "landscapes." Positions on such landscapes are specified by coordinates whose specific values reflect the values of key independent system variables and the corresponding values of the dependent variable(s) of the system. The IWCAT project has used these geometrical structures to express the relationships between the nature of the intelligence and other information input to I&W analysts (the independent system variables) and their assessment (the dependent system variable(s)) of the perceived level of OMG threat corresponding to these inputs.

Test data sets corresponding to different values of selected independent variables associated with OMGs are presented to selected I&W analysts and others in a knowledge development activity where these individuals are asked to describe their perceptions of the

Exhibit 1-2

Overview of the IWCAT Concept of Operations



level of OMG threat reflected in these data. The information obtained during this activity is then analyzed and an attempt made to fit these data to the cusp catastrophe manifold with the aid of a statistical catastrophe theory-based computer program. This process uses methods which are rigorous extensions of the techniques of linear regression. When the catastrophe manifold has been created in this way, it can be used to assess the nature of I&W-related data sets.

There are other factors which influence perception but which are not tractable within this scheme. These are variously referred to as context variables or tacit knowledge. These variables include general knowledge about the world which influences how judgments are made. Examples of this kind of knowledge include the time of year at which the judgment has occurred and the politico-military context of the problem.

To provide as realistic a problem environment as possible, sets of context variables were defined and presented to the individuals performing the OMG assessment task as written scenarios. These scenarios are called "Treaty Obligation," "Friendly Ally," and "Third Party Hostilities" scenarios. The first of these scenarios would require direct United States military involvement under particular circumstances; the second places a high level of obligation on the United States to provide information to an ally and failure to do this would result in damage to United States political and other interests; the third scenario is one in which the United States has no direct interest in the conflict, but monitors it to recognize changes which impact on United States interests as they occur.

1.4.2 A CATASTROPHE THEORY-BASED KNOWLEDGE DEVELOPMENT ENVIRONMENT

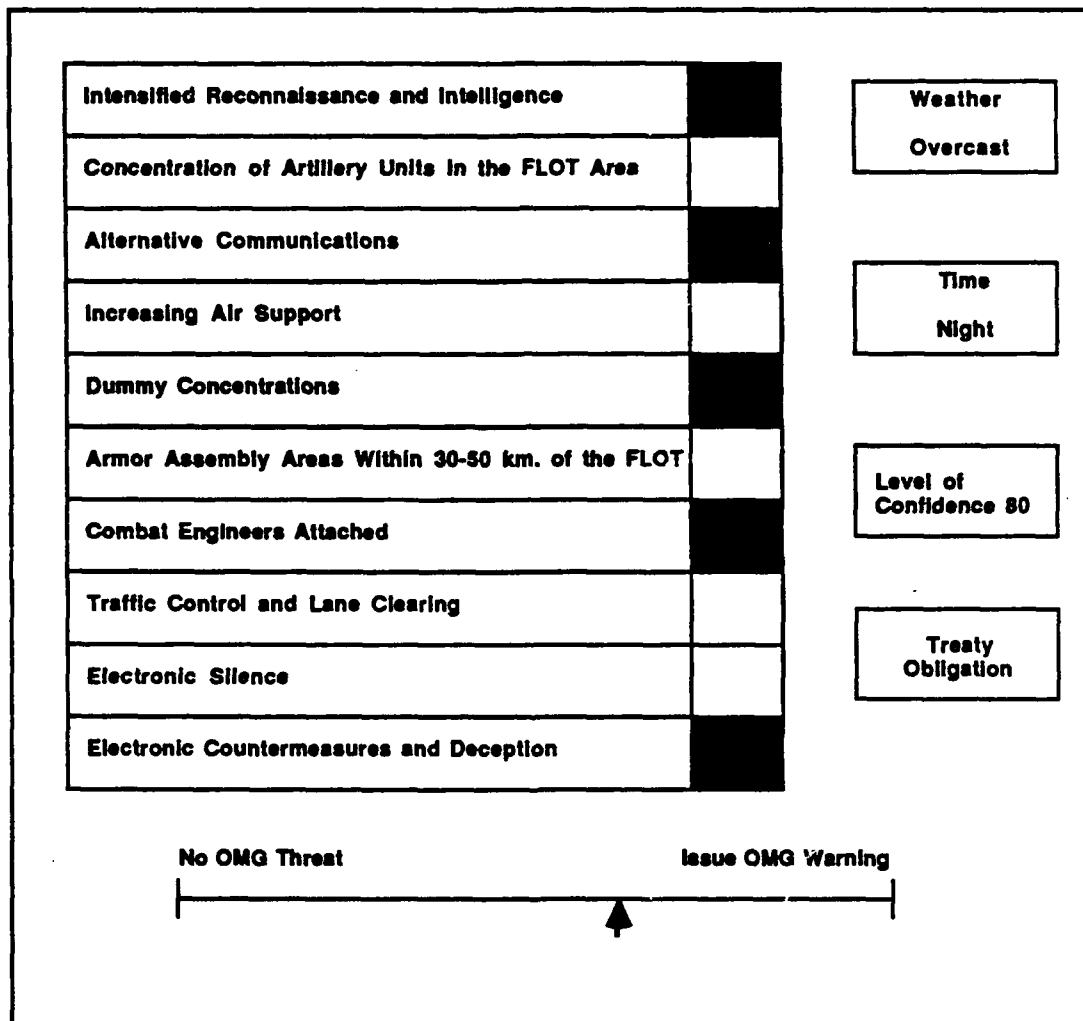
A series of activities, described in the IWCAT proposal as knowledge development activities, were undertaken to determine the responses of I&W analysts when faced with the task of assessing the likelihood that an OMG has been formed, or is in the process of being formed. Individuals were given a set of data which were designed to resemble as closely as possible in a study environment the type of data that would be available to an I&W analyst in an operational environment. The data produced during this activity was analyzed with the aid of a nonlinear statistical procedure based on catastrophe theory (developed by Cobb (1978, 1980)) in order to investigate conditions under which ambiguous identifications and sudden and gradual changes in I&W assessments can take place.

Analysis of the I&W environment and discussions with the government have led the IWCAT project team to the identification of the following key control factors and behavior variables associated with I&W analysis activities.

1. The control factors represent the inputs to the process of I&W analysis. The IWCAT project team selected two control variables (number of active indicators and level of confidence) to represent these inputs. The "number of active indicators" variable represents the number of indicators which are activated when the analyst makes a judgment. The "level of confidence" variable represents a measure of the degree to which a particular set of indicators can be considered to be a true representation of actual military behavior. Additional information including weather, time of day, and scenario type is also presented to the analyst during the OMG threat assessment task.
2. The behavior variable represents the result of the process of I&W analyst assessment. The IWCAT project team named this variable the analyst's OMG threat assessment since this variable represents the perception of the I&W analyst of the likelihood of the formation of an OMG from an apparently "normal" pattern of military advance.

Exhibit 1-3

Components of the IWCAT Analyst Computer Display



The two major control variables described above ("number of active indicators" and "level of confidence") are considered to be independent variables whose values determine the value of the dependent or behavior variable, an assumption that can be tested with the aid of actual analyst assessments and the cusp surface analysis program, as described below. Thus, the number of active indicators and the level of confidence in these indicators provide information that can permit an I&W analyst to assess the likelihood of the formation of an OMG. The independent (number of active indicators and level of confidence) variables can be represented as orthogonal axes and all sets of I&W-related data could be assigned a value with respect to these axes (see Exhibit 1-1, for example).

In the event that some sets of independent variables create different or ambiguous perceptions of the threat of OMG formation, such behavior could be illustrated with the aid of the cusp catastrophe manifold, which is multivalued for some ranges of its control factor values. Under these circumstances, some sets of control factor values (the number of active indicators and level of confidence conditions associated with the I&W-related indicators) generate multiple behavior variable (or OMG threat perception) values while other sets generate a single behavior variable value.

1.4.2.1 Test Data Sets

During the OMG threat assessment activities, the I&W analyst is presented with a sequence of different data sets each with a different number of active indicators and level of confidence properties which have been chosen to reflect indications of different adversarial status conditions that might be presented to I&W analysts during an investigation of whether or not an OMG was in the process of formation.

Analysis performed by Synectics personnel and a review of several unclassified documents which describe the properties of Operational Maneuver Groups (OMGs) has led to the identification of the following ten OMG-related indicators (Exhibit 1-3). These indicators are presented in no particular order to avoid implying any preestablished ranking, importance, or priority of a particular indicator, or sets of indicators, in the data displays.

1. Intensified reconnaissance and intelligence.
2. Concentration of artillery units in FLOT (Front Line Of Troops) area.
3. Alternative communications.
4. Increasing air support.
5. Dummy concentrations.
6. Armor assembly areas within 30-50 km of the FLOT.
7. Combat engineers attached.
8. Traffic control units and lane clearing.
9. Electronic silence.
10. Electronic countermeasures and deception.

When using the IWCAT system, the I&W analyst is presented with a level of confidence number the value of which reflects the degree to which the particular set of data elements is considered to represent an "actual" situation of interest (Exhibit 1-3). In each display, only the indicators listed would be "turned on." The type of scenario (Treaty Obligation, Ally Support or Third Party Hostilities), and the attendant weather and day/night conditions are also presented on the warning display screen.

The analyst forms a judgment as to his or her own relative degree of certainty that the display indicates that an OMG activity is impending, and enters this on a sliding scale at the bottom of the display with the aid of the "arrow" (< and >) keys. The display software then captures this registration as a decimal number (0.0 to 1.0), stores it for subsequent statistical analysis, and advances to the next situation in the series to be displayed (Exhibit 1-4).

1.4.2.2 The Collection of Test Assessments

In order to determine the reaction of I&W analysts to particular types of data, a selection of test data sets, each with different numbers of active indicators and level of confidence information, is presented to the individuals participating in the knowledge development activity. These individuals can undergo an initial period of training and can be asked to review each element of the data set for a short time and then provide an assessment of the OMG-related posture of an adversary as reflected in these data. This assessment is recorded by indicating a position on the scale as mentioned above and these assessments are stored in the OMG Threat Assessment Data Base (Exhibits 1-4 and 1-5).

1.4.2.3 The Processing of OMG Assessment Data

The results for each individual participant are tabulated, recorded, and analyzed with the aid of a statistical catastrophe theory-based computer program (Cobb, 1980) in order to determine whether the data can be described with the aid of a linear model, or whether the data could be described more appropriately with the aid of a nonlinear model based on the cusp catastrophe manifold. Exhibit 1-5 describes the conditions of four different sets of indicators with their associated level of confidence values. These four different data sets are considered to have generated four distinct assessments of the likely formation of an OMG and this assessment is assumed to have been recorded in the OMG Threat Assessment Data Base and statistically analyzed (Exhibits 1-5 and 1-6). The number of active indicators and level of confidence parameters can form the axes of the control space associated with the statistical catastrophe model (Exhibit 1-6).

Exhibit 1-4

Analyst OMG Threat Assessment and Data Base Formation Activities

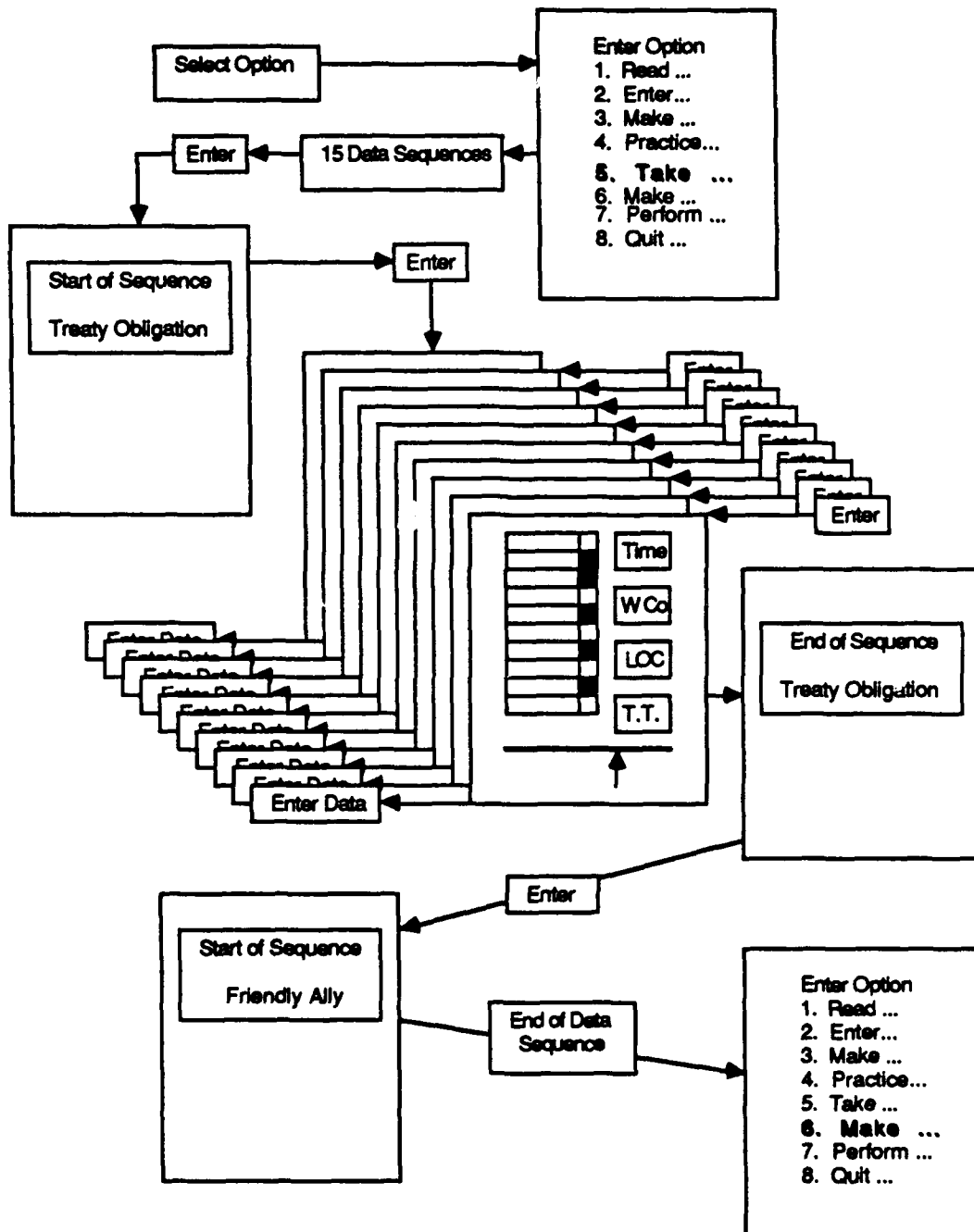


Exhibit 1-5

Relationship of the OMG Threat Assessment Data Base to the Cusp Catastrophe Manifold Control Plane

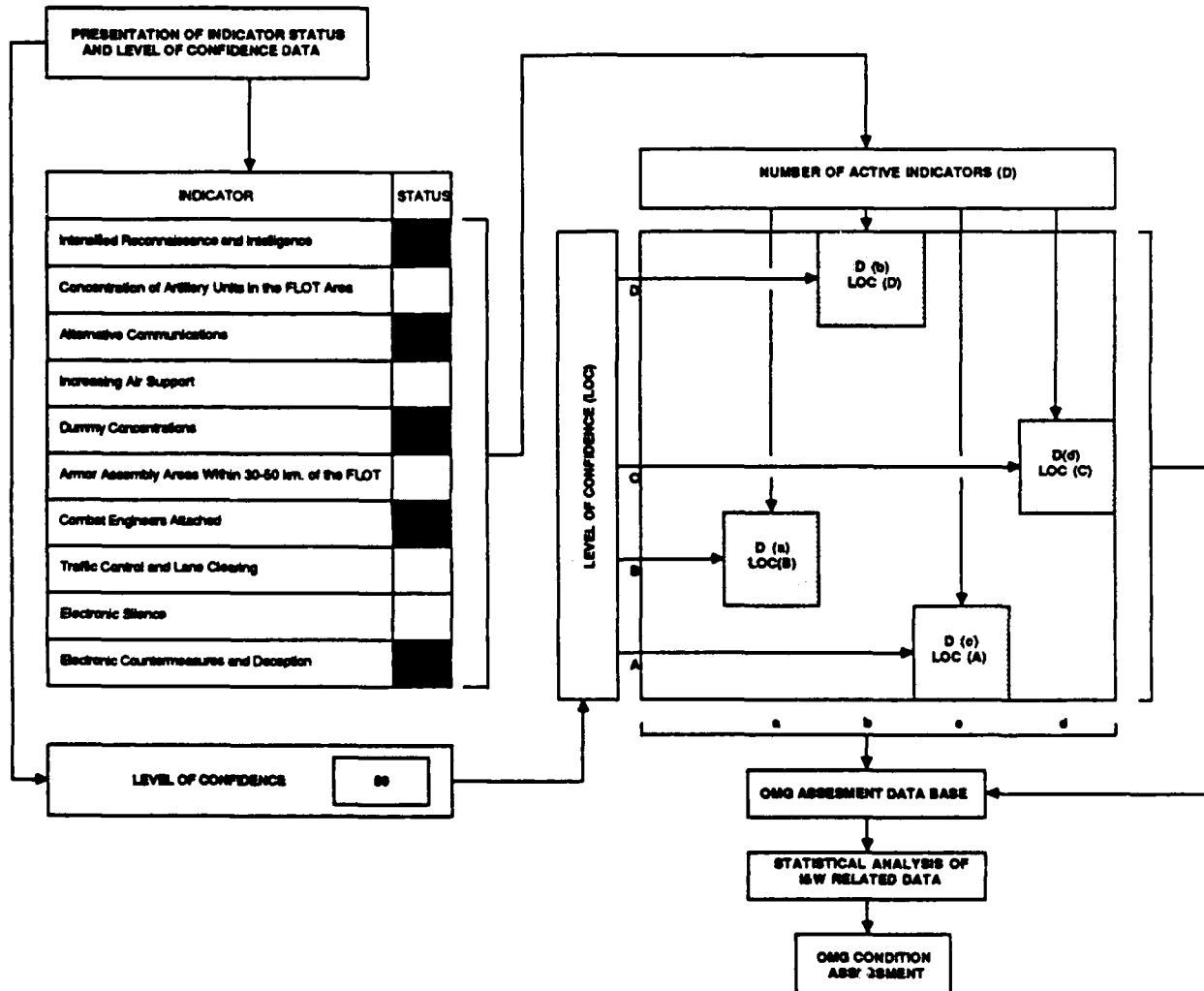
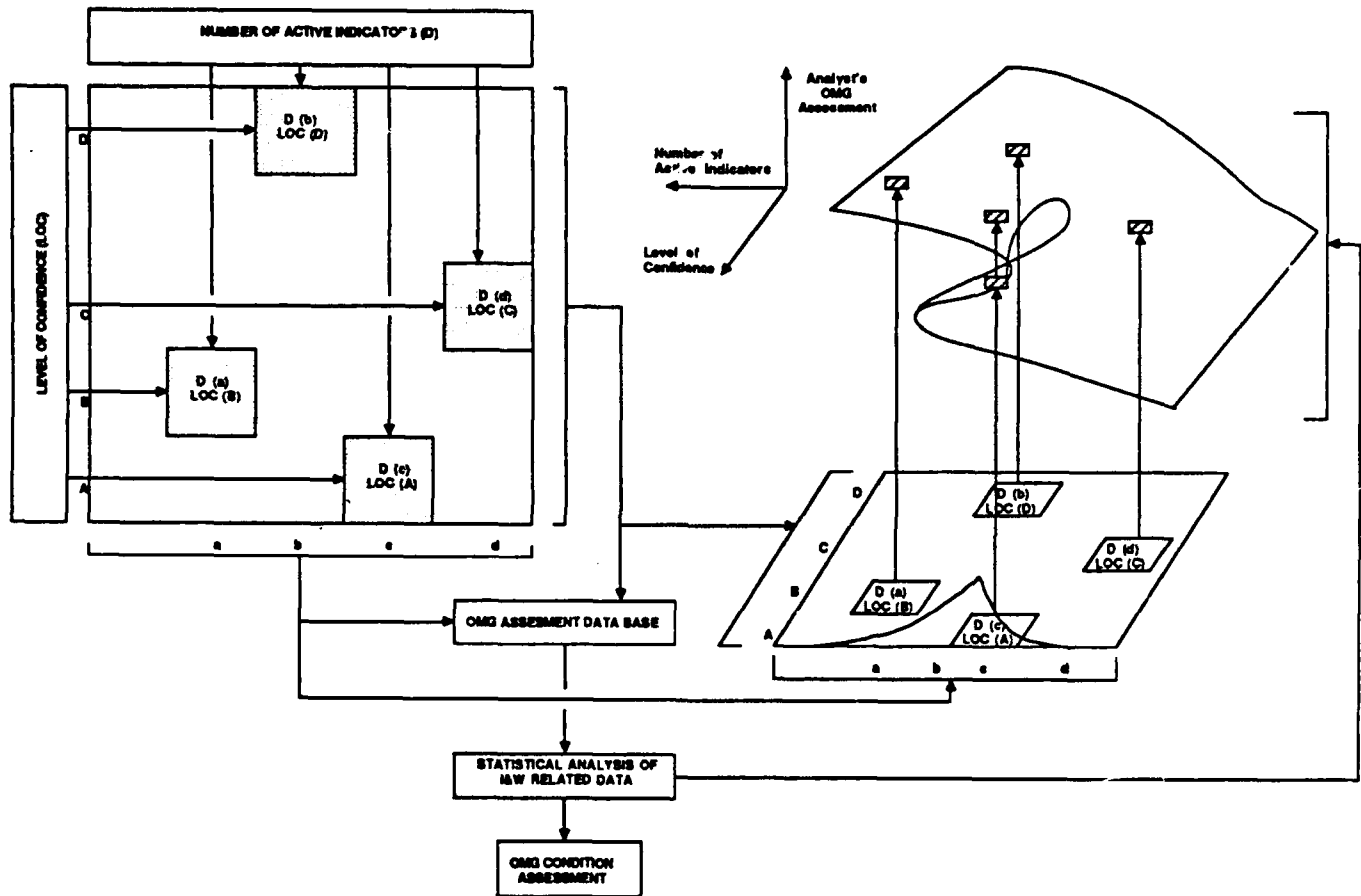


Exhibit 1-6

Fitting OMG Threat Assessment Data Base to the Cusp Manifold Surface



1.5 CUSP SURFACE ANALYSIS

The analysis of the OMG Threat Assessment Data Base in the IWCAT system is performed with the aid of a program based on statistical catastrophe theory. A catastrophe manifold or "cusp surface" is a statistical model derived from catastrophe theory with one dependent variable and an arbitrary number of independent variables. The cusp catastrophe model is a response surface that contains a smooth pleat in which the original control variables and the original behavioral variable have been transformed by a mathematical process which adjusts the coordinate system so that the shape of the original response surface matches that of the cusp surface near the cusp catastrophe point, which serves as the origin of the pleat.

Thus, the canonical behavior variable for the cusp surface model is a function of both the original behavior variable and the original control variables, while the canonical control variables (a) and (b) are each functions of all of the original control variables. This is the principal difference between the statistical model and the topological model.

1.5.1 ESTIMATING PARAMETERS

The cusp surface analysis procedures of the IWCAT computer system use the method of maximum likelihood to estimate the parameters of the cusp model. The conditional probability density function (PDF) for the behavioral variable (which Woodcock has also called the property distribution function) has either one mode or two modes separated by an antimode. Therefore the predictions made by the cusp model are the modal values of the conditional probability density function. The antimode is an "*antiprediction*" – a value that is specifically identified as that which is "not likely to be seen."

The differences and similarities between linear regression and cusp surface analysis are worth careful examination. The conditional PDF of a function in linear regression is a normal or Gaussian shaped curve generated from a function that is the exponential of a quadratic. By contrast, the conditional PDF of the function of cusp surface analysis is a bimodal curve and is generated from a function that is the exponential of a quartic. The predicted values in linear regression are the means of the conditional densities, which also happen to be modes, while in cusp surface analysis, the predicted values are modes and the densities are frequently bimodal, yielding two, ambiguous, predictions of system behavior. Lastly, in linear regression the formulas for the sampling variance of the estimators are known exactly, while in cusp surface analysis the corresponding formulas are approximations.

The cusp surface analysis program begins with the estimated coefficients of the linear regression model, and iterates towards the parameter vector that maximizes the likelihood of the cusp model given the observed data. The iterative scheme is a modified Newton-Raphson method. If the very first iteration yields a decrease in the likelihood function, the program immediately halts with a message indicating that the linear model is preferable to any cusp model (this is not a rare occurrence).

1.5.2 MAKING PREDICTIONS

The parameter estimates reported by the cusp surface analysis program are useful for generating predictions, but their values indicate nothing about their statistical significance. Therefore the program also reports an approximate t-statistic for each coefficient, with a given

number of degrees of freedom. These can be interpreted in the usual fashion: magnitudes in excess of the critical value indicate that the coefficient is significantly different from zero at the specified significance level, however, these t-statistics are only approximate. Of course, these statistics can also be misinterpreted in the usual ways. For example, it is a mistake to pay attention to any of these values unless the overall model has passed all of its tests for acceptability.

There is no single definitive statistical test for the acceptability of a catastrophe model. Part of the difficulty stems from the fact that a catastrophe model generally offers more than one predicted value for a behavior, or dependent, variable given a set of control, or independent, variables. This makes it difficult to find a tractable definition for prediction error, without which all goodness-of-fit measure that are based on the concept of prediction error (e.g., mean squared error) are nearly useless. Another part of the difficulty arises from the fact that the statistical model is not linear in its parameters. And finally, of course, it is scientifically unsound to base any definitive statement on the analysis of a single data set, no matter what its statistics show. Confirmation must always be sought in the independent replication of results. In spite of these difficulties and caveats however, there are a variety of ways in which a catastrophe model may be tested through statistical means.

Cusp surface analysis offers three separate tests to assist the user in evaluating the overall acceptability of the cusp catastrophe model (Exhibits 1-7, 1-8, and 1-9). The first test is based on a comparison of the likelihood of the cusp model with the likelihood of the linear model. The test statistic is an "asymptotic chi-square," which means that as the sample size increases the distribution of the test statistic converges to the chi-square distribution. The degrees of freedom for this chi-square statistic is the difference in the degrees of freedom for the two models being compared. Sufficiently large values of this statistic indicate that the cusp model has a significantly greater likelihood of being "correct" than has the linear model.

The cusp catastrophe model may be said to describe the relationship between a dependent variable and vector of independent variables if all of these three conditions hold:

1. The chi-square test shows that the likelihood of the cusp model is significantly higher than that of the linear model.
2. The coefficient for the cubic term and at least one of the coefficients of the factors A and B are significantly different from zero.
3. At least 10% of the data points in the estimated model fall in the bimodal zone.

In the literature on applications of catastrophe theory there are two distinct ways of calculating predicted values from a catastrophe model. In the Maxwell Convention the predicted value is the most likely value, i.e., the position of the highest mode of the probability density function. In the Delay Convention a mode is also the predicted value, but the mode occupied by the system is not necessarily the highest-valued mode. Instead, the predicted value is the mode that is located on the same side of the antinode as the observed value of the state variable. Thus the delay convention uses as its predicted value the equilibrium point towards which the equivalent dynamical system would have "moved." The delay convention is most commonly adopted in applications of catastrophe theory, but there are circumstances in which the Maxwell convention is more appropriate.

Exhibit 1-7

A Conventional or a Catastrophe Theory-Based Approach?

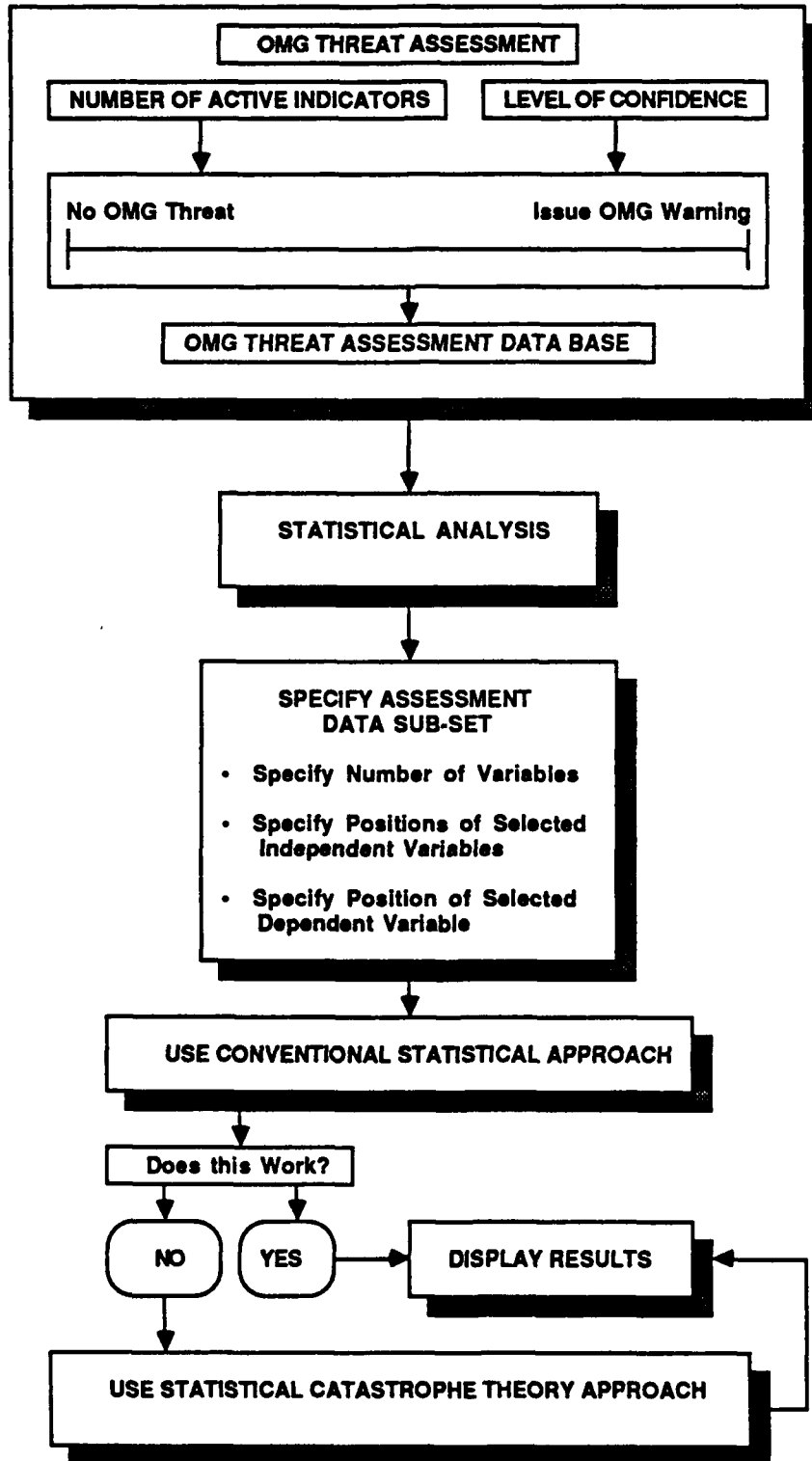


Exhibit 1-8

Criteria for Acceptance of the Cusp Catastrophe-Based Model

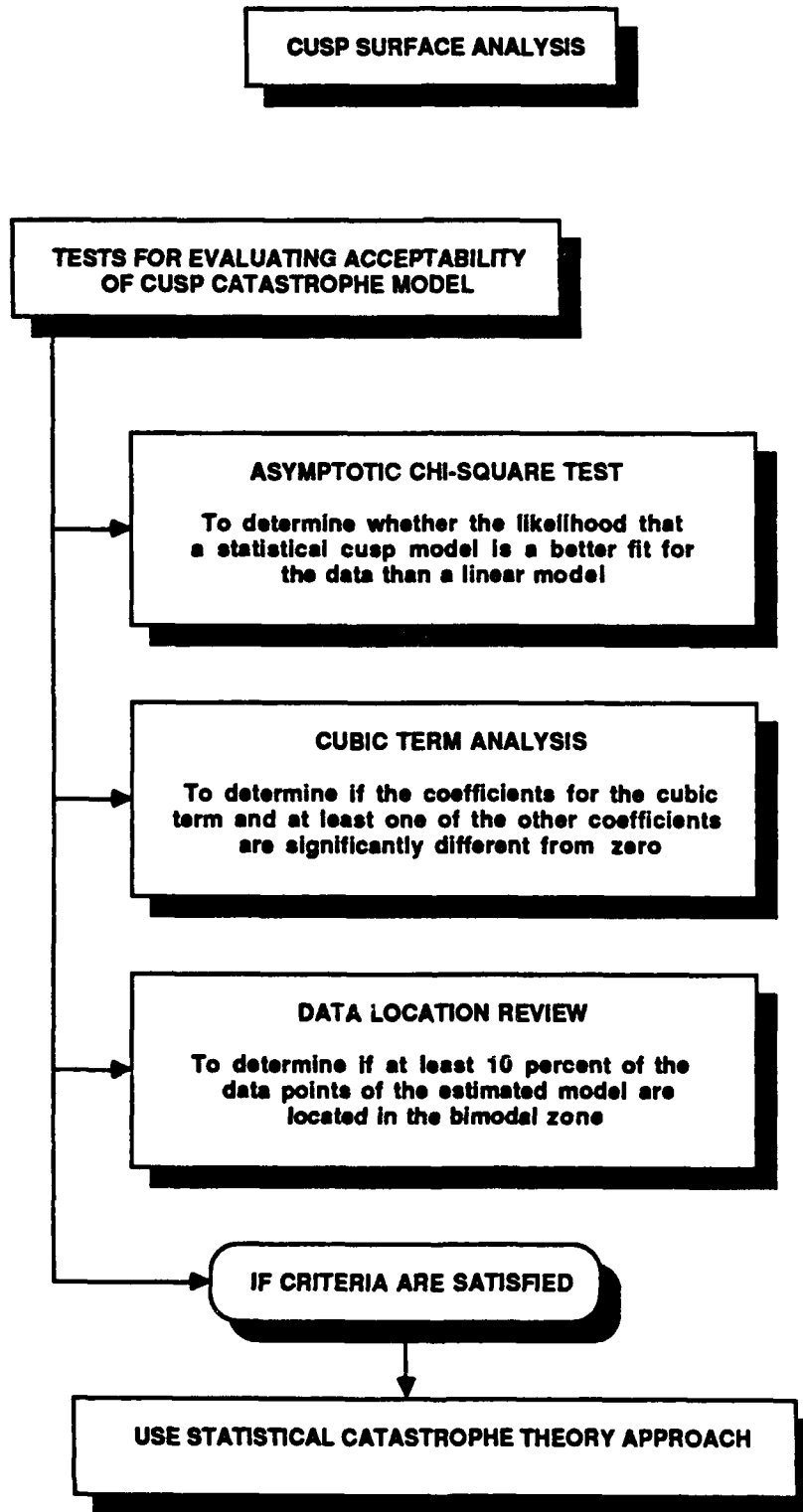
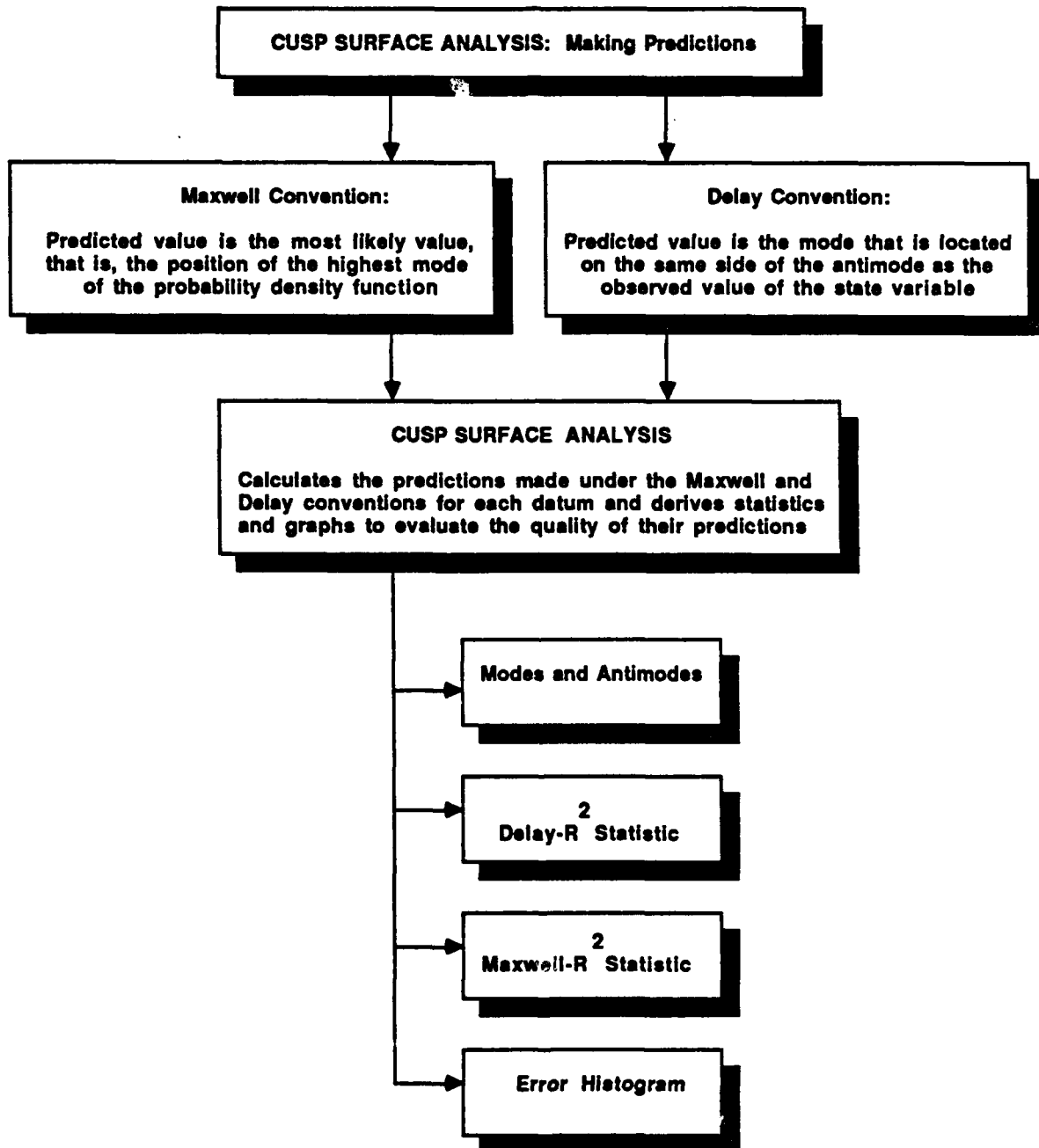


Exhibit 1-9

Cusp Surface Analysis: Making Predictions



1.6 OMG THREAT ASSESSMENT ANALYSES

The IWCAT system software was used in a series of tests during which individuals with experience in the indications and warning and intelligence analysis areas were asked to assess the perceived level of Operational Maneuver Group (OMG) threat associated with a series of sets of OMG-related indicators (Exhibit 1-10). The IWCAT system permits the creation of an OMG threat assessment data base and its subsequent analysis with a nonlinear statistical program based on statistical catastrophe theory in order to construct a mathematical model of the data which could be used as the basis for further analysis of the responses of I&W analysts to situations of interest.

When a nonlinear model can be constructed from the analysts threat assessment data, it is possible to describe a range of different I&W analyst response behaviors such as sudden and gradual perceptual changes, divergence, ambiguity, hysteresis, perceptual "trapping," and counter-intuitive or paradoxical behavior. One particularly interesting discovery provides statistical evidence which suggests that I&W analysts with different types of training and previous mission responsibilities appear to respond to different features of the OMG threat assessment data set.

1.6.1 MAPPING DATA TO THE CATASTROPHE MODEL SURFACE

The IWCAT system uses the method of maximum likelihood to estimate the parameters of a model based on the observed data and, when a cusp-based model can be constructed, performs statistical tests to determine whether a linear or the cusp-based model provide a better description of the data. A cusp-based model of the data is accepted when the chi-square test shows the likelihood of the cusp model to be significantly higher than that of the linear model; the coefficient of the cubic term and one of the other coefficients of the cusp model are significantly different from zero; and at least 10% of the data points in the estimated model fall in the bimodal region (see Section 4., for example).

In the process of constructing the cusp model, the system transforms the input data to fit a cusp surface. This surface is an inherently three-dimensional object which can be drawn as a structure with three axes, each of which represents a component of the cusp model. Two of these axes represent the control factors or input variables and the third axis represents the behavior or output variable of the system of interest and positions on the surface can be located with respect to the values of these three axes. The two control factors, which may themselves be a function of other variables, define a plane called the control plane.

The IWCAT system provides the user with a series of diagrams which display the features of the cusp model. In one display (see Exhibit 1-11, for example), the transformed data are presented as locations on the control plane formed from (transformed) versions of the control factors called the bifurcation (or splitting) and asymmetry (or normal) factors. Based on this analysis, it is possible to construct a cusp catastrophe model that is the best available "fit" for the I&W analyst-derived data (see Exhibit 1-12, for example). The (transformed) actual data is located within the circle drawn on the control plane formed from transformations of the number of active indicators and level of confidence control factors. In this particular case, some of these data lie inside, and the remainder lie outside the region of bimodality or ambiguity on the control plane. A linear model of the data is highly appropriate when all the data points lie outside the region of ambiguity.

Exhibit 1-10

IWCAT System Activities

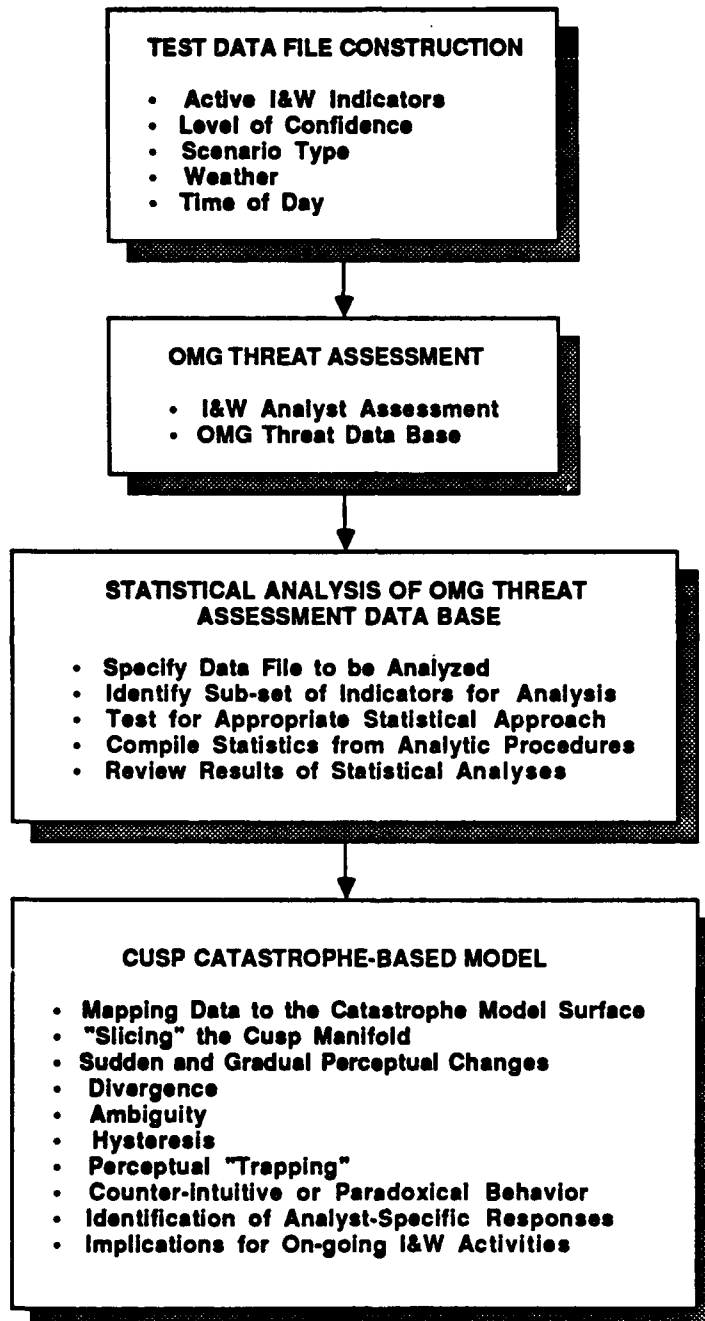
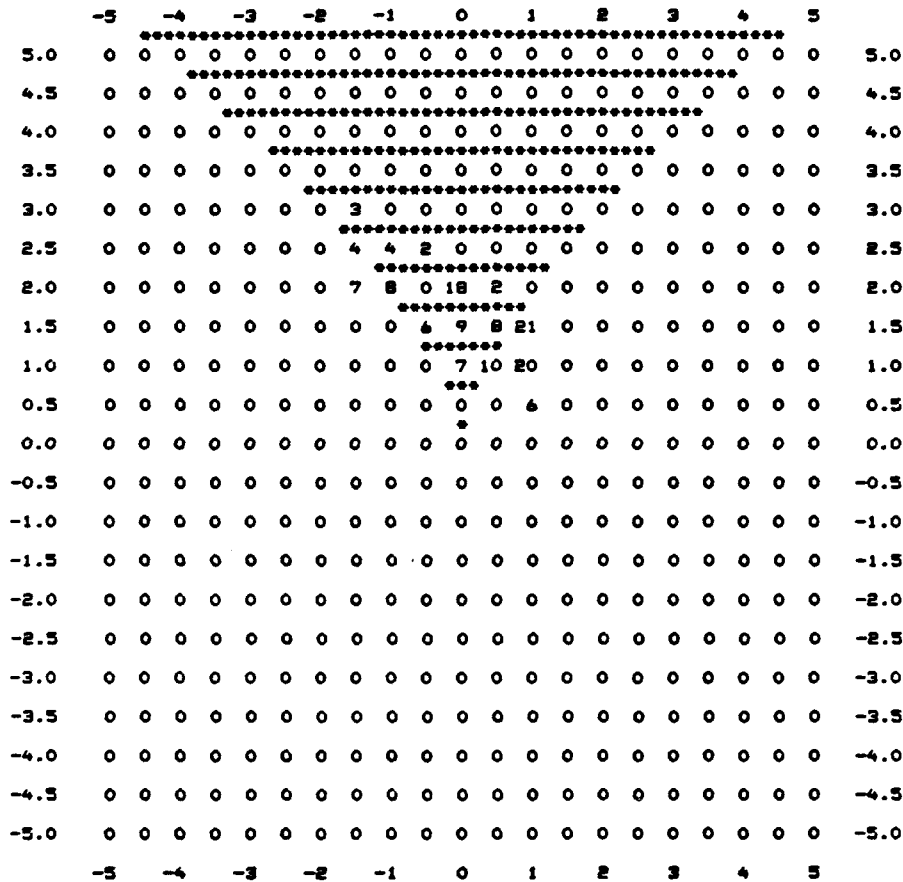


Exhibit 1-11

I&W Analyst-Derived Data Plotted on the Control Plane of the Cusp Model

Location of data in the control space:

Vertical axis: Bifurcation (splitting) factor
Horizontal axis: Asymmetry (normal) factor
Asterisks: Bimodal zone



0 cases did not fit in the above figure.

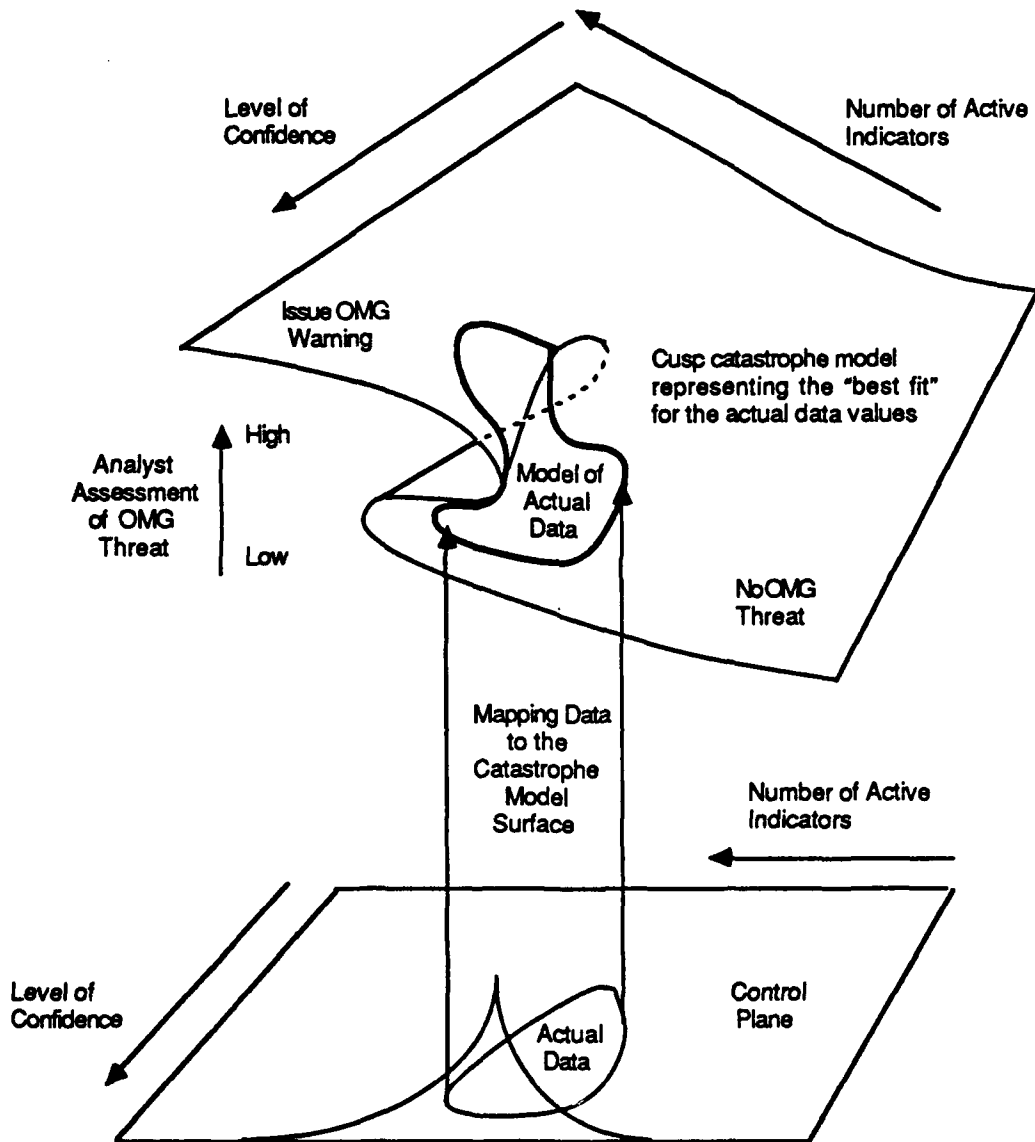
Linear $R^2 = 0.285$ (Multiple regression)
Delay $R^2 = 0.472$ (Attracting-mode convention)
Maxwell $R^2 = -0.299$ (Most-likely-mode convention)

* Negative R^2 values occur when the cusp model is worse than a constant.

>>>>> Fraction of cases in bimodal zone: 0.415 <<<<<

Exhibit 1-12

Mapping Data to the Cusp Model Surface



1. Sudden and Gradual Perceptual Changes. The cusp model constructed from I&W analyst-derived data describes conditions under which sudden and gradual changes in analyst perceptions can take place (see Exhibit 1-13, for example).
2. Divergence. The cusp model can also illustrate the property of perceptual divergence, as shown in Exhibit 1-14) where relatively small differences in the initial level of detail can have a profound impact on the nature of the analysts OMG threat assessment.
3. Ambiguity. Preconditioning can lead to perceptual ambiguity, a phenomena which is illustrated with the aid of the cusp surface model shown in Exhibit 1-15.
4. "Slicing" the Cusp Surface. The IWCAT system provides the user with a series of diagrams representing "slices" of the cusp model surface in which all but one of the control factors are held fixed at their mean values and the effect of changes in the remaining factor on the shape of the surface is displayed (see Exhibits 1-16a and 1-16b, for example).
5. Perceptual Hysteresis. The phenomena of "perceptual hysteresis," may be illustrated with the aid of the cusp surface model (Exhibit 1-17).
6. Perceptual "Trapping." Use of the IWCAT system by a series of analysts has led to the characterization of a phenomenon which Woodcock has called "perceptual trapping." Exhibit 1-18 illustrates the phenomena of partial perceptual trapping while Exhibit 1-19 illustrates complete perceptual trapping.
7. Counter-Intuitive or Paradoxical Behavior. Cusp surface models based on analyst's perceptions of OMG threat suggest that the analyst's perceptions may exhibit patterns of counter-intuitive or paradoxical behavior, as shown in Exhibit 1-20.

1.6.2 SPECIFIC ANALYST ASSESSMENTS

As mentioned above, several members of Synectics staff who have been involved in various forms of I&W and intelligence analysis activity participated in testing the IWCAT system. The following constitutes a summary discussion of the results of these different tests and a detailed analysis of these results can serve as a starting point for further research investigations and for the development of an operational IWCAT facility.

In each case the analyst was presented with a test data set of indicators and other information described in Section 5, and asked to assess the level of OMG threat that they appear to reflected to the analyst. Following this task, the analyst was asked to designate which of the indicators were of primary importance and which were of secondary importance in determining the level of perceived OMG threat. Information generated by this process was then subjected to analysis using the cusp analysis program.

It is a suggestive finding of this statistical analyses performed during this investigation that the nature of the response of the different analysts to the OMG threat test data appeared to depend upon their background and experience. Thus analysts with extensive active duty military experience (Analysts B and C below) appeared to pay almost exclusive attention to the number of active indicators; another analyst (Analyst D below) with much more national level intelligence analytic experience appeared to pay almost exclusive attention to the patterns (or sequence type) of the displayed indicators. Analyst A, with extensive military experience

Exhibit 1-13

Cusp Model of Sudden and Gradual Changes in Analyst Perceptions

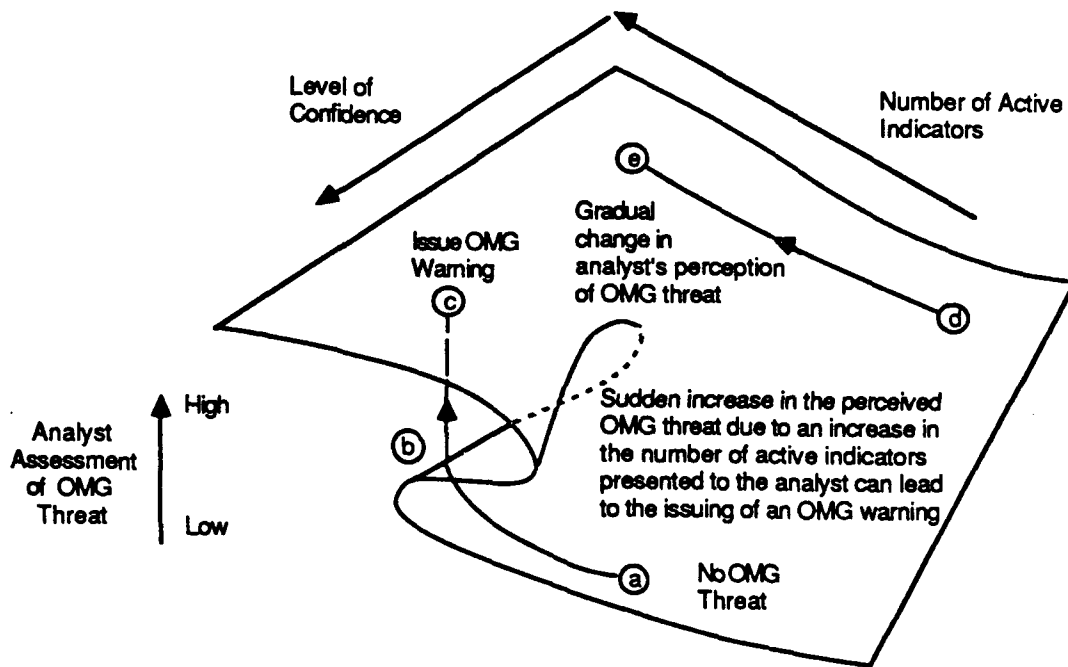


Exhibit 1-14

Cusp Model of Divergent Perceptions

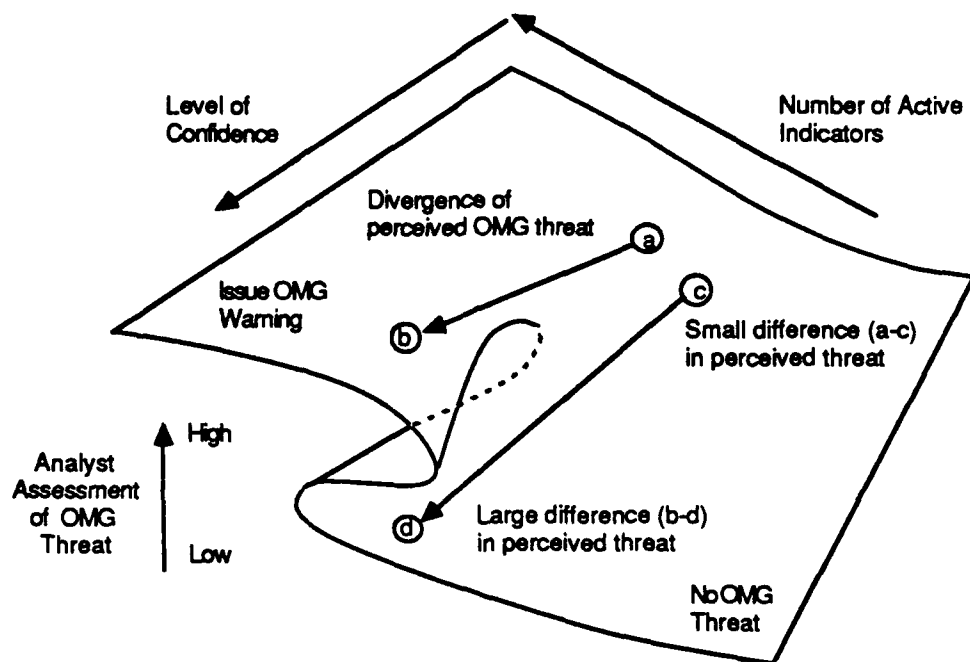


Exhibit 1-15

Cusp Model Can Provide a New Understanding of the Causes of Perceptual Ambiguity

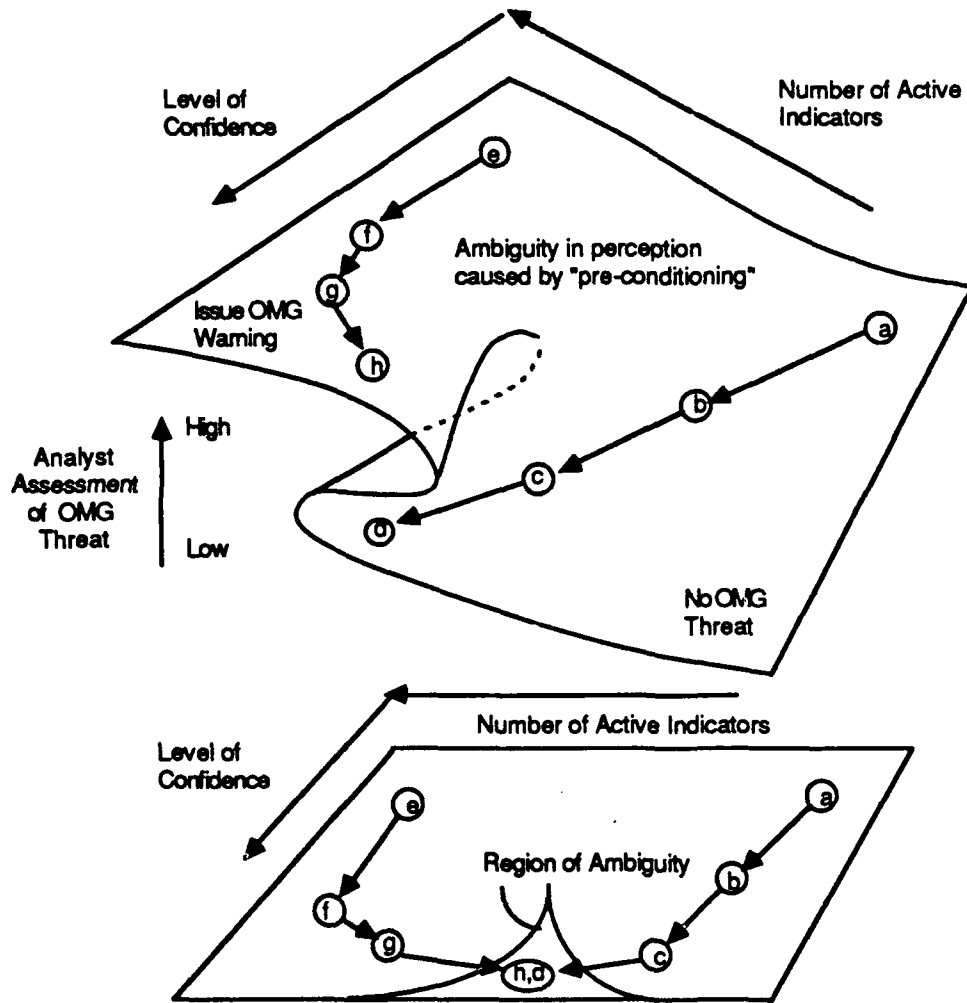


Exhibit 1-16

"Slicing" the Cusp Surface

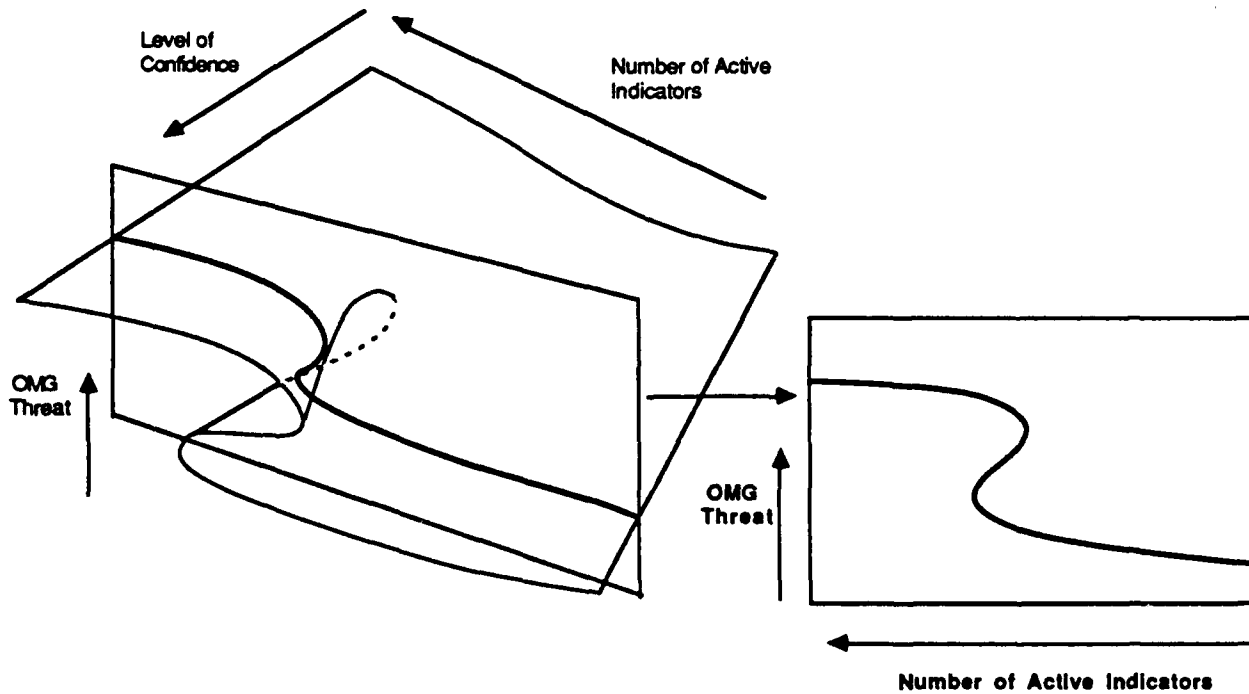
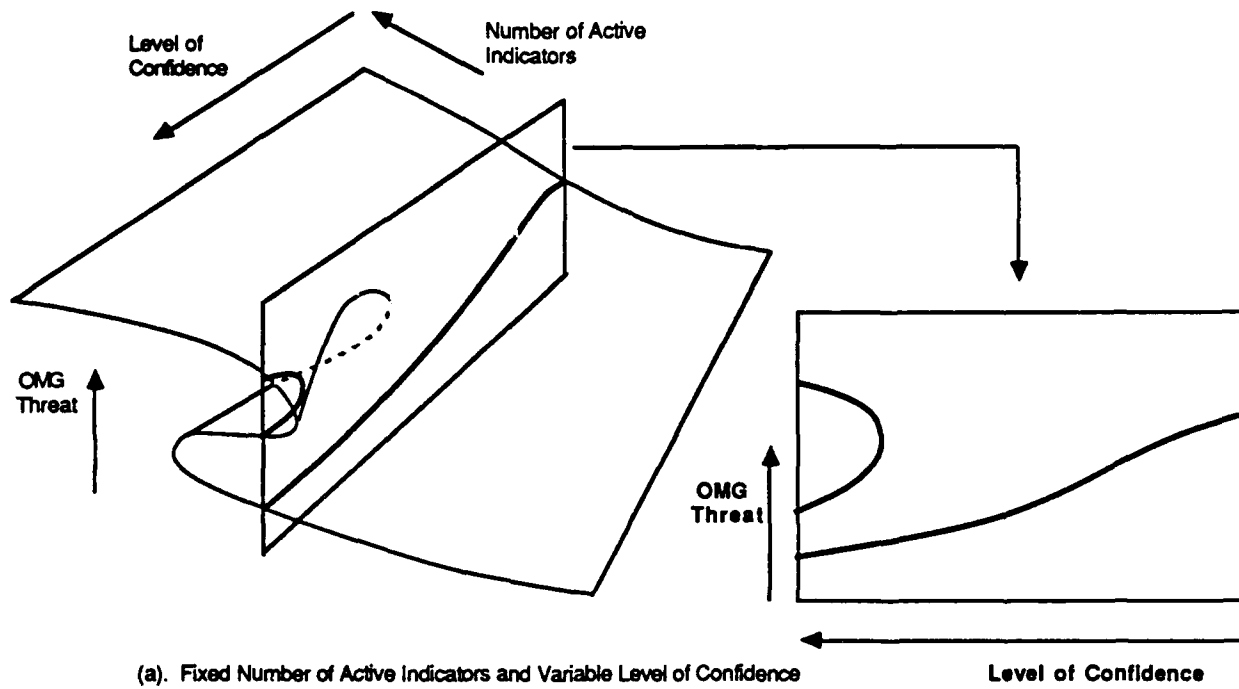


Exhibit 1-17

Perceptual Hysteresis

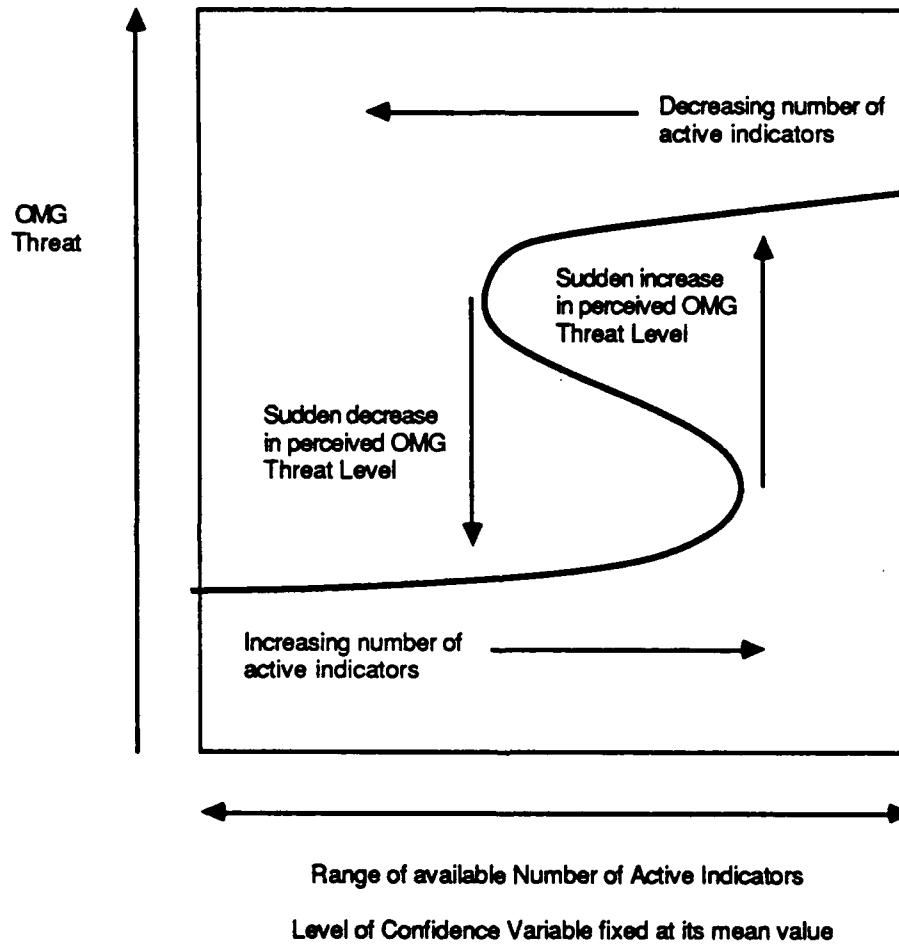


Exhibit 1-18
Partial Perceptual "Trapping"

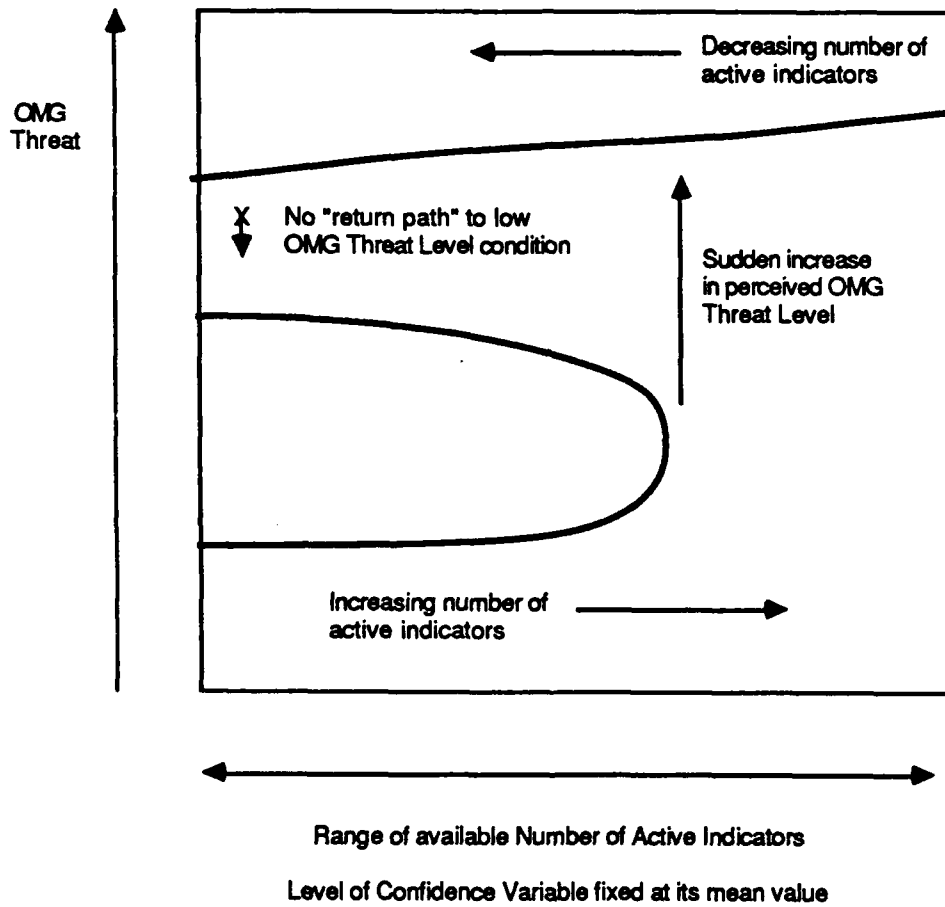


Exhibit 1-19
Complete Perceptual "Trapping"

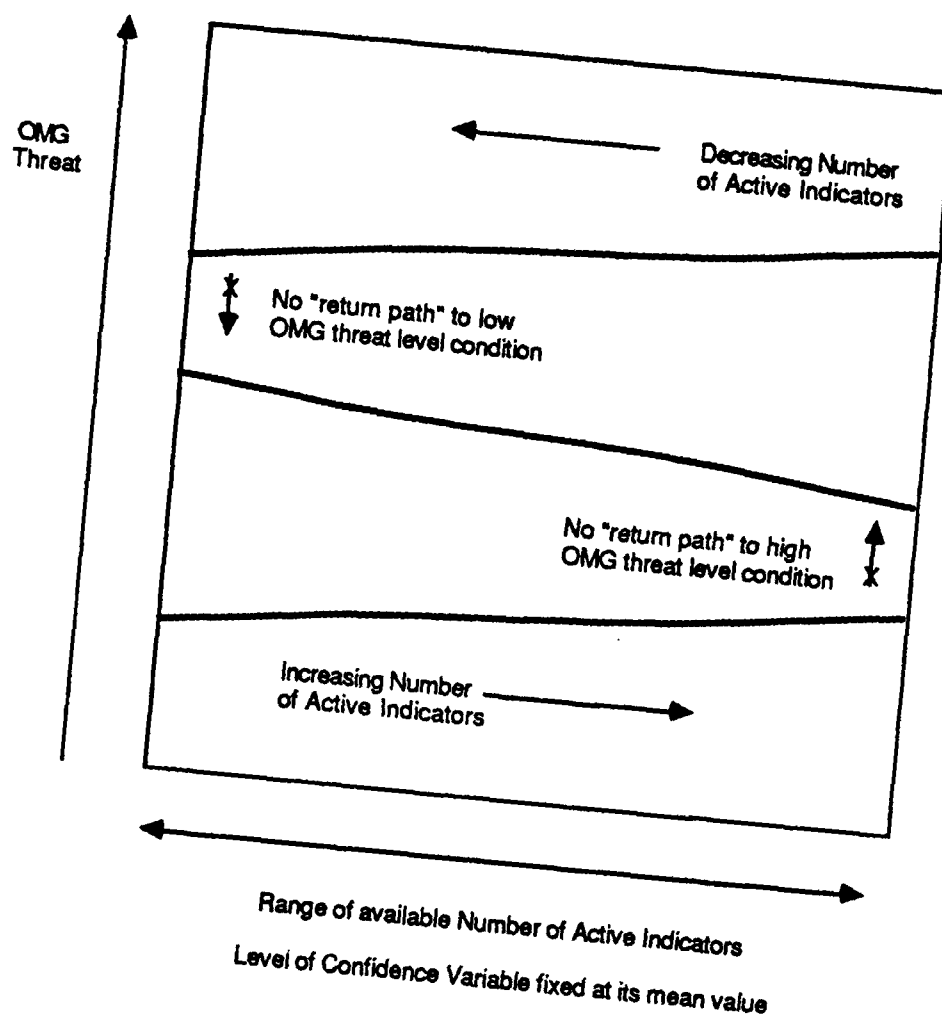
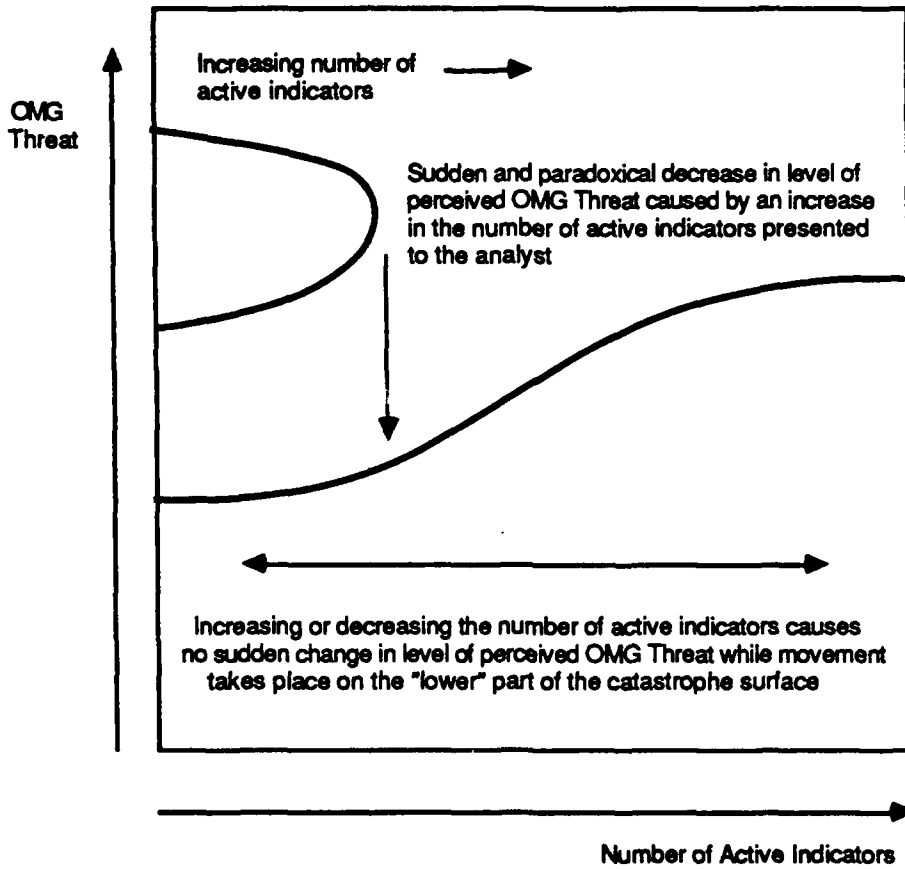


Exhibit 1-20

Counter-Intuitive or Paradoxical Behavior



Level of Confidence fixed at its mean value

and involvement in more national level intelligence analytic activity appeared to pay attention to both number of active indicators and their pattern. Analyst E, with national level weapons targeting experience, performed the test. However, the data collected in this process appeared to form a linear model since the cusp analysis program terminated its activities because no cubic term was detected.

While the observation that the nature of analyst perceptions of OMG threat is predicated by the nature of the experience and training of the analysts, is a tentative finding due to the small sample size of analysts that were used in the experiment, such a suggestion can have profound implications on the way that I&W and other forms of intelligence analyses are performed. Results of the IWCAT project suggest that analysts (who could be referred to as "front-line" analysts) closely associated with the more immediate or tactical aspects of the combat environment concentrate on the number of active indicators while those analysts (who could be referred to as "headquarters" analysts) who are involved in the analysis of the more strategic aspects of combat, and who may receive most of their intelligence input from the front-line analysts, appear to pay more attention to detecting a pattern in the indicator series.

If substantiated by further work and analysis, such a finding can have an important impact on the relationships between these front-line and headquarters analysts since the first type of analyst acts as a perceptual filter for the information that is presented to the second type of analyst. The fact that these different types of analysts concentrate on different aspects of the available intelligence information (such as the number of active indicators or the pattern of indicators, for example) could introduce unexpected and unintentional biases in the interpretation of this information and lead to a misunderstanding of the nature of particular combat situations. The possibilities of such perceptual disconnects and their impact on I&W and command and control (C²) should be of concern to I&W analysts and others and further investigations of this suggestion with the IWCAT system would appear to be appropriate.

1.6.2.1 The Perceptions of "Analyst A"

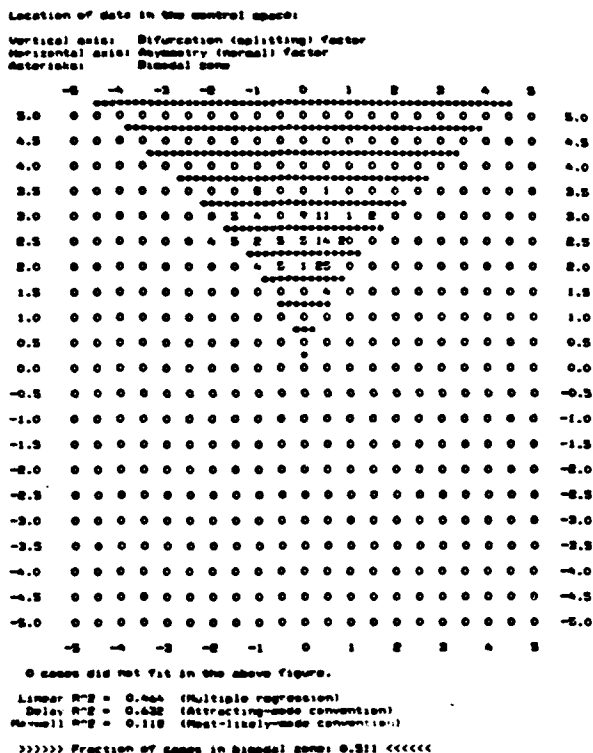
The following is a brief review of the analysis of the I&W OMG threat data collected from one of the five analysts (designated here as "Analyst A"), who are members of Synectics staff and who have all had experience as intelligence analysts. More detailed information concerning the results of this analysis are presented in Section 6.

1. Impact of the number of active primary indicators and level of confidence on OMG threat perception. Exhibit 1-21 presents the analysis of the effect of the number of active primary indicators and level of confidence on OMG threat perception. The control plane plot (Exhibit 1-21a) shows that 51.1% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Slices of the catastrophe model surface for a range of numbers of active primary indicators and with the level of confidence fixed at its mean value reveal situations in which partial perceptual trapping can occur (Exhibit 1-21b). Slices of this surface for a range of values of the level of confidence variable and with the number of active primary indicators fixed at their mean value reveals situations in which perceptual hysteresis can occur (Exhibit 1-21c) as the level of confidence is increased or decreased.
2. Impact of the number of active secondary indicators and level of confidence on OMG threat perception. Exhibit 1-22 presents the analysis of the effect of number of secondary indicators and level of confidence on OMG threat perception. The control

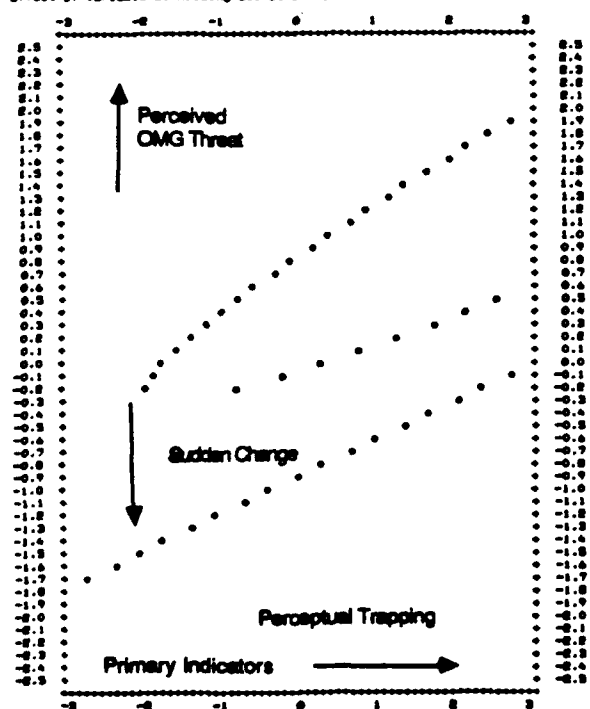
Exhibit 1-21

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot



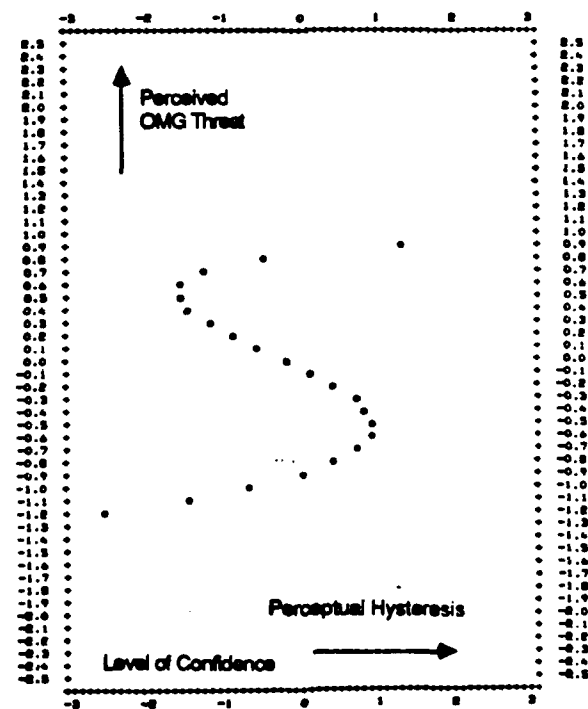
Effect of variable 2, holding all others constant at their mean values.



• Mode symbol
• Attracted symbol

(b) Variable Primary Indicators, Fixed Level of Confidence

Effect of variable 10, holding all others constant at their mean values.



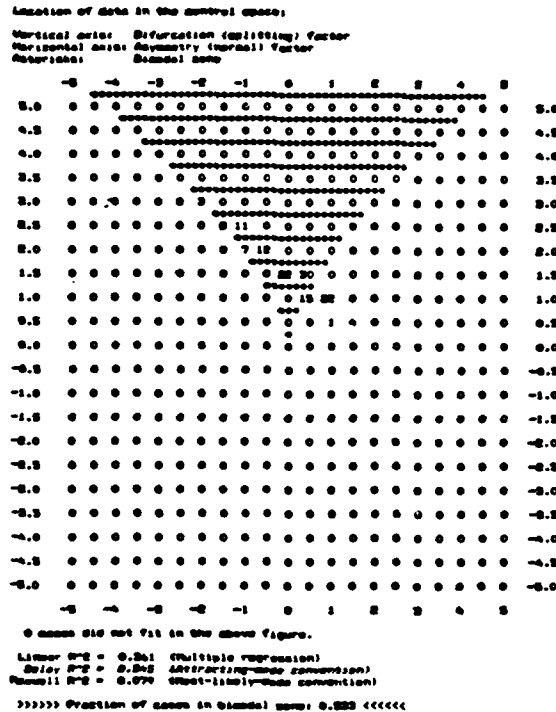
• Mode symbol
• Attracted symbol

(c) Variable Level of Confidence, Fixed Primary Indicators

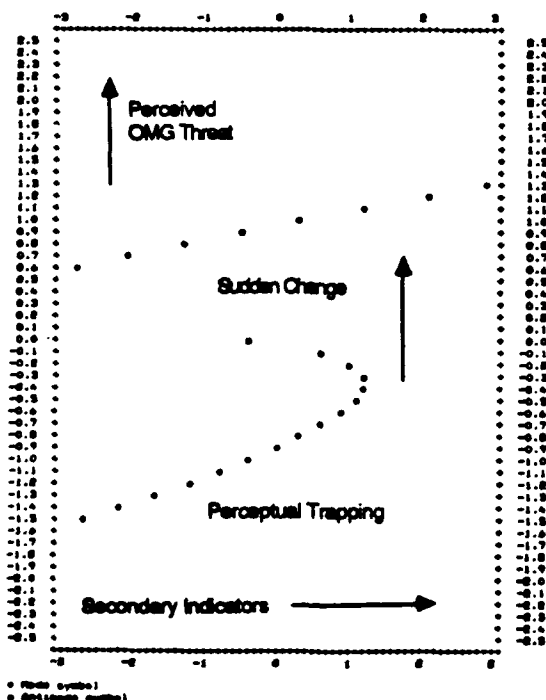
Exhibit 1-22

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot

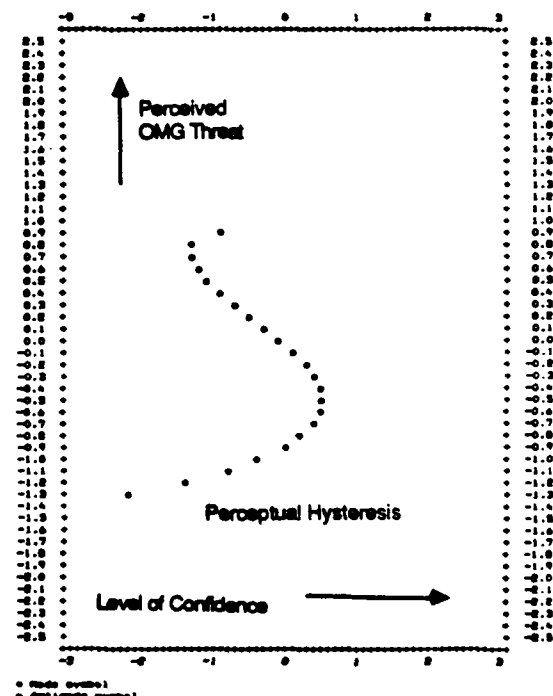


Effect of variable 2, holding all others constant at their mean values.



(b) Variable Secondary Indicators, Fixed Level of Confidence

Effect of variable 2, holding all others constant at their mean values.



(c) Variable Level of Confidence, Fixed Secondary Indicators

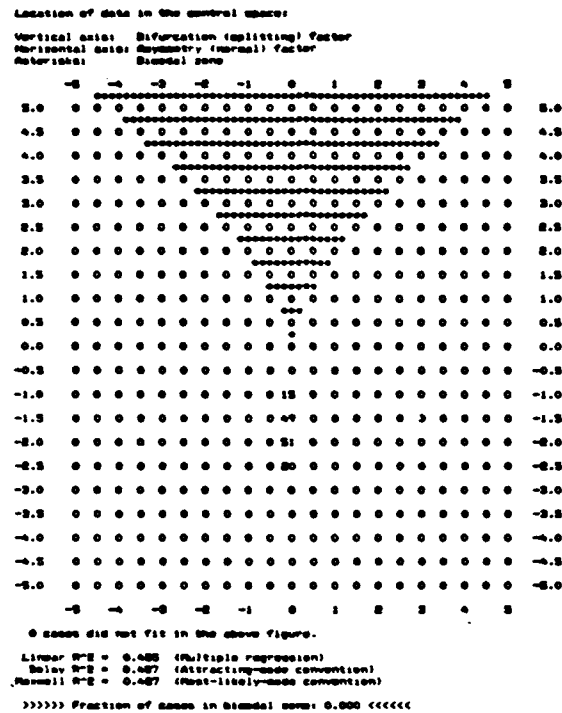
plane plot (Exhibit 1-22a) shows that 53.3% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. A slice of the cusp surface at a fixed level of confidence and variable number of active secondary indicators reveals situations in which perceptual trapping can occur. Increasing the number of these active indicators from a low to a higher level causes a gradual increase in perceived OMG threat to a condition at which a sudden change in these levels can take place (Exhibit 1-22b). A slice of the cusp model surface for a range of values of the level of confidence and with the number of active secondary indicators fixed at their mean value reveal situations in which perceptual hysteresis can occur (Exhibit 1-22c) as the level of confidence is increased or decreased.

3. Impact of number of combined active primary and secondary indicators and level of confidence on OMG threat perception. Exhibit 1-23 presents the analysis of the effect of the activity number of all indicators and level of confidence on OMG threat perception. The control plane plot (Exhibit 1-23a) shows that none of the data are located within the bimodal zone. This finding is also confirmed by the data presented in Exhibits 1-23b and 1-23c. The comparison of the material presented in Exhibits 1-21, 1-22 and 1-23 is revealing. While analysis of the data set in which the primary or secondary active indicators provides graphs which suggest that sudden changes in perception can take place, analyzing the effect of these indicators as a whole reveals a linear response characteristic. Under such circumstances, the combination of the responses generated by the simultaneous consideration of the effect all the indicators can be described with the aid of a simple linear model. This demonstrates a major difficulty that might arise when data of different levels of importance to an analyst are combined for statistical or other purposes. Such observations should be a matter for further consideration.
4. Impact of sequence type and level of confidence on OMG threat perception. Exhibit 1-24 presents the analysis of the effect of sequence type and level of confidence on OMG threat perception. The control plane plot (Exhibit 1-24a) shows that 41.5% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. A slice of the cusp surface at a fixed level of confidence and variable sequence type reveals situations in which perceptual trapping can occur with the analyst's OMG threat perceptions restricted either to a high value range or a low value range (Exhibit 1-24b). A slice of the cusp model surface for a range of values of the level of confidence and with the sequence type fixed at their mean value reveal situations in which perceptual hysteresis can occur (Exhibit 1-24c) as the level of confidence is increased or decreased.
5. Impact of number of active primary indicators, sequence type, and level of confidence on OMG threat perception. Exhibit 1-25 presents the analysis of the effect of number of active primary indicators, sequence type, and level of confidence on OMG threat perception. The control plane plot (Exhibit 1-25a) shows that 54.8% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. A slice of the cusp model surface for a range of values of number of active primary indicators and with the sequence type and level of confidence variables fixed at their mean values reveals situations in which partial perceptual trapping can occur (Exhibit 1-25b). A slice of the cusp surface at a fixed level of confidence and number of active primary indicators and variable sequence type reveals situations in which perceptual trapping can occur with the analyst's OMG threat perceptions restricted either to a high value range or a low value range for all ranges of sequence type values (Exhibit 1-25c). A slice of the cusp model surface for a range of values of the level of confidence and with the numbers of active primary

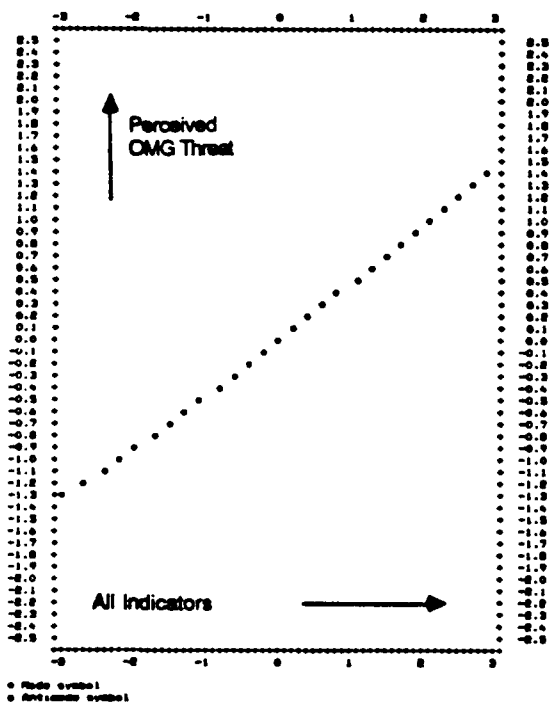
Exhibit 1-23

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot

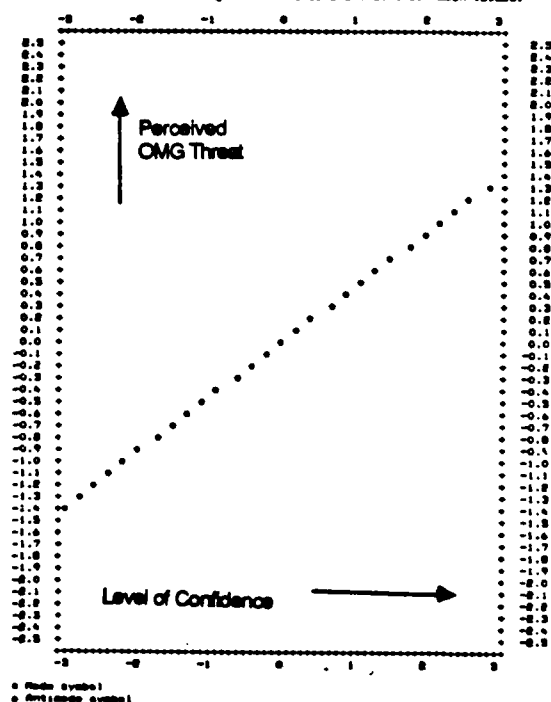


Effect of variable 4, holding all others constant at their mean values.



(b) Variable All Indicators, Fixed Level of Confidence

Effect of variable 10, holding all others constant at their mean values.

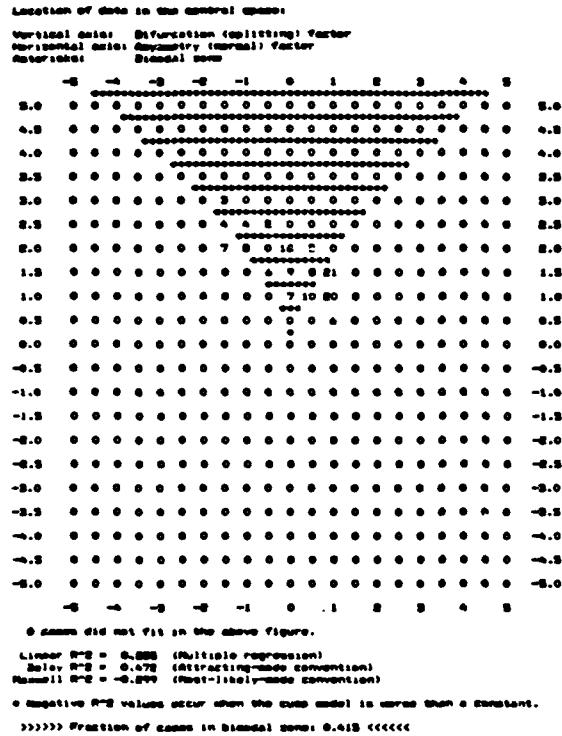


(c) Variable Level of Confidence, Fixed All Indicators

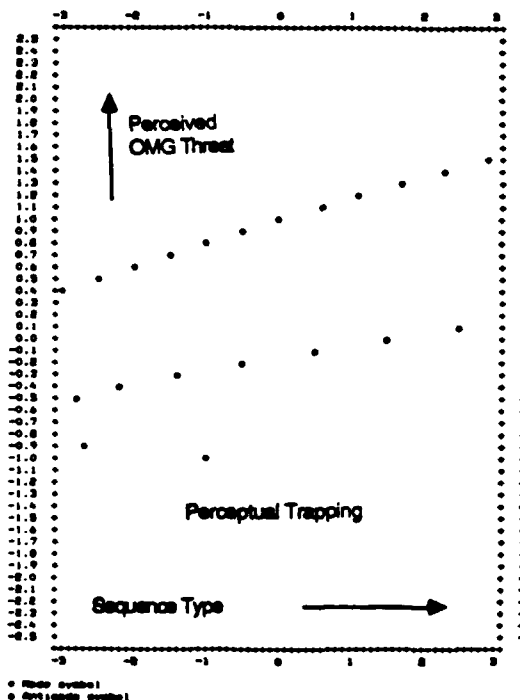
Exhibit 1-24

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot

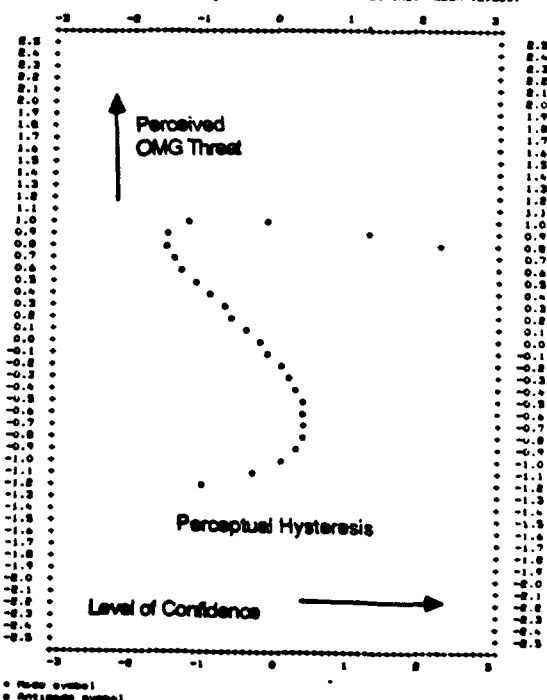


Effect of variable 8, holding all others constant at their mean values.



(b) Variable Sequence Type, Fixed Level of Confidence

Effect of variable 10, holding all others constant at their mean values.

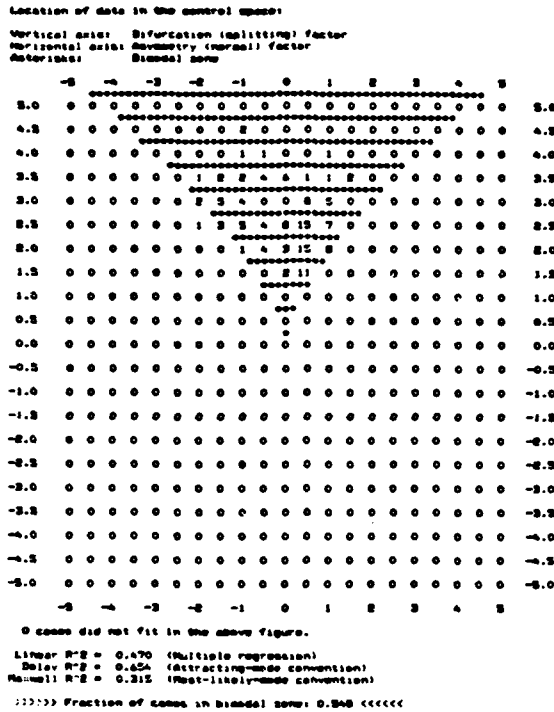


(c) Variable Level of Confidence, Fixed Sequence Type

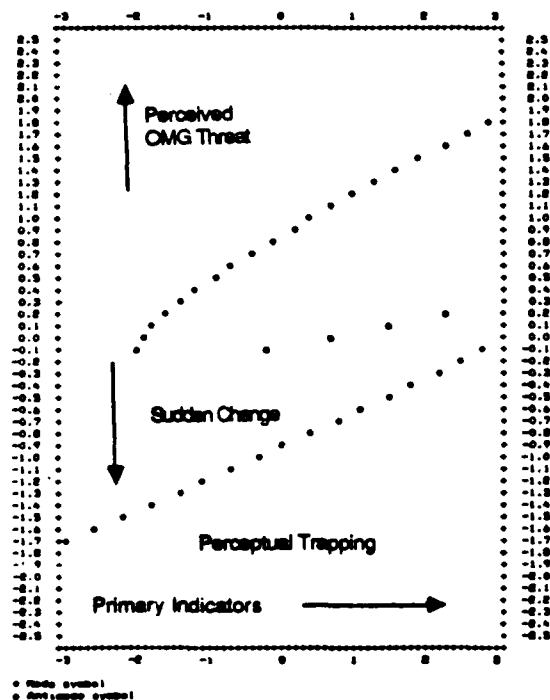
Exhibit 1-25

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot



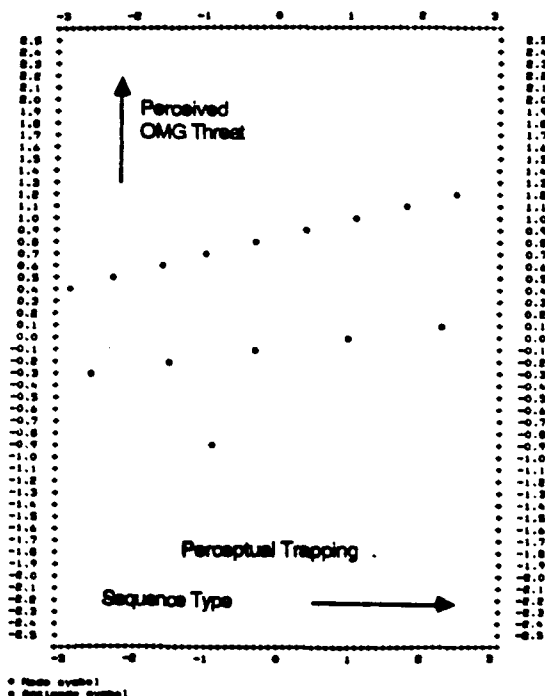
Effect of variable B, holding all others constant at their mean values.



• Mode symbol
• Asymmetry symbol

(b) Variable Primary Indicators, Fixed Sequence Type
and Level of Confidence

Effect of variable B, holding all others constant at their mean values.

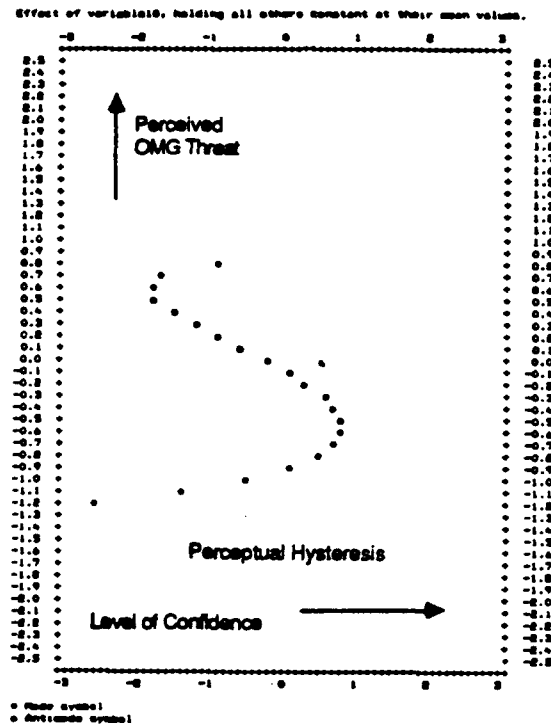


• Mode symbol
• Asymmetry symbol

(c) Variable Sequence Type, Fixed Primary Indicators
and Level of Confidence

Exhibit 1-25 (Continued)

Analysis of OMG Threat Assessment Data



(c) Variable Level of Confidence, Fixed Primary Indicators and Sequence Type

indicators and sequence type fixed at their mean values reveals situations in which perceptual hysteresis can occur (Exhibit 1-25d) as the level of confidence is increased or decreased.

SECTION 2. THE IWCAT EFFORT IN CONTEXT

The Indications and Warning Applications of Catastrophe Theory (IWCAT) effort has determined that it is feasible to use catastrophe theory and related state-of-the-art mathematical techniques to provide new facilities to support the activities of I&W analysts in areas of importance to the United States Air Force. Investigations have concentrated on the use of indicators related to the formation of Operational Maneuver Groups (OMGs) as a test of the system. This effort has involved the development of a new flexible and adaptable problem-solving and decision-making environment that is able to capture and use small, and apparently insignificant, changes in information that are the precursors of dramatic changes in overall system behavior.

This final technical report identifies a specific I&W-related problem which is amenable to analysis with the techniques of catastrophe theory. In general, suitable problems will be those in which several key influences determine system behavior. I&W problems of interest exhibit some or all of the following properties:

1. Gradual and sudden changes, divergence, bimodality, and hysteresis that are characteristic of the behavior exhibited by the elementary catastrophes.
2. Small changes in the information provided to an analyst can give rise to either small or large changes in perception under the same conditions.
3. Small biases in information can give rise to dramatically different analyses.

After extensive discussions with the government, the IWCAT team selected an I&W problem which involved the recognition of an OMG, one of the most difficult problems in tactical analyses. A set of ten indicators predicting the development of a Soviet OMG was developed. Settings of these indicators were presented to military analysts who were asked to assess the probability of OMG development and the resulting assessments were captured and analyzed with the aid of a statistical program based on catastrophe theory.

2.1 NEW FACILITIES ARE NEEDED TO SUPPORT THE I&W ANALYST

Military analysts and decision-makers in the Indications and Warning (I&W) area are faced with the need to analyze and understand large amounts of often conflicting and contradictory data derived from sensors, communications systems, and other sources. These tasks often have to be performed under severe time-pressure and if used in the field, at some actual physical risk to the analysts and decision-makers themselves. Faced with the problem of information overload in critical periods of combat, such individuals will have to resort to the use of analytic methods that capture the essence of system behavior and "friendly" graphics devices that can facilitate the understanding, reasoning, and decision-making activities of analysts in ways that develop and reinforce their perceptions. The IWCAT effort has produced a prototype computer-based system that can support the I&W analyst by providing a new technology for capturing I&W analysts' perceptions of situations of interest and communicating their understanding to battlefield commanders. The IWCAT technology can also be of value of I&W analysts and decision-makers by:

1. Alerting individuals to conditions where small changes in indicator input can give rise to either gradual or sudden changes of perception in the same situation under different conditions.

2. Clarifying the causes and effects of different perceptions of the same situation.
3. Providing an analytic capability that can give rational interpretations of nonlinear and apparently counter-intuitive behavior.
4. Identifying and characterizing the different types of responses of I&W analyses and others to features of I&W-related data sets.
5. Providing methods that can be used to support the training of such analysts and the interpretation of their assessments of particular sets of indicators.

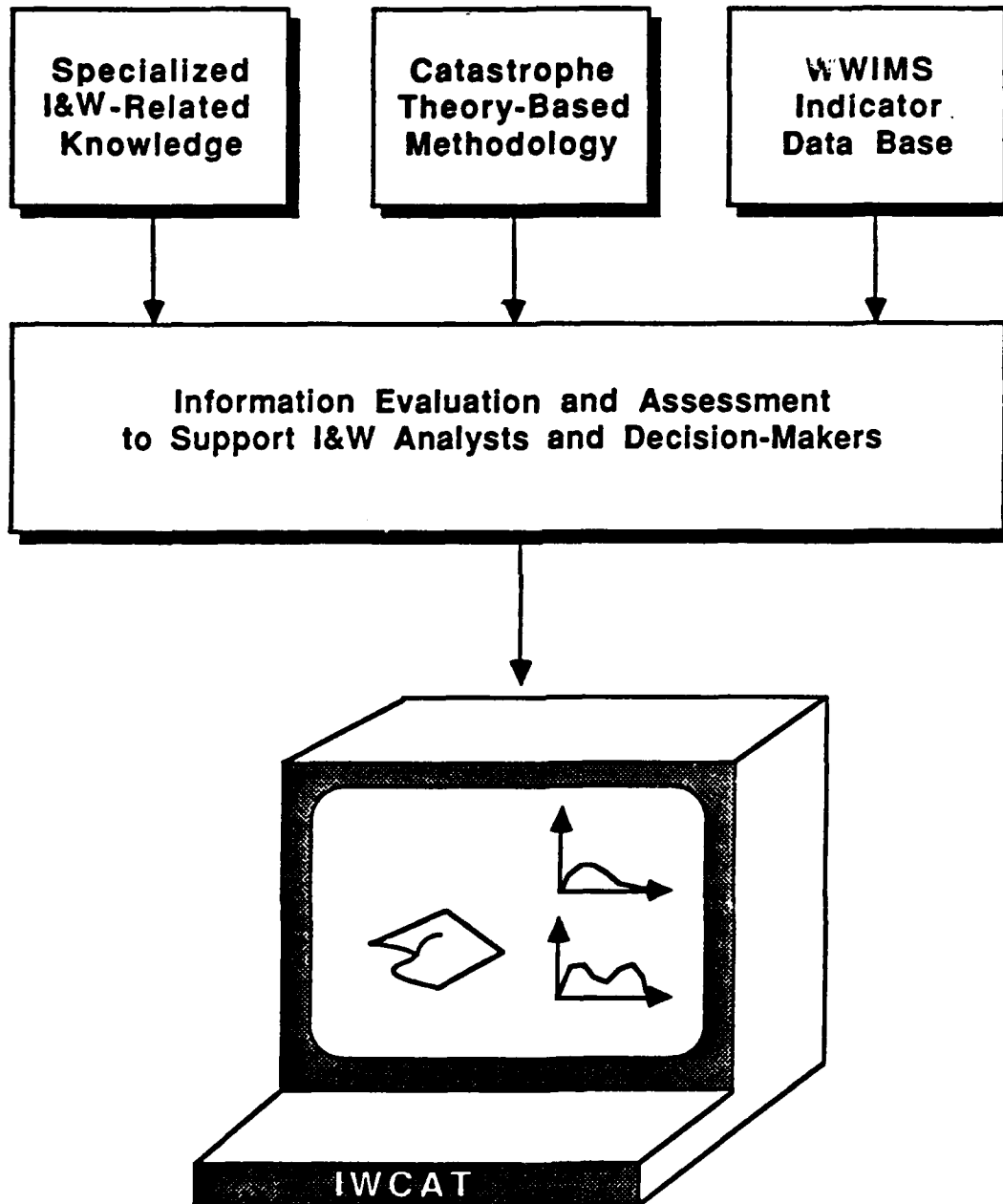
The IWCAT system provides a synthetic environment in which different indicators, representing the different key factors used by I&W analysts to make assessments and provide warning of an OMG are combined in a mathematically rigorous manner to provide an overall perception of the situation of interest. These key indicators determine position on a geometric structure (technically known as the catastrophe manifold and referred to in places in this report as the cusp surface) which consists of regions which can be described as flat plains and cliffs. The flat plains represent regions in which the perception of the analyst is unambiguous. The cliffs, by contrast, represent conditions (represented by a particular set of indicator values) under which sudden change perceptual changes can take place (Exhibit 2-1). Furthermore, these cliffs mark the boundaries of regions where the analyst's perceptions are ambiguous and where incorrect and misleading assessments can be made.

2.2 ACTIVITIES OF I&W INTELLIGENCE ANALYSTS

The main duties of an I&W analyst is to receive, screen, process, analyze, and evaluate potentially significant intelligence information. Information on the world situation comes to the I&W analyst mainly in the form of intelligence messages, which are received in both electronic and hard copy form in the I&W center 24 hours a day. The volume of message traffic is very high and increases tremendously during the time of crisis. It is the job of the analyst to separate useful data from irrelevant data and then to assimilate and correlate these items of information into a pattern of events having indications significance. Products of this analysis include briefings and reports describing the situation at hand. Specific responsibilities include:

1. Maintenance of an intelligence watch over activities in a particular area.
2. Identification of foreign activities which may adversely affect national security.
3. Prompt communication of indications of such activity to appropriate command elements and other I&W centers.
4. Analysis of event indications in depth.
5. Deduction of enemy motivations and intentions.
6. Assessment of probable developments.
7. Assessment of events in terms of impact on the imminence of hostilities.

Exhibit 2-1
IWCAT Environment



8. Timely communication of all significant information and perceptions to commanders and other decision-makers.

Much of the analysis performed to determine the significance of an incoming piece of information depends upon the context of the data. Things are not always as they appear and a knowledge of prior events, foreign doctrine, enemy procedures, and history must be taken into account when analyzing a particular activity, for example.

2.3 OPERATIONAL MANEUVER GROUP CHARACTERISTICS

An OMG is a highly mobile military unit which is designed to operate behind NATO lines and now appears to be an integral part of Soviet tactical strategy. Its mission is to attack or raid valuable targets, destroy or limit the nuclear capability of the west, disrupt reinforcement supply lines, and maintain close proximity with NATO troops making the introduction of tactical nuclear weapons difficult.

OMGs evolved from the Soviet Mobile Group, a highly mobile tank formation used extensively during World War II. This kind of military unit has developed into an OMG and has been reintroduced by the Soviet Military for several reasons. First, NATO's current tactical strategy is vulnerable to an OMG. NATO's current strategy is one of Active Defense. Donnelly (1982) characterizes an Active Defense as having the purpose of compelling the attacker to: "make repeated and systematic attempts to break into the defended line, so that the attacker should expend forces and time without gaining momentum." The defender identifies where the attacker is making the main thrust. Once this portion of the front is identified, additional units are moved up from quieter sectors. This provides a counter concentration needed to block the main thrust. The obvious weakness of this tactic is that it weakens one part of the line in order to strengthen another. This creates an opportunity for a highly mobile force such as an OMG to attack the weakened portion of the front, punch a hole in the line, and engage in operations behind NATO lines.

The second reason for the introduction of an OMG is the realization by Soviet military planners that if a war with the west is to be won, it must be won within a short period of time after hostilities begin. Donnelly (1982) states that if the war drags on, there is a high risk that:

1. It will develop into a catastrophic strategic nuclear exchange; or
2. It will cause undo strain on the Soviet social system causing it to be destroyed from the inside.

An OMG is designed to facilitate a quick win by destroying NATO defensive capacity and opening a second front during the offensive.

Finally, the introduction of an OMG was designed to deter the west from deploying tactical nuclear weapons once hostilities start. NATO uses tactical nuclear weapons to partially offset the imbalance of conventional forces that exists between its forces and those of the Warsaw Pact. However, to use tactical nuclear weapons, Warsaw Pact forces must be well separated from NATO forces. If the two forces are in close proximity to each other, there is a risk that friendly forces will be destroyed if tactical nuclear weapons were used.

2.3.1 OMGs AND OTHER MILITARY UNITS

OMG's do not operate in isolation. They depend upon other military units for support. For the OMG to be most effective, it must arrive behind NATO lines intact. Because of this, an OMG will attempt to penetrate an opponent's defenses only after they have been weakened or diverted by first echelon forces. OMG completes the breakthrough started by the first echelon forces.

While attempting to break through NATO lines, the OMG will be supported by heavy artillery preparation and covering fire barrage. Artillery and air support are considered decisive elements in modern combat. The two major artillery units supporting the OMG are the Division Artillery Group (DAG) and the Regimental Artillery Group (RAG). Both groups are usually reinforced with nondivisional artillery battalions.

Air defense for the OMG is provided by integrated systems of antiaircraft artillery, surface-to-air missiles (SAMs) and interceptor aircraft of frontal aviation. They provide air coverage at all altitudes.

2.3.2 IDENTIFYING OMGs

There are several classes of criteria which can be used to identify an OMG. They are the time at which OMGs will be inserted into battle, the location at which the OMG will be inserted, the activities of other units that will be done in support of the OMG, and the changes that occur to a military unit prior to its operation as an OMG. These will be described in more detail below.

2.3.2.1 Time and Location of OMG Insertion

The key to operational success of an OMG is surprise. If NATO forces have time to develop a credible defense, then the resources of the OMG can become quickly exhausted. To maximize the amount of surprise that the introduction of an OMG will create, it is expected to be introduced very early in the battle. This will leave NATO with insufficient time to complete mobilization or create an organized defense. Furthermore, it will make it difficult for NATO to use nuclear weapons since the decision to introduce them into an operation is a relatively slow process. Because of this, Dick (1983) expects that the OMG will be introduced during the first or second day of operations and states that there is some expectation that this introduction will occur at night at which time "surprise and shock are maximized."

The places where the OMG is inserted will be weak points in the NATO defense. It is expected that OMGs will be inserted in at least two places. These places will be characterized as having low combat power, lack of defense in depth, and low force density.

2.3.2.2 Concomitant Activity of Other Military Units

There are a number of activities in which Soviet forces will engage in support of the OMG's penetration of NATO defensive lines. Among them are:

1. The introduction of jammers to disrupt NATO air and fire support nets and command and control in the sector at which the break will occur.
2. The introduction of ground based air defense in support of a breakthrough operation. Specifically:
 - a. SA-7/14s protecting every concentration.
 - b. ZSU-23-4 forward with the lead battalions no more than 400 meters in the rear.
 - c. SA-9s between the first and second echelons of the regiment (approximately 10-15 km behind the lead battalions and in close proximity to the Regional Artillery Group (RAG)).
 - d. SA-6/8 centered in the division main area 10-20 km back from the lead battalion.
3. The start of heavy artillery preparation and covering fire barrage immediately before penetration. This would be done with a RAG positioned 2-4 km behind and up to 15 km on either side of the breakthrough point. It would also be supported by a DAG which would be positioned 4-5 km behind the RAG.

2.3.2.3 Changes to Military Units Becoming OMGs

An OMG can be a formation of division, corps, or army size. For a military unit to operate as an OMG, it must undergo a variety of changes to increase its mobility (an OMG must be able to move rapidly) and self-sufficiency (an OMG must be able to operate independently of other friendly forces deep behind enemy lines). Among these changes are:

1. A potential OMG will detach a number of its capabilities. These include:
 - a. Frog units.
 - b. Forward artillery.
 - c. Surface to surface missiles (considered the most noticeable event).
2. It will make the following kinds of attachments:
 - a. Self-propelled artillery.
 - b. Combat engineers.
 - c. Lift capacity particularly to carry additional POL.
 - d. Signal troops to provide long range communication.
3. Potential OMG's will take on increased amounts of fuel and ammunition before penetration.
4. They will increase their communications and possibly swap radars with distinctive signatures between formations to confuse NATO's intelligence picture.

During the IWCAT project, a series of notional, unclassified, indicators, considered by the IWCAT project team to reflect OMG activities and characteristics, were identified and sets of these indicators were presented to test analysts in order to determine their assessment of OMG threat. Data generated by this process was analyzed with the aid of nonlinear statistical procedures based on catastrophe theory.

2.4 CATASTROPHE THEORY CAN PROVIDE NEW TOOLS FOR THE I&W ANALYST

Advances in mathematics in such areas as catastrophe theory have provided a new understanding of the nature of highly complicated and inherently nonlinear systems. These advances have paved the way for the application of new mathematical techniques to such problems as those associated with I&W. These applications can be supported through the development and use of new analytic "tools" based upon catastrophe theory. However, in order to avoid prohibitively long training periods, such tools should be made available to I&W analysts and decision-makers in such a way that these individuals are not required to understand their mathematical details. The IWCAT system has achieved such "mathematical transparency" through the use of menus, other forms of man-machine interface techniques, and self-documentation by means of appropriate text files.

The IWCAT project has used catastrophe theory and related mathematical techniques to develop new methods for complimenting and supporting the cognitive activities of the military analyst. This will lead to the development of a suitable form of man-machine interface that uses the facilities provided by catastrophe theory to analyze the following types of behavior:

1. Small changes in environmental conditions can lead to either large or small changes in perception under different conditions.
2. Small biases in the information that is provided to an analyst can lead to the production of markedly different results of the analysis.
3. The same information presented to two different analysts can lead to dramatically different assessments of object identity, for example.

Catastrophe theory is a relatively new type of method for the analysis of complicated systems. Elementary catastrophe theory is based on a theorem due to Thom (1969, 1975). This theorem provides a classification of the nature of the stationary states of those systems which have up to four key inputs (or control or conflicting factors), two outputs (or behavior variables), and which consist of cooperating elements whose actions seek to minimize some form of potential energy-like property associated with the system. In applications where the elementary catastrophes are used the theory provides a series of diagrams, called catastrophe manifolds (or popularly, "catastrophe landscapes"), whose use makes possible the performance of a series of "thought-experiments" on the behavior of a system of interest. Aspects of the theory are described in Appendix A.

Use of these state-of-the-art mathematical techniques in the IWCAT effort has made possible the development of a wide range of new I&W analytic tools that can be used to support the activities of military analysts and decision-makers. The process of mutual reinforcement between identifying and transferring advances in mathematics to the I&W arena, and in developing a new type of understanding of the I&W environment is a key feature to the approach used in the IWCAT effort.

This approach has drawn on existing mathematical, I&W, and combat knowledge (particularly that relating to the use of I&W techniques) as well as applicable computer-based technologies. It has also supported the design of a computer-based I&W capability, targeted for installation in the RADC intelligence workstation (IWS) environment, in which system development and the transitioning of the results obtained by independent mathematical research and development can proceed in parallel.

2.5 MODELS OF PERCEPTION CAN BE BASED ON CATASTROPHE THEORY

Systems that exhibit some or all of the properties of gradual and sudden changes in behavior, divergence, bimodality, and hysteresis have an underlying dynamical nature to which the catastrophe theory-based analysis can be applied. Such properties are associated with the phenomena of perception and the following is an illustration of the use of catastrophe theory to describe and explain such phenomena.

Two control, or input factors, whose actions determine the nature of the perception of the object by an observer and which serve as the control factors of a model of perception based on catastrophe theory, are identified in the application of the theory to modeling the perception of I&W analysts. These factors are:

1. The number of active indicators in the sets of indicators presented to the I&W analyst.
2. The level of confidence that a particular set of indicators represent actual military activities based on an assumed knowledge of the capabilities of intelligence collection and processing capabilities, for example (see section 3.3, for example).

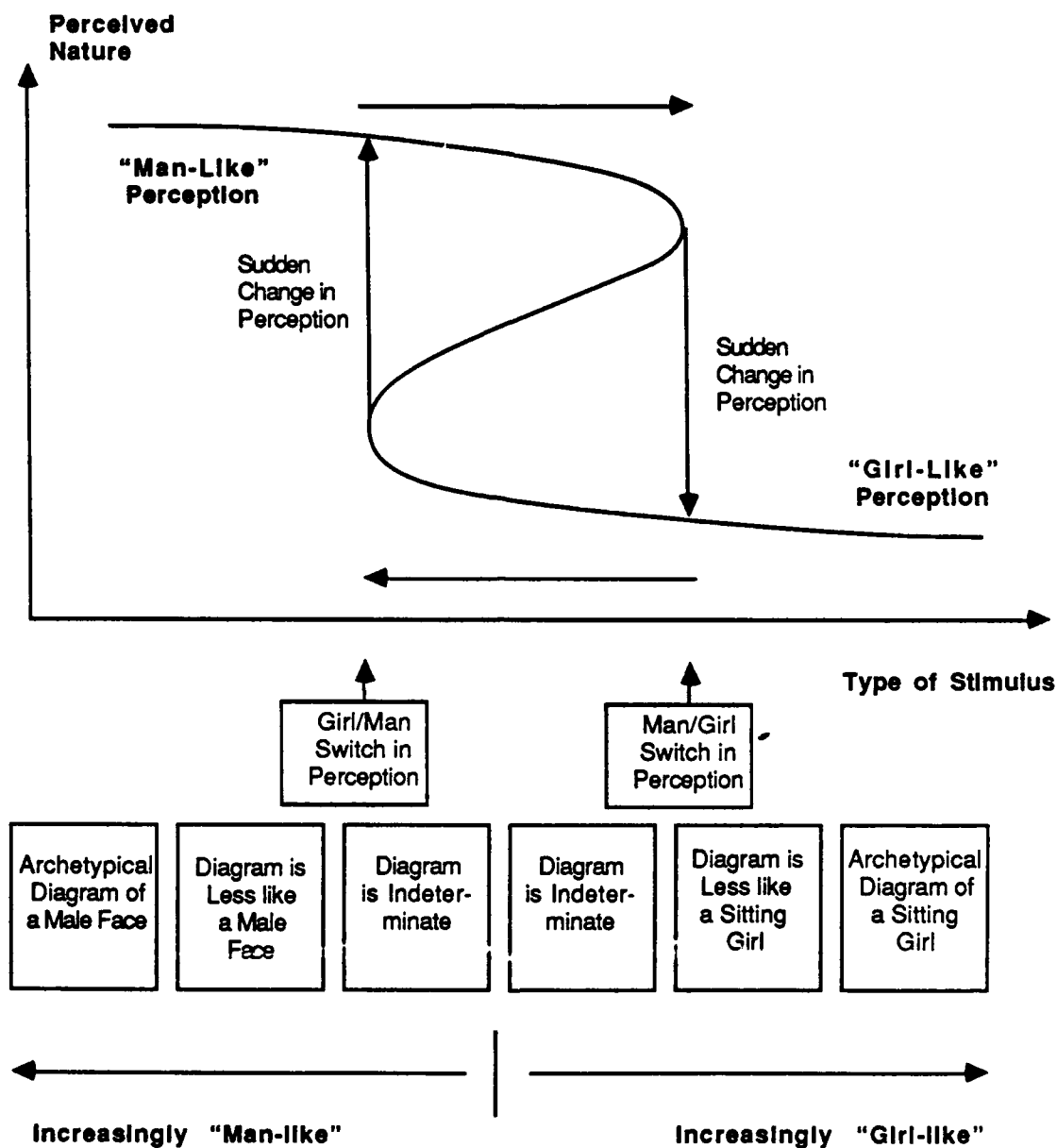
The action of these factors will determine the perception of the object or scene by an observer and this perception will be described as the behavior or output variable of the system in the catastrophe-theoretic model of perception as described below.

Poston and Stewart (1978b) have used the ambiguous figure described earlier by Fisher to illustrate the phenomenon of bistable perception. In this case, an object can appear as one of two alternatives; the face of a man, or a sitting girl. The sequence of line diagrams presented in Exhibit 2-2 presents the biasing sequence described by Poston and Stewart (1978b) in which an ambiguous or "neutral" object can be transformed either into the face of a man or a sitting girl by the addition of more and more characteristic features of one or the other type. The ability to perceive an object also depends upon the level of detail presented to the observer. In their analysis of bistable perception, Poston and Stewart (1978b) show this impact by presenting a series of figures with different numbers of active indicators (Exhibit 2-3). Diagrams with large amounts of detail are readily recognizable as a man's face or a sitting girl, or as some combination of these entities. However, diagrams with much less detail cannot be recognized as a particular entity or combination of entities with any degree of ease.

The factors of input characteristics and detail associated with an object act to determine the nature of the object as perceived by an observer, and changes in these factors can lead to changes in the perceived nature of such an object. The causes and effects of such changes in perception can be investigated with the aid of catastrophe theory, as shown below.

Exhibit 2-2

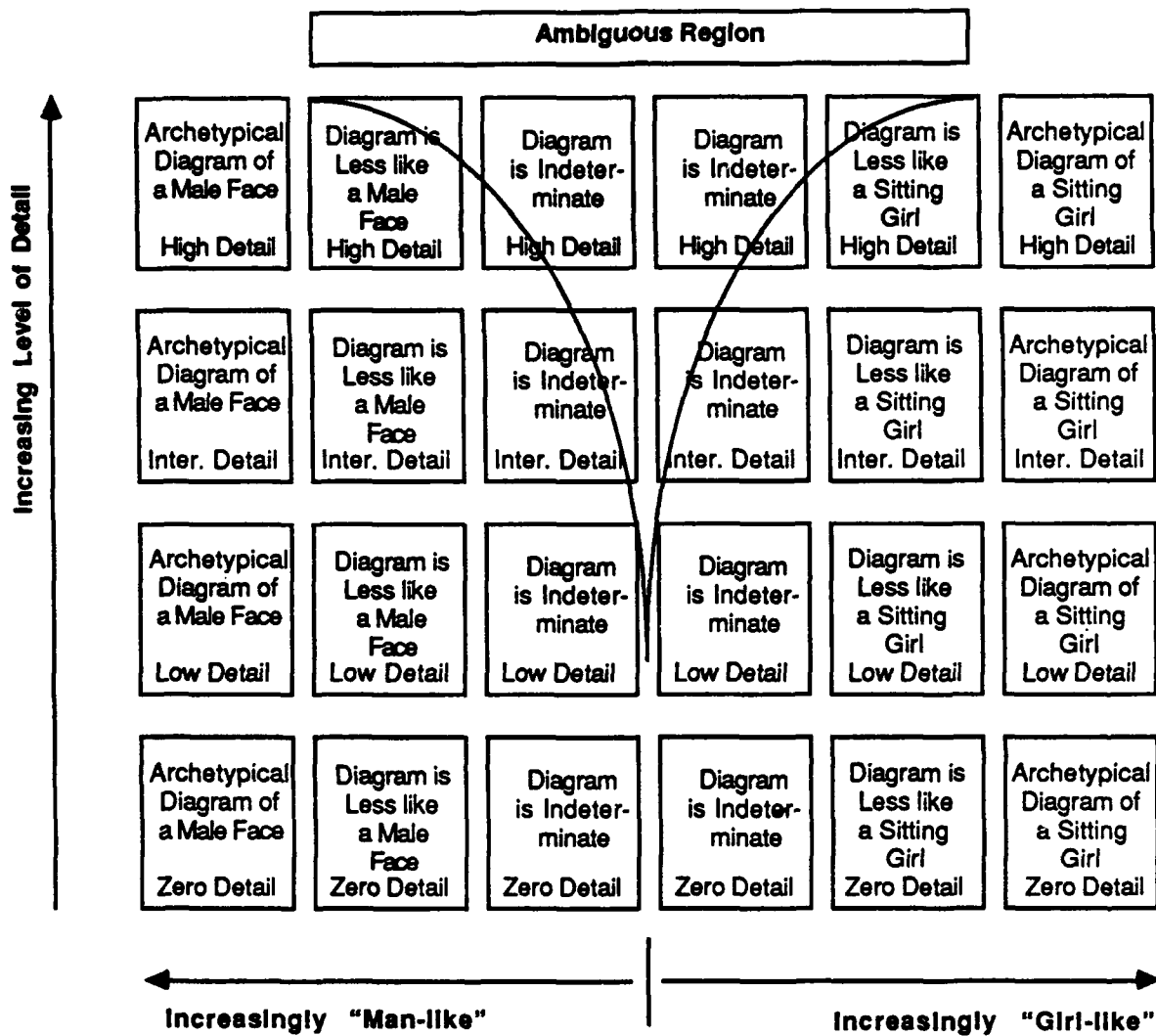
Input Characteristics



Input characteristics can range from "Man-like" to "Girl-like" with an ambiguous intermediate condition where the object could be either "Man- or Girl-like."

Exhibit 2-3

Increasing Detail



(Modified and re-drawn after Poston and Stewart, 1978)

Increasing the level of detail of an object can influence the perception of that object.

2.5.1 SUDDEN CHANGES IN PERCEPTION

In this illustration the object will be identified as either "A-like" or "B-like" and changes in the type and detail of information presented to the observer will influence the nature of the perception of these objects. When presented with a large amount of "A-like" detail, an observer would develop a strong perception that the object was "A-like" (position (a), Exhibit 2-4). However, modifying this information so that it was increasingly "B-like" could lead to a condition in which the observer's perception undergoes a sudden change and is now perceived to be "B-like" (path (a-b-c), Exhibit 2-4). This phenomena can be observed by scanning the figures in Exhibit 2-2. Moving from right-to-left can lead you to switch your perception from a figure that appears to be a sitting girl to an object that appears to be the face of a man at approximately the sixth or seventh figure, for example.

2.5.2 DIVERGENCE

The perception of objects can exhibit the phenomena of divergence under which a small difference in the type of information initially presented to the observer can have a profound impact on the nature of the subsequent perception of the object as additional elements of information are obtained and made available to this observer. Thus, the presentation of increasing amounts of detail can lead to the development of a strong perception that the object is "A-like" (path (a-c), Exhibit 2-5). By contrast, a small difference in the initial type of information presented to the observer (represented by position (b) compared with position (a), (Exhibit 2-5), for example) can lead to the emergence of a large perceptual difference in the nature of the object (path (b-d), Exhibit 2-5, for example).

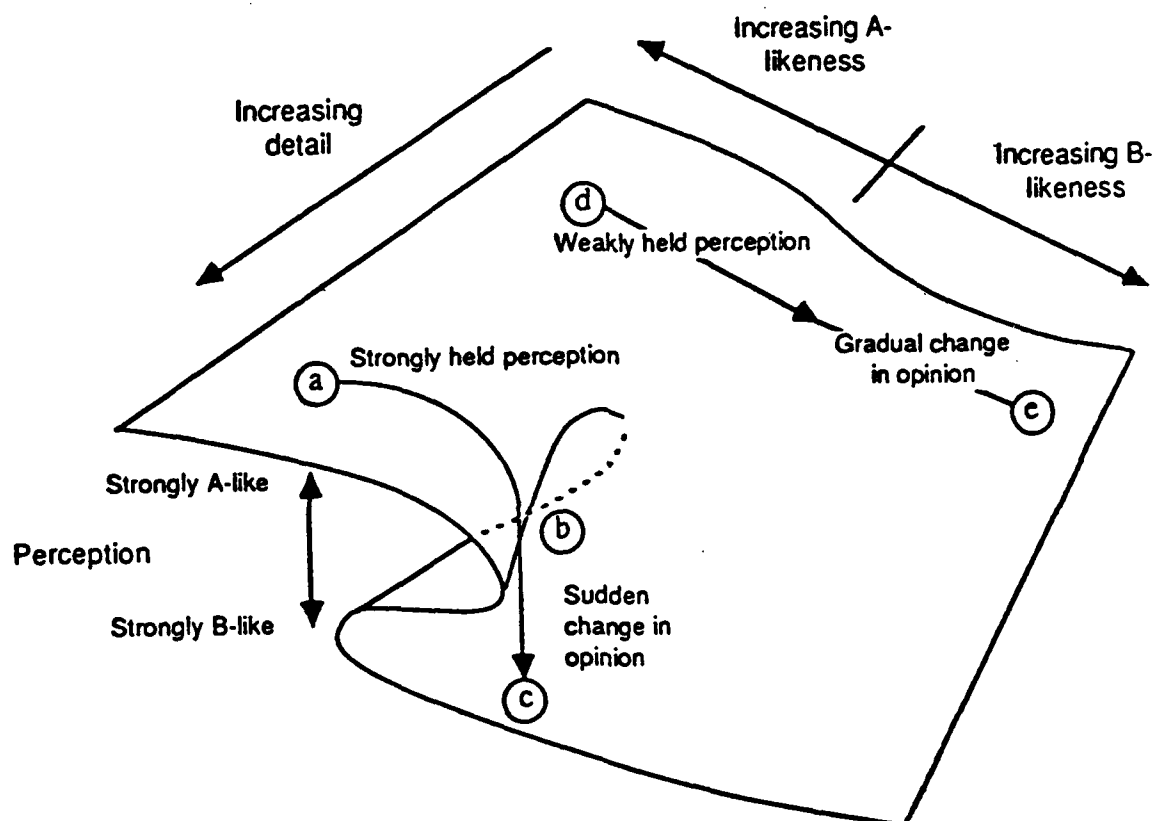
2.5.3 BIMODALITY

The phenomena of divergence also illustrates the phenomenon of bimodality. Thus, two different observers could be presented with the same information and, as a result of the way in which the previous information was presented, or due to different experiences, training, or other influences acting on these observers, they would perceive the information to represent completely different objects. Thus, one observer might conclude that the object was strongly "A-like" (position (a), Exhibit 2-6) while another observer might conclude that the object was strongly "B-like" (position (b), Exhibit 2-6). Such behavior illustrates the situation that occurs when two individuals use the same body of information to come to diametrically opposed conclusions about a situation of interest such as that associated with the I&W environment.

2.5.4 HYSTERESIS

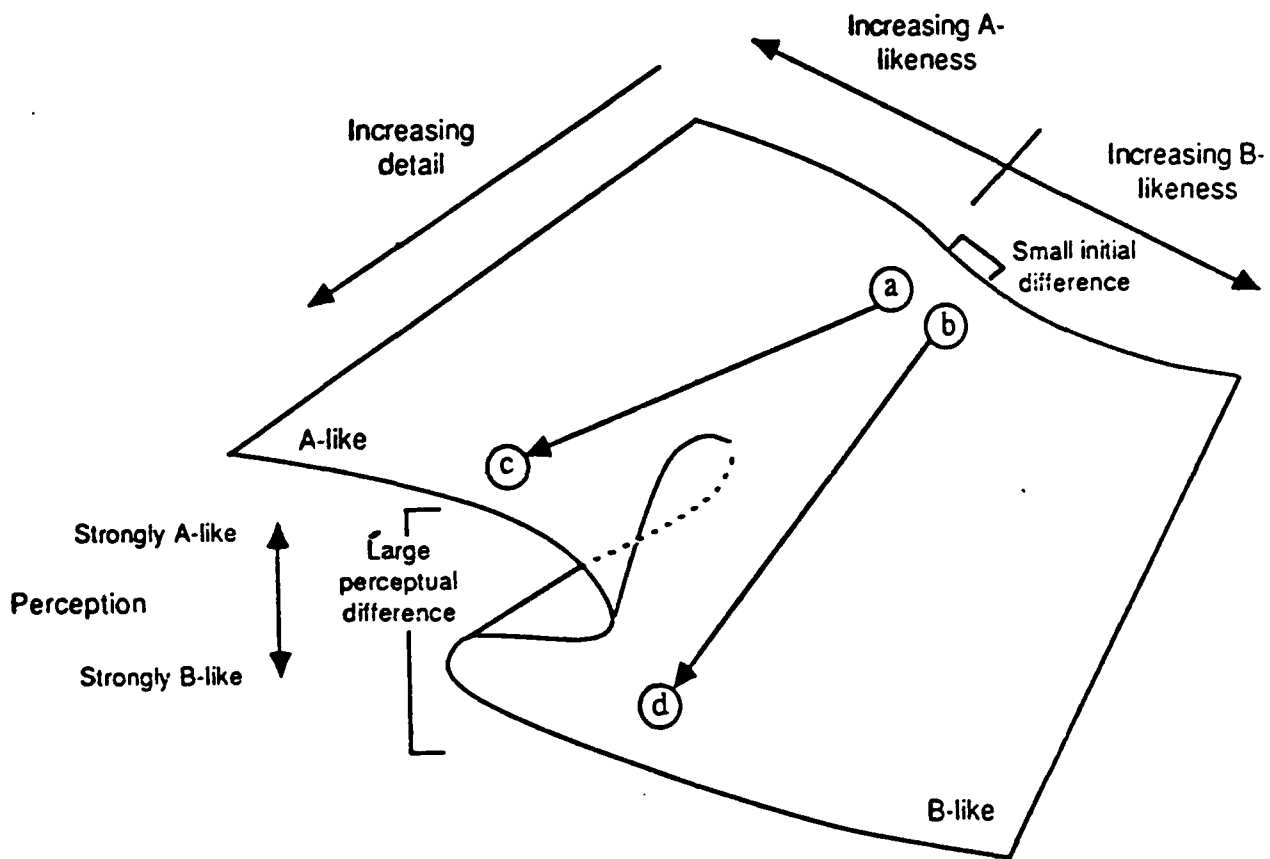
The perception of the nature of an object can exhibit the phenomenon of hysteresis. This can take place as the nature of the information presented to an observer is changed from strongly "A-like" to strongly "B-like" and back again. A change in the type of information from strongly "A-like" to strongly "B-like" can lead to a sudden transformation in the nature of the perceived object from "A-like" to "B-like" (path (a-b-c), Exhibit 2-7). A subsequent increase in the amount of "A-like" information can lead to a further sudden change in the nature of the perceived object from "B-like" to "A-like" (path (c-d-a), Exhibit 2-7). These sudden changes in perception generally will take place when the observer is presented with a sequence of different levels of "A-like" and "B-like" information, for example.

Exhibit 2-4
Sudden Changes



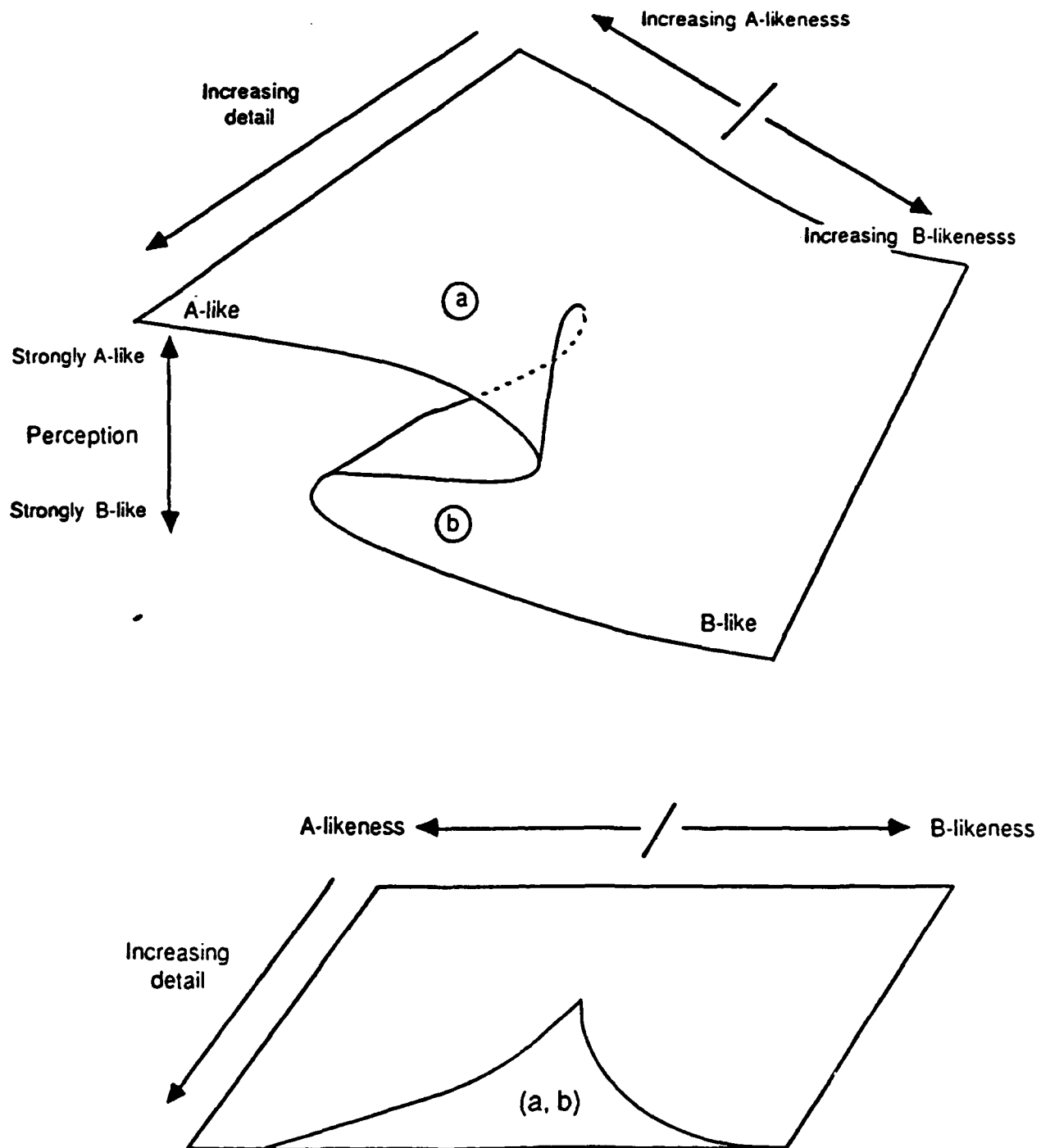
A strongly held perception can be overcome as the result of large amounts of new, definitive information and result in a dramatic change in opinion. A weakly held perception can be transformed without such a dramatic shift in opinion.

Exhibit 2-5
The Property of Divergence



Small initial differences in perception formed when small amounts of information are available can have a profound effect on subsequent perceptions as more information is made available.

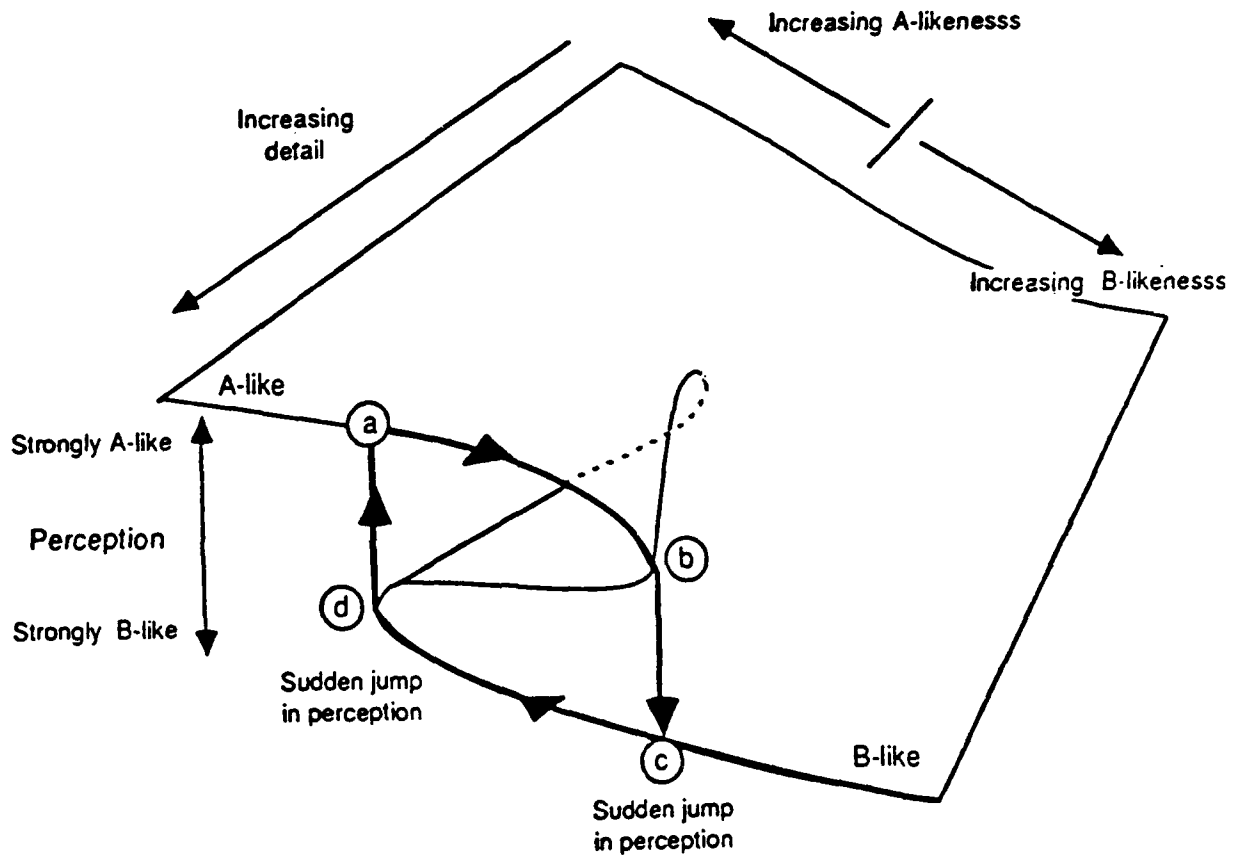
Exhibit 2-6
The Property of Bimodality



Given the same information, two different analysts can arrive at diametrically opposed assessments of a situation.

Exhibit 2-7

Perceptions Can Exhibit Hysteresis



Decision-makers can change their minds about a particular situation as they are presented with a sequence of different types of evidence.

This section has reviewed the need for new interactive qualitative computer-based facilities to support the activities of I&W analysts. The Operational Maneuver Group (OMG) was selected as the test case for analysis during the IWCAT project. Aspects of the application of catastrophe theory to the modeling of perception and some of the key features of a new I&W environment that is based on catastrophe theory developed during the IWCAT effort have been outlined in this section. Activities leading to the design and implementation of a computer-based system for OMG threat assessment and nonlinear data analysis are described in the next section of the report.

SECTION 3. IWCAT CONCEPT OF OPERATIONS

This section of the report describes the concept of operations for the IWCAT software system. United States Air Force I&W analysts have the mission of providing a timely recognition and reporting of changes in military events that are of interest to the United States. Activities performed under the IWCAT contract have reviewed typical I&W activities and have led to the identification of those classes of problems which are amenable to analytic procedures based on catastrophe theory. Syntectics' IWCAT project staff has determined, in collaboration with the government, that the conditions under which an Operational Maneuver Group (OMG) is formed from an otherwise "normal" pattern of soviet military advance are of sufficient interest to the government to warrant its selection as the appropriate "I&W situation" as specified in the IWCAT statement of work.

The IWCAT effort has provided an environment for the analysis of those conditions responsible for the production of ambiguous I&W analyst perceptions and where gradual changes in the nature of the data presented to such individuals can give rise to gradual or sudden changes in perception under different conditions, for example. Thus, the IWCAT effort has lead to:

1. The development of new techniques for the presentation of data to I&W analysts.
2. Methods for capturing analysts' perceptions of these data.
3. The use of nonlinear techniques based on statistical catastrophe theory to reveal conditions under which ambiguous or conflicting perceptions may occur and to identify situations where sudden perceptual changes may take place.

The concept of operations outlines a series of activities associated with the knowledge development environment. These involve the generation and analysis of intelligence data sets which are representative of such activities performed by I&W analysts and which also use newly developed statistical analytic procedures based on catastrophe theory and related techniques for data manipulation.

The design and implementation of this environment is responsive to the need to provide I&W analysts with access to advanced analytic procedures in as "transparent" a manner as possible and where, as a consequence, the necessary mathematical operations will have to be performed automatically, or with as minimal a level of user involvement as possible.

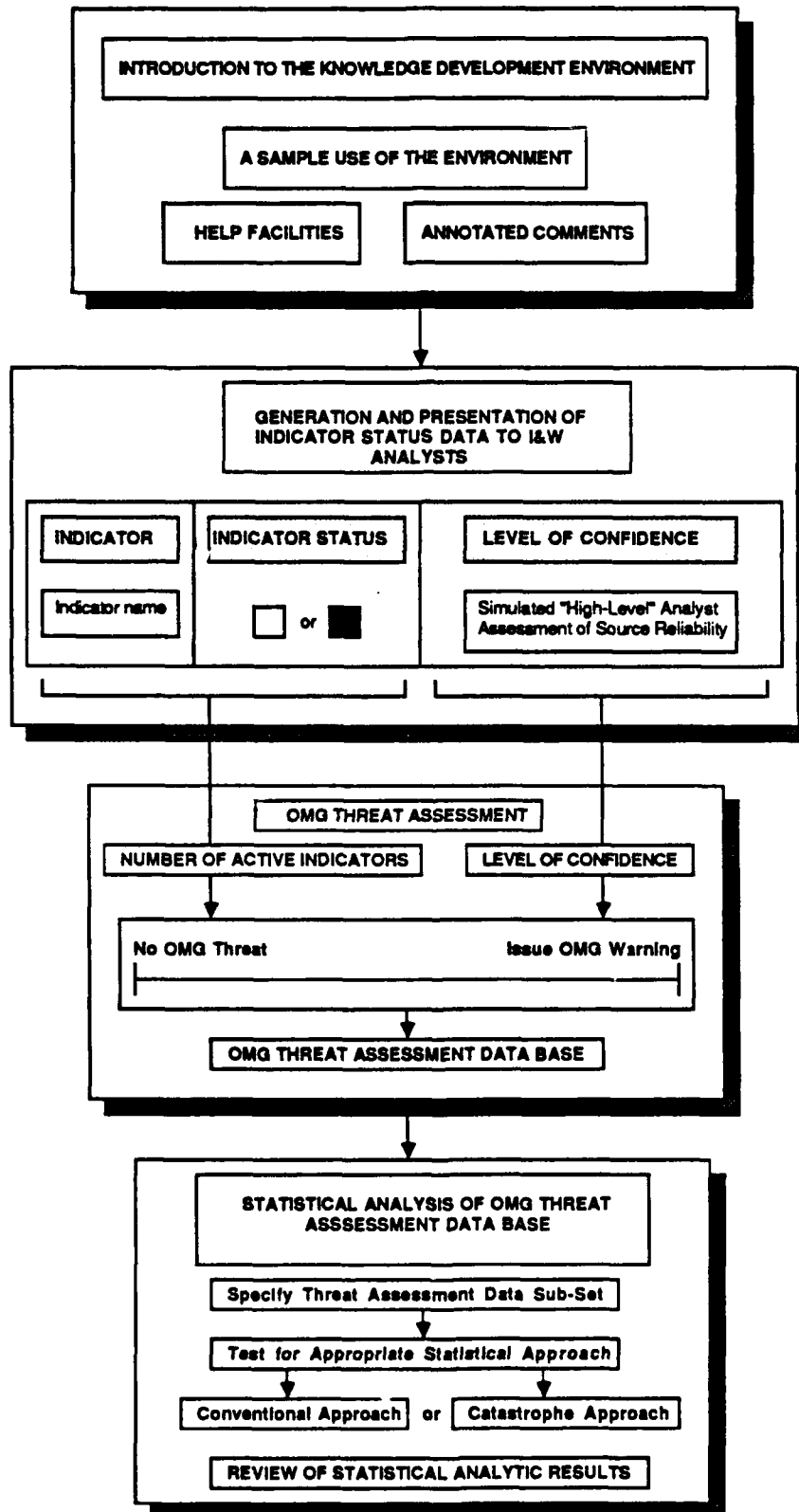
3.1 OVERVIEW

The overall concept of operations for the IWCAT project is illustrated in Exhibit 3-1. The operation of the IWCAT knowledge development environment involves several major phases of activity, including the following:

1. The introduction to the knowledge development environment facility provides the user of the IWCAT facility with a sample use of the environment, help facilities, and annotated comments that aid in their use of this facility.

Exhibit 3-1

Overview of the IWCAT Concept of Operations



2. The generation of OMG-related test data sets involves a dedicated scenario generator that produces groups of indicators whose properties have been chosen to reflect those of an OMG. The test data sets consist of a list of OMG-related indicators with an overall measure of confidence value and statements concerning the assumed weather conditions, time of day, and politico-military background against which the OMG assessment is to be performed.
3. The presentation of test data sets to I&W analysts permits intelligence analysts to undertake OMG-related threat assessment activities and the construction of an OMG Threat Assessment Data Base. The analysts have access to the list of indicators which are structured to present different numbers of active indicators and level of confidence data as well as information on weather, time of day, and the politico-military background against which the simulated assessment problem is supposed to be taking place. This information will be used to generate an assessment of the likelihood that an OMG has been or is being formed. The set of indicators and the analyst assessment will form the basis for the OMG Threat Assessment Data Base.
4. The statistical analysis of the OMG Threat Assessment Data Base. Creation of this data base as outlined above sets the scene for its analysis with the aid of techniques based on statistical catastrophe theory. These techniques are used to investigate the properties of the I&W analyst-derived data in order to determine the nature of those conditions under which perceptual ambiguities may arise, and where sudden changes in perceived OMG threat levels may occur, for example.
5. The review of the results of this statistical analysis provides a new level of insight into the processes of perception and threat assessment undertaken by I&W analysts and can set the scene for the development of new types of operational facilities for OMG threat analysis, for example.

3.2 MAPPING I&W PROBLEMS TO CATASTROPHE THEORY SURFACES

Catastrophe theory describes a series of structures called catastrophe manifolds (or surfaces) which resemble stylized "landscapes." Positions on such landscapes are specified by coordinates whose specific values reflect the values of key independent system variables and the corresponding values of the dependent variable(s) of the system. The IWCAT project has used these geometrical structures to express the relationships between the nature of the intelligence and other information input to I&W analysts (the independent system variables) and their assessment (the dependent system variable(s)) of the perceived level of threat corresponding to these inputs. This section of the report outlines the underlying mathematical technology and describes how it was used in the IWCAT project.

3.2.1 SINGLE-VALUED AND MULTI-VALUED FUNCTIONS

Catastrophe theory provides a method for expressing functional or cause and effect relationships in terms of a particular type of graph known as the catastrophe manifold. Thom's (1972) theorem shows that almost all graphs could be discussed in terms of seven basic shapes which are known as the elementary catastrophes. Single-valued functional relationships between dependent and independent variables are those in which one value of the dependent variable relates to one and only one value of the independent variable. Thus, each value of the independent variable x gives rise to a unique value, $f(x)$ of the dependent variable (Exhibit 3-2(a)).

Catastrophe theory provides the basis for describing situations in which no single value functional relationship exists between system variables. The graph shown in Exhibit 3-2(b) possesses regions (the interval between x_1 and x_2) in which there are three values of the dependent variable for one value of the independent variable, x . Although the curve is continuous, the behavior of the dependent variable is not. As the value of the independent variable is changed from x_0 to x_3 , the value of the dependent variable jumps from the upper to the lower part of the curve at point x_2 , for example. Under these circumstances, a basically continuous model (represented by the curve) can produce discontinuous behavior (the jumps between the layers of the curve).

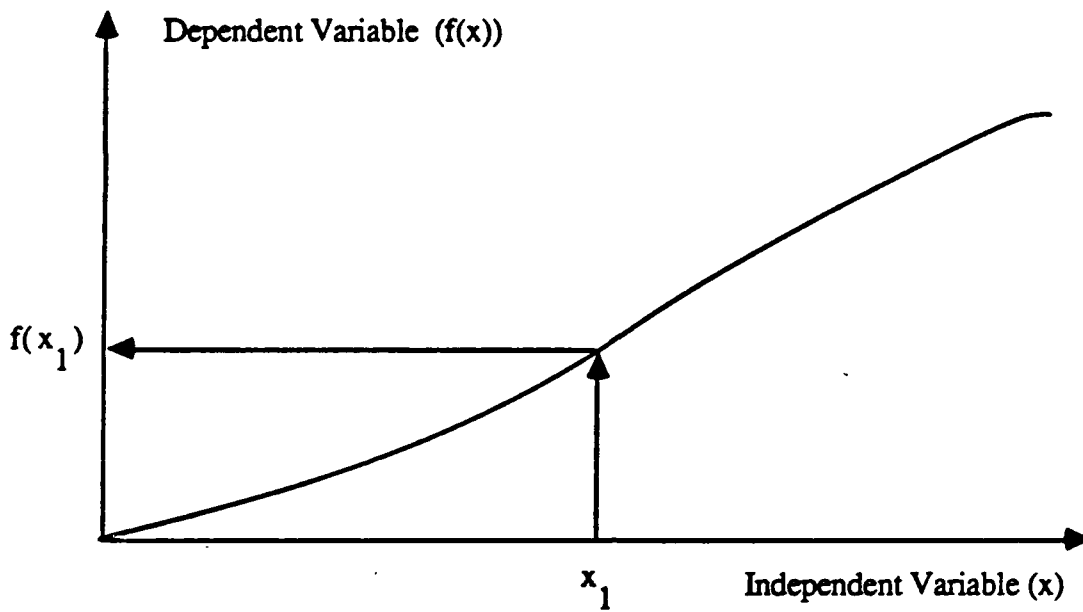
Catastrophe theory studies such processes from a general mathematical viewpoint. The IWCAT effort is involved in identifying circumstances under which continuous changes in such information as an intelligence data set gives rise to a sudden or discontinuous change in the perception of an I&W analyst. Catastrophe theory provides a framework for studying the impact of a limited number (between one and four) of independent (control, or key) factors on the behavior of a system of interest. This behavior is reflected in the values of one or two dependent (or behavior) variables. An obvious method for solving the problem of multi-valued functions would provide methods for identifying the key types of information examined by the analyst in the process of making a judgment of a particular situation with the independent or control variables. The judgment itself could then be identified with the dependent, or behavior, variable of the system of interest. Such a method was described in the IWCAT proposal and serves to guide project research activities.

3.2.2 SINGLY STABLE AND B₁-STABLE PERCEPTIONS

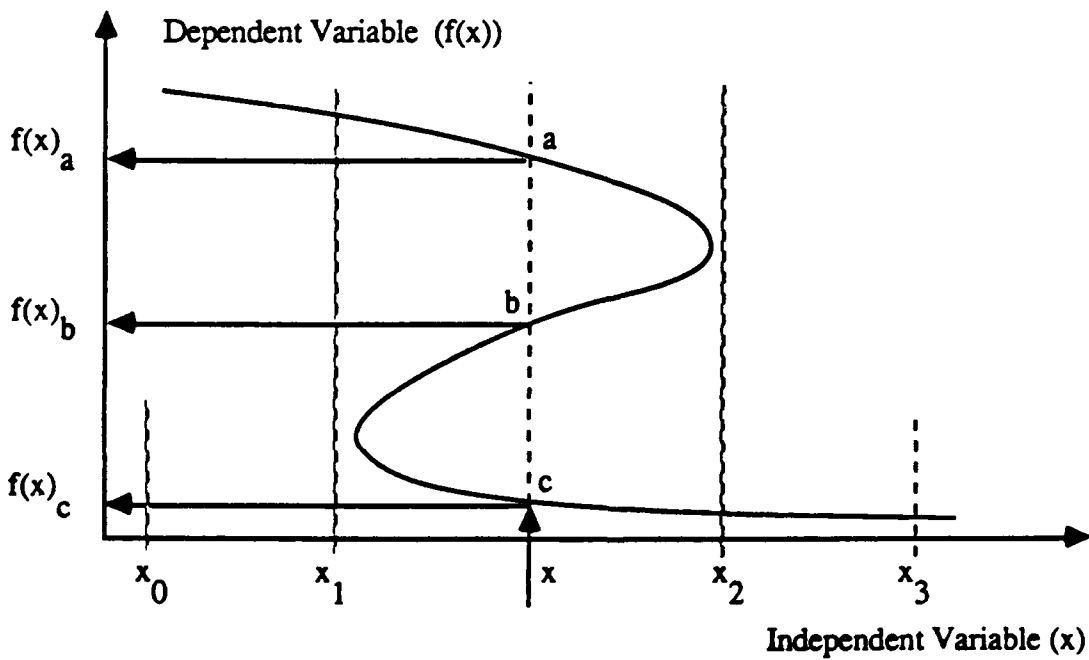
The IWCAT effort involves the identification of several independent variables which represent some properties of the OMG-related information presented to I&W analysts. Test data sets corresponding to different values of these independent variables were presented to selected I&W analysts and other individuals in a knowledge development activity where these individuals were asked to record their perception of these data for inclusion in a data base. In actual uses of the IWCAT system, this method has permitted the identification of conditions of perceptual bistability in which the same data set can generate two different types of perception (see Exhibits 3-3 and 3-4, for example). Thus, while test data sets corresponding to values of the independent variable of x_4 and x_6 may generate single perceptual responses ($p(x_4)$ and $p(x_6)$, respectively), a data set corresponding to a value of the independent variable of x_5 can give rise to two markedly different types of perception ($p(x_5)_1$ and $p(x_5)_2$, respectively).

Exhibit 3-2

Functional Relationships



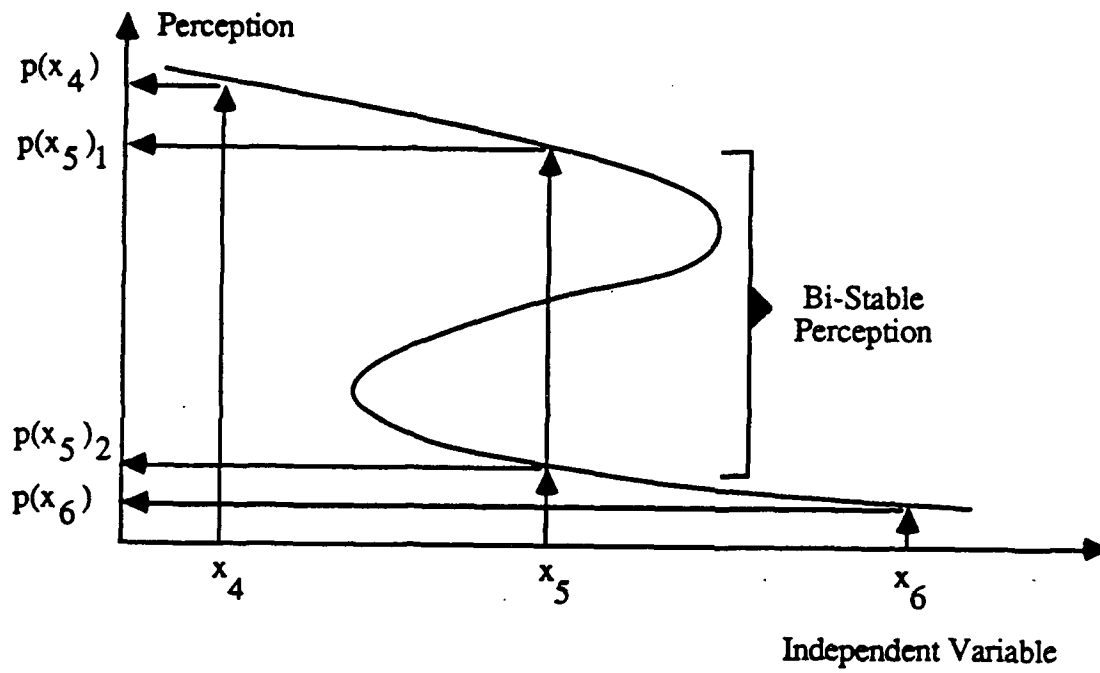
(a) A single valued functional relationship



(b) A multivalued functional relationship

Exhibit 3-3

Singly-Stable and Bi-Stable Perceptions



Stewart and Peregoy (1983) observed such behavior in a series of perceptual investigations in which illustrations were presented to a series of test subjects. It is the aim of the IWCAT project to determine whether a similar type of bistable perceptual behavior can be generated with data sets derived from information contained in the I&W test data set. The IWCAT activities will involve the use of I&W test data to create an appropriate set of data elements that will be presented to the test subjects during the use of the IWCAT system.

The information obtained during this activity will be analyzed and an attempt will be made to fit these data to the cusp catastrophe manifold with the aid of a specialized computer program which is based on statistical catastrophe theory. When the catastrophe manifold has been created in this way, it can be used to assess the nature of I&W-related data sets (Exhibit 3-4). Thus, a "type #1" input would provide an unambiguous estimated position on the manifold while another data set input "type #2" would give rise to an ambiguous perception by the analyst. Such an ambiguous result could be used to alert I&W analysts to the need for extra caution in the interpretation of the data and the possibility that a warning signal should be issued.

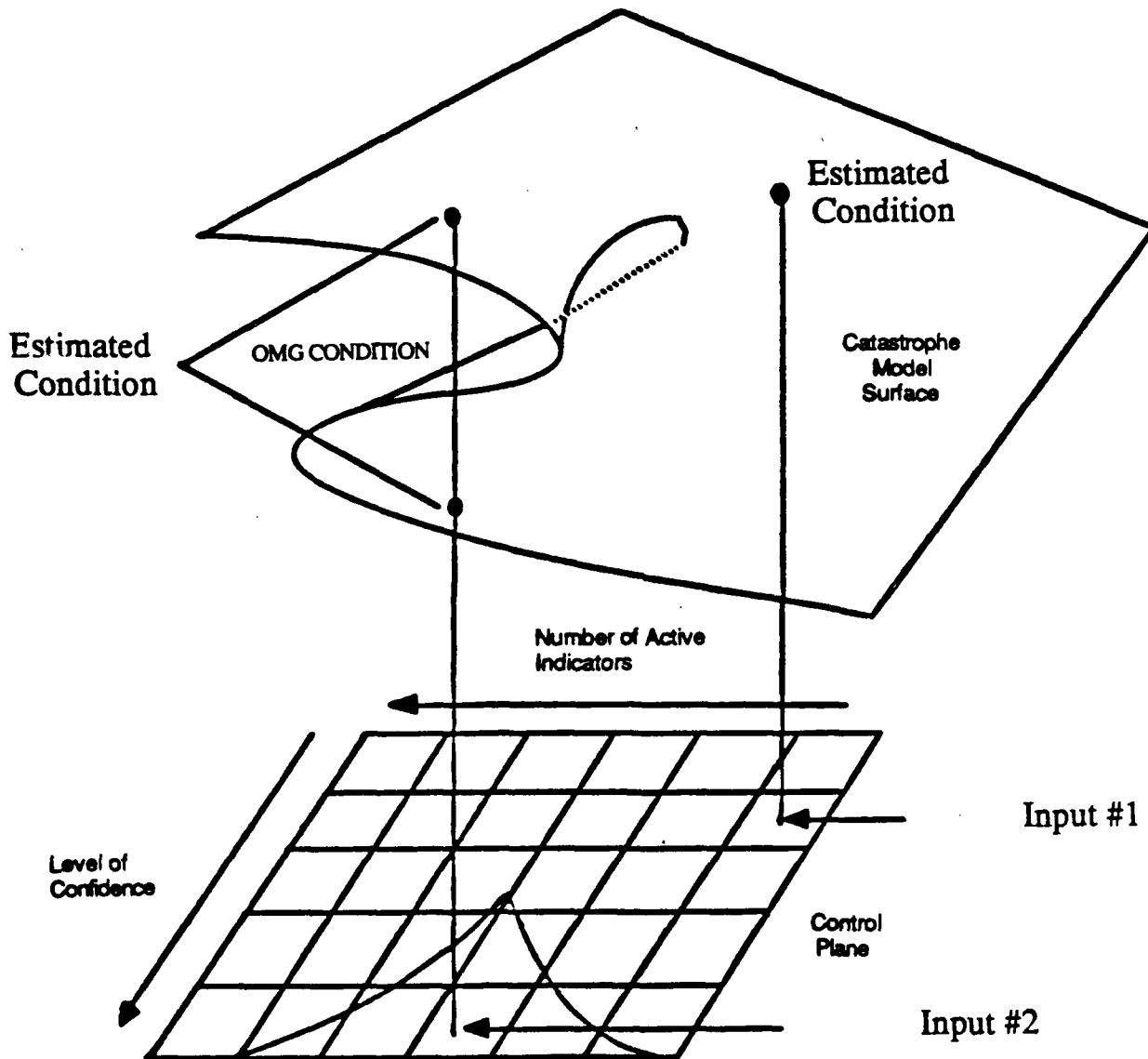
There are other factors which influence perception but which are not tractable within this scheme. These are variously referred to as context variables or tacit knowledge. This is general knowledge about the world which influences how judgments are made. Examples of this kind of knowledge include the time of year at which the judgment has occurred and the existence of certain military and/or political events (such as the ascension to power of a new leader). This information cannot be represented easily within the above coordinate system for two reasons:

1. This information is not represented by clusters of indicators. It is additional knowledge that the analyst considers when judgments are made.
2. The information is not easily quantified or even collected. There are numerous context variables which can affect judgment making it difficult if not impossible to collect some kind of comprehensive list of them. Furthermore, they do not cluster easily into dimensions. Thus, it is difficult to assign some number to them. They are instead, features, which are either present or absent at the time that a judgment is made.

To circumvent this problem, we propose to control the context variables by presenting the individuals performing the judgment task with a written scenario. The scenario will contain information which will be sufficient to define the values of a subset of these context variables. For example, prior to performing the judgment task, a person will be presented with descriptions of three different politico-military scenarios each reflecting different levels of United States interest and potential involvement. These scenarios are called "Treaty Obligation," "Friendly Ally," and "Third Party Hostilities" scenarios. The first of these scenarios would require direct United States military involvement; the second places a high level of obligation on the United States to provide information to an ally and failure to do this would result in damage to United States political and other interests; the third scenario places the United States in the position of not participating in hostilities, but wanting to keep track of ongoing activities in order to protect her interests and those of her allies.

Exhibit 3-4

Catastrophe Theory-Based I&W Assessment Activities



3.3 A CATASTROPHE THEORY-BASED KNOWLEDGE DEVELOPMENT ENVIRONMENT

A series of activities, described in the IWCAT proposal as knowledge development activities, were undertaken to determine the responses of I&W analysts when faced with the task of assessing the likelihood that an OMG has been formed, or is in the process of being formed. When using the IWCAT system, individuals are given a set of notional unclassified indicators and other data which have been designed to resemble as closely as possible in a study environment the type of data that would be available to an I&W analyst in an operational environment. The data produced during this activity can be analyzed with the aid of a nonlinear statistical procedure based on catastrophe theory (developed by Cobb (1978, 1980)) in order to investigate conditions under which ambiguous identifications and sudden and gradual changes in I&W assessments can take place.

Analysis of the I&W environment and discussions with the government have led the IWCAT project team to the identification of the following control factors and behavior variables that are associated with the major features of the activities performed by I&W analysts.

1. The control factors represent the inputs to the process of I&W analysis. The IWCAT project team selected two control variables (number of active indicators and level of confidence) to represent these inputs. The "number of active indicators" variable represents the number of indicators which are activated when the analyst makes a judgment. The "level of confidence" variable represents a measure of the degree to which a particular set of indicators can be considered to be a true representation of actual military behavior. Additional information including weather, time of day, and scenario type is also presented to the analyst during the OMG threat assessment task.
2. The behavior variable represents the result of the I&W analyst assessment process. The IWCAT project team named this variable the analyst's OMG threat assessment since this variable represents the perception of the I&W analyst of the likelihood of the formation of an OMG from an apparently "normal" pattern of military advance.

The two major control variables described above ("number of active indicators" and "level of confidence") are considered to be independent variables whose values determine the value of the dependent or behavior variable. This assumption can be tested with the aid of the cusp surface analysis program and actual analyst OMG threat assessments which reflect the impact of the number of active indicators and the level of confidence in these indicators.

In the event that some sets of independent variables create different perceptions of threat of OMG formation, such multi-valued behavior could be illustrated with the aid of the catastrophe manifolds. The catastrophe manifolds are multivalued for some ranges of the control factor values and their shape serves as a model of the results of the analyst OMG threat assessment activity. Under these circumstances, some sets of control factor values may be responsible for multiple perceptual states while other sets of values of these variables may generate a single behavior variable or perceptual state.

3.3.1 THE OMG INDICATIONS AND WARNING PROBLEM

The Operational Maneuver Group relies upon very rapid deployment and penetration behind the battlefield on the first or second day of hostilities. The attacking OMG can achieve the necessary element of surprise if issuance of "warning" can be avoided until the operation begins. The task for the I&W analyst is to detect, recognize, and issue the warning of an impending OMG operation as far in advance as possible.

1. **Detection.** By its very nature, the formation of an Operational Maneuver Group presents an unusually difficult detection problem for the I&W analyst. The purpose of the OMG is to employ surprise and shock to achieve significant objectives before defenses can react. Thus, if an OMG operation is to be undertaken at all, exceptional efforts to conceal, deceive, and confuse are virtually certain in the formative stages.
2. **Recognition.** The second obstacle to early isolation of facts which might indicate OMG activity is that some of the indicators which signal OMG development are similar to, or the same as, the indicators which might portend a more conventional offensive operation. Particularly in a context of rapidly intensifying tension and formation of opposing lines along facing borders, the OMG indicators could become lost in the "noise level."

3.3.2 THE SOURCE EVALUATION SYSTEM

The fundamental task of the Indications and Warning analyst is to determine the implications of information which may often be incomplete, inconclusive, and contradictory. In addition to the expected obscuration of data which attends any analytical task, the I&W analyst faces an additional difficulty in that most, if not all, of the information which he seeks may be being deliberately concealed or distorted by the enemy.

Given the information "as-reported," the I&W analyst must next apply deductive and inductive reasoning to integrate this data in context with other and *a priori* knowledge in order to discern a current meaning and, possibly, provide a prognosis of coming events. A major variable in this process is the analyst's personal estimation as to the validity of the reported data which he has not personally "seen."

While I&W analysts are generally knowledgeable in the technical aspects of collection sources, it is virtually impossible for the analyst to maintain detailed and current expertise in all systems. For example, in the HUMINT area, the analyst would certainly place higher confidence in a report of "out of garrison assembly of a combat engineering unit" which was submitted by an experienced U.S. Attaché who had observed and reported reliably on the installations, units and deployments in the area over the past 18 months, as compared to a report from a defector who had incidentally transited the area, or even from another attaché who was newly assigned in-country. In the case of imagery-derived information, for example, such factors as specific resolution, haze, lighting, sun angle, observation direction, or the level of expertise of the imagery interpreter may be considerations. Such source reliability information is not generally available to the I&W analyst.

The purpose of the IWCAT "Source Evaluation System" concept is to isolate one of the variables by providing the analyst with an "expert" assessment of the validity of the collected information. The "system" is represented by a "black box" containing an assemblage of

collection technology expertise – human or automated, or a combination of both. The system is assumed to have the detailed technical knowledge and day-to-day cognizance of the collection sources required to assign a quantified level of confidence to the raw data collected by each of the available sources, and to perform a high-level aggregation of these individual assessments into a single “Confidence Level” for the combined set of indications (Exhibit 3-5).

The “Source Evaluation System” is assumed to possess the following attributes:

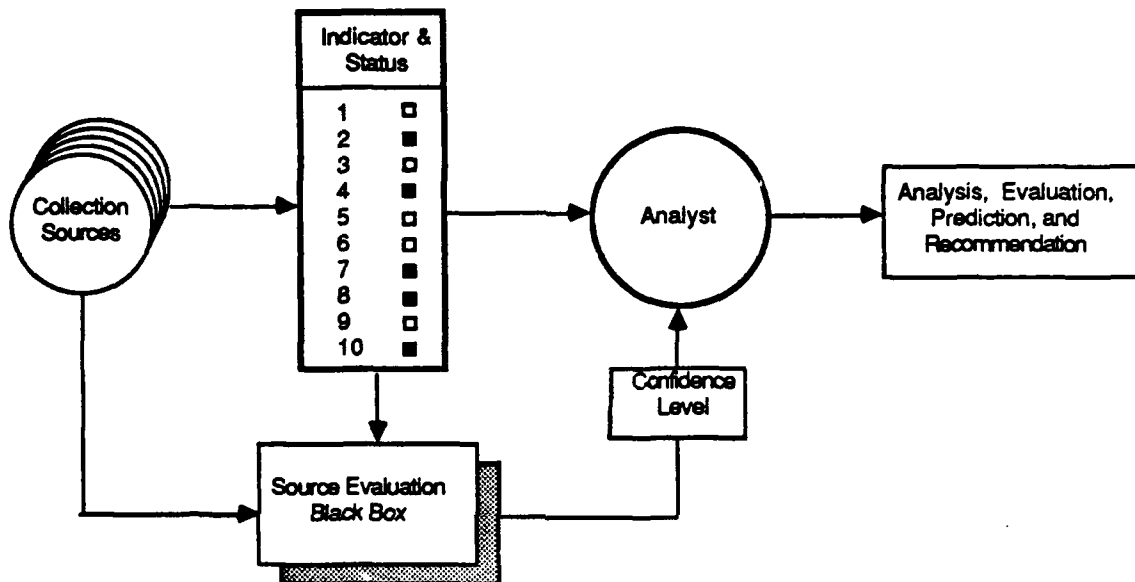
1. Collection system expertise. The “Source Evaluator” system’s expertise is solely in the area of assessing the quality and technical validity of source information. There is no capability for assigning meaning to the information or predicting the likelihood of an event.
2. Access to all available information. All facts may not be available, but the system has “seen” everything which has been reported.
3. Objectivity. Absence of optimistic or pessimistic tendencies. No outside motivations, preconceived opinions or personal agendas.
4. Integration of information. Confirmations and disconfirmations have been considered and weighted within each indicator. Ambiguities and conflicts between indicators have not been considered. (This is the analyst’s task.)
5. Familiarity with deception techniques. The vulnerability of each source and the potential for deception (maskirovka) have been considered for each indicator and have been factored into the confidence levels presented.
6. Absence of source bias. The overall “system” has equal expertise in each of the collection and exploitation techniques employed, and of the limitations implicit in each of the sources. There is no “favorite discipline” bias.
7. Normalized output. The output of the system is a percentage (1 to 100) expressing the overall level of confidence that the combined “Indicator Status” display is valid as presented.

During the OMG threat assessment activities, the analyst will be presented with a level of confidence number the value of which reflects the degree to which the particular set of data elements is considered to represent an “actual” situation of interest.

3.3.3 THE TEST DATA SETS

During the OMG threat assessment activities, the analyst will be presented with a sequence of different data sets each with a different number of active indicators and level of confidence properties which have been chosen to reflect indications of different adversarial status conditions as might be presented to I&W analysts during an investigation of whether or not an OMG was in the process of formation.

Exhibit 3-5
The "Source Evaluation System"



3.3.3.1 Number of Active Indicators

Analysis performed by Synectics personnel and a review of several unclassified documents which describe the properties of Operational Maneuver Groups (OMGs) has led to the identification of the following ten OMG-related indicators (Exhibit 3-6). These indicators are presented in a random sequence to avoid implying any preestablished order of importance or priority in the data displays.

1. Intensified reconnaissance and intelligence. Surprise, rapid movement and exploitation of weaknesses will require comprehensive current knowledge of the adversarial situation, deployments and weaknesses. While essential to effective OMG preparation, this indicator could also herald the preparations for some other type of offensive action.
2. Concentration of artillery units in FLOT (Front Line Of Troops) area. Advancement of the OMG will be preceded and accompanied by massive artillery fire in the penetration area. Assembly of artillery units in a concentrated area of the FLOT would precede the actual concentrated bombardment.
3. Alternative communications. Unexpected routing and/or "skip echelon" electronics communications to and from the suspected unit, or to and from a nearby "cover" unit indicates some "unusual" mission to be operated outside of the existing chain of command.
4. Increasing air support. Activation of reserve and alternate airfields, increased presence or movement of fighter-bombers and/or attack helicopters will be necessary to support OMG operations. These may, however, also accompany preparations for more conventional offensive operations.
5. Dummy concentrations. Establishment of bogus concentrations and feinting operations may be intended to both conceal and cover the formation of an OMG.
6. Armor assembly areas within 30-50 km of the FLOT. The OMG would be built up in an area 30 to 50 kilometers behind the enemy's own Front Line of Troops. Size of the area ranges from 200 to 400 square kilometers. Extensive concealment and cover measures used.
7. Combat engineers attached. Fast and deep movement into the area behind the FEBA will require an attached capability for rapid bridging for water crossing and other obstacle breaching operations.
8. Traffic control units and lane clearing. Surprise and rapid forward movement will require intensive traffic control if the OMG is to pass quickly among and through concentrated first echelon enemy forces on the FLOT.
9. Electronic silence. Concealment and deception will be paramount in achieving surprise. All attempts will be made to hide the presence of the forming OMG by suppressing detectable electronic emissions.
10. Electronic countermeasures and deception. Employment of deception, confusion, corner reflectors, etc., to mask the presence of the assembly area. Employment of active countermeasures, jamming, chaff, etc., as movement advances.

Exhibit 3-6

Selected I&W-Related Indicators

INDICATOR	STATUS
Intensified Reconnaissance and Intelligence	
Concentration of Artillery Units in the FLOT Area	
Alternative Communications	
Increasing Air Support	
Dummy Concentrations	
Armor Assembly Areas Within 30-50 km. of the FLOT	
Combat Engineers Attached	
Traffic Control and Lane Clearing	
Electronic Silence	
Electronic Countermeasures and Deception	

A selection of sample indicators is presented in Exhibit 3-7 which displays three samples of indicator status conditions for each number of active indicators with each indicator represented by a simulated light that can be either "on" (white rectangle) or "off" (black rectangle). Thus under number of active indicators level 10 conditions, all indicators are "on," under number of active indicators level 5, five indicators are "on," while for number of active indicators level 0, no indicators are "on." It is readily apparent from this exhibit that the same number of active indicators can support markedly different indicator patterns.

3.3.3.2 Level of Confidence

As mentioned above, the analyst will be presented with a level of confidence number the value of which reflects the degree to which the particular set of data elements is considered to represent an "actual" situation of interest (Exhibit 3-8).

3.3.3.3 Scenario Presentation

In the initial tests of the IWCAT system with analyst personnel, a single "baseline scenario" was presented to the analyst to define a hypothetical politico-military context for the warning indicator sequences. Preliminary tests, using only this one scenario (which is now described as the "Treaty Obligation" scenario, see below), resulted in an apparent distortion caused by the high "penalty" associated with issuing a false alarm. In order to provide an assessment of this factor, the data presentation program and analyst's briefing were modified to present scenarios with the three different levels of "error penalty" described below.

During the initial testing phase, and in response to analyst feedback concerning the realism of the sequences, two other changes were made to the program. Firstly, the range of minimum and maximum confidence levels was restricted and the amount of change in confidence level between any two consecutive indications was constrained in order to prevent erratic swings in the data which might have affected a change in analyst perceptions. Secondly, weather and day/night indications were added to the warning display and were programmed to show deteriorating conditions as confidence levels decreased, for example. The purpose was to provide some degree of "explanation" for the changing level of confidence values for the analyst.

As testing progressed, it became apparent that each analyst was assigning a different significance to subsets of the ten indicators as the result of the development of a subjective interpretation of the relative importance of each indicator. In order to capture and use such information, the IWCAT program was modified to allow the analyst to rank order his own interpretation of the relative importance of the indicators. This information was recorded by the program and made available in the analysis of the analyst-derived OMG threat assessment data.

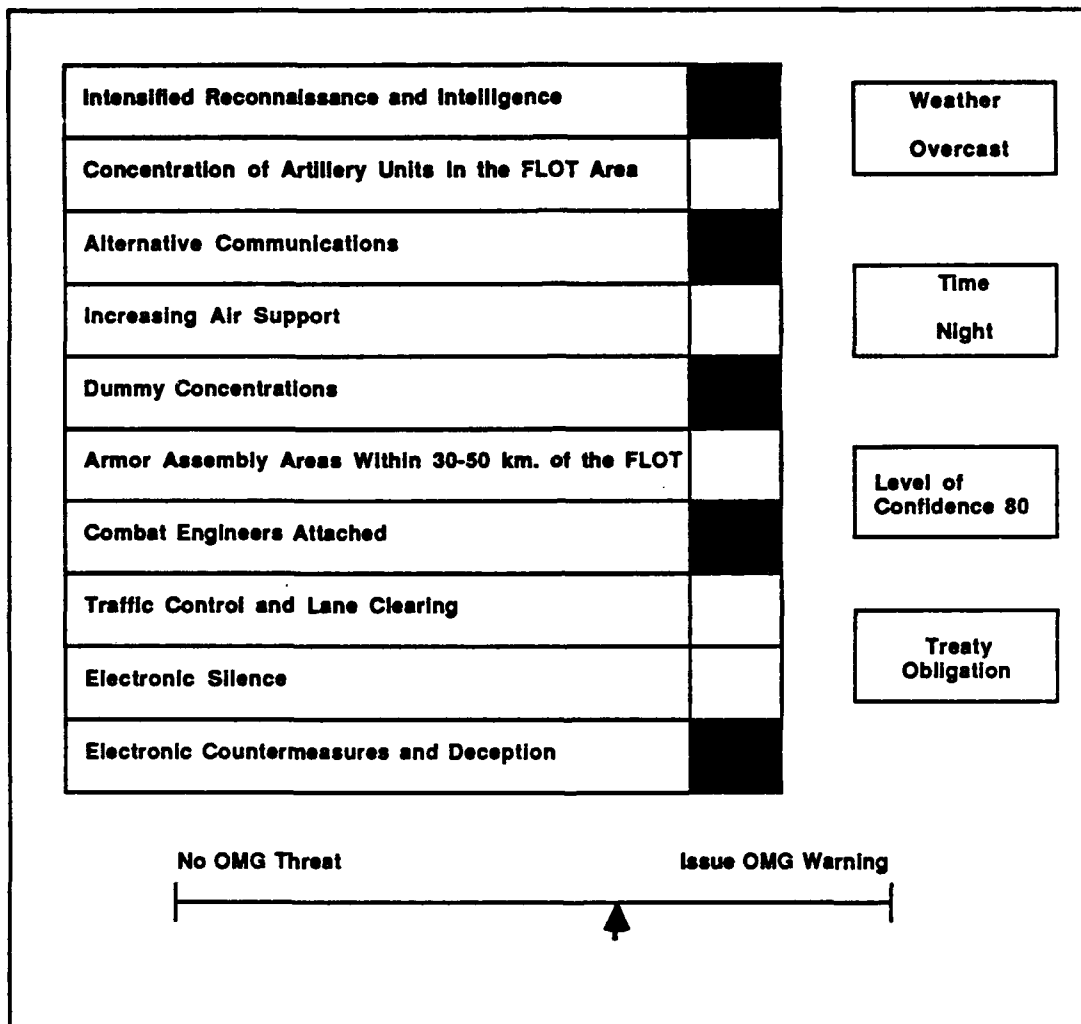
Exhibit 3-7

Indicator Activity Patterns as a Function of Number of Active Indicators



Exhibit 3-8

Components of the IWCAT Analyst Computer Display



The politico-military background for each sequence was presented as one of the following three scenario subgroup describing treaty obligation, friendly ally, and third party hostility conditions. Each of the situations represented by the indicators and other information contained in a single computer display was designed to be considered in sequence with other displays within the same scenario subgroup.

"TREATY OBLIGATION" SCENARIO

"Tensions between Aquatania and Nemonia have heightened to dangerous levels in recent weeks. Both nations, and their respective allies, are on full alert. There have been confirmed and reliable reports of massive military movements along both sides of the common border for the past four days. Nemonia has sealed all access to the Aquatanian city of Captiville, which is located entirely within Nemonia. Aquatania has sought our assistance in providing the earliest warning of any possible formation of Nemonian Operational Maneuver Groups. Under the terms of the Mutual Defense Pact, attack by a Nemonian OMG requires immediate U.S. military intervention."

"The obligation for immediate engagement by our military forces makes it imperative that the warning of impending Nemonian OMG activity be accurate and timely. Issuance of any 'false alarm' will result in our intervention, which would almost certainly precipitate a full scale engagement of the superpower military forces. Alternatively, a successful Nemonian OMG operation into Aquatania without adequate warning time to prepare defensive reaction will virtually ensure irreparable losses and collapse of the relatively weaker Aquatanian forces. Thus, there is neither margin nor place for conservative or 'safe side' analysis."

"Appropriate levels of intelligence resources have been committed to this task. The results of the most recent collections are presented. Overall confidence level and validity of the collected intelligence has been assigned by the 'Source Evaluation Expert.' Fluctuations in these confidence levels indicate the incorporation of new and possibly conflicting data elements that tend to lower the overall level of confidence."

"FRIENDLY ALLY" SCENARIO

"Mountainia is a long term ally of the United States, but no formal treaty obligations exist between the two countries for military assistance in the event of hostilities with a third party. In recent weeks, tensions between Mountainia and its aggressive neighbor Swampovia have heightened to dangerous levels. Both nations are on full alert. There have been confirmed and reliable reports of massive military movements along both sides of the common border for the past week. Mountainia has requested U.S. intelligence assistance in providing the earliest warning of any possible formation of Swampovian Operational Maneuver Groups."

"While there is no obligation to provide direct military assistance to Mountainia, it is recognized that a successful Swampovian OMG operation into Mountainia, without adequate warning time to prepare defensive reaction, would almost certainly result in collapse of the Mountainian government. For this reason, it is essential that adequate warning be provided in order to alert Mountainian military forces in time to defend against OMG operations. Given enough time, it may be possible to nullify or even dissuade such operations."

"Appropriate levels of intelligence resources have been committed to this task. The results of the most recent collections are presented. Overall confidence level and validity of the

collected intelligence has been assigned by the 'Source Evaluation Expert.' Fluctuations in these confidence levels indicate the incorporation of new and possibly conflicting data elements that tend to lower the overall level of confidence."

"THIRD PARTY HOSTILITIES" SCENARIO

"South Aquaba and Baluchistan, countries in the Middle East, have begun hostilities and appear to be entering a phase of increased military activity against each other. Both countries have received substantial amounts of military aid from the Soviet Union and the United States during the past five years. America has no treaty or other obligations to either country, but has significant levels of interest in maintaining the stability of the region. South Aquaba has asked the United States to provide intelligence information on Baluchistani troop movements in return for future price discounts in the crude oil that it produces in great abundance. Detection of a Baluchistani OMG has no direct impact on United States security, but would be of great value to the South Aquaban government."

"There is no immediate obligation for a United States military involvement in the conflict. However, an accurate and timely warning would enhance the reputation of the United States and would be of potentially great economic benefit to us. Issuance of a 'false alarm' will precipitate full-scale hostilities, with the aggression blamed on the Aquabans. By contrast, failure to detect an actual OMG operation will virtually guarantee the defeat of the Aquabans. While neither error has a direct military implication for the United States, each could result in potential long term damage to American economic and political interests."

"Appropriate levels of intelligence resources have been committed to this task. The results of the most recent collections are presented. Overall confidence level and validity of the collected intelligence has been assigned by the 'Source Evaluation Expert.' Fluctuations in these confidence levels indicate the incorporation of new and possibly conflicting data elements that tend to lower the overall level of confidence."

3.3.3.4 Sample Sequences

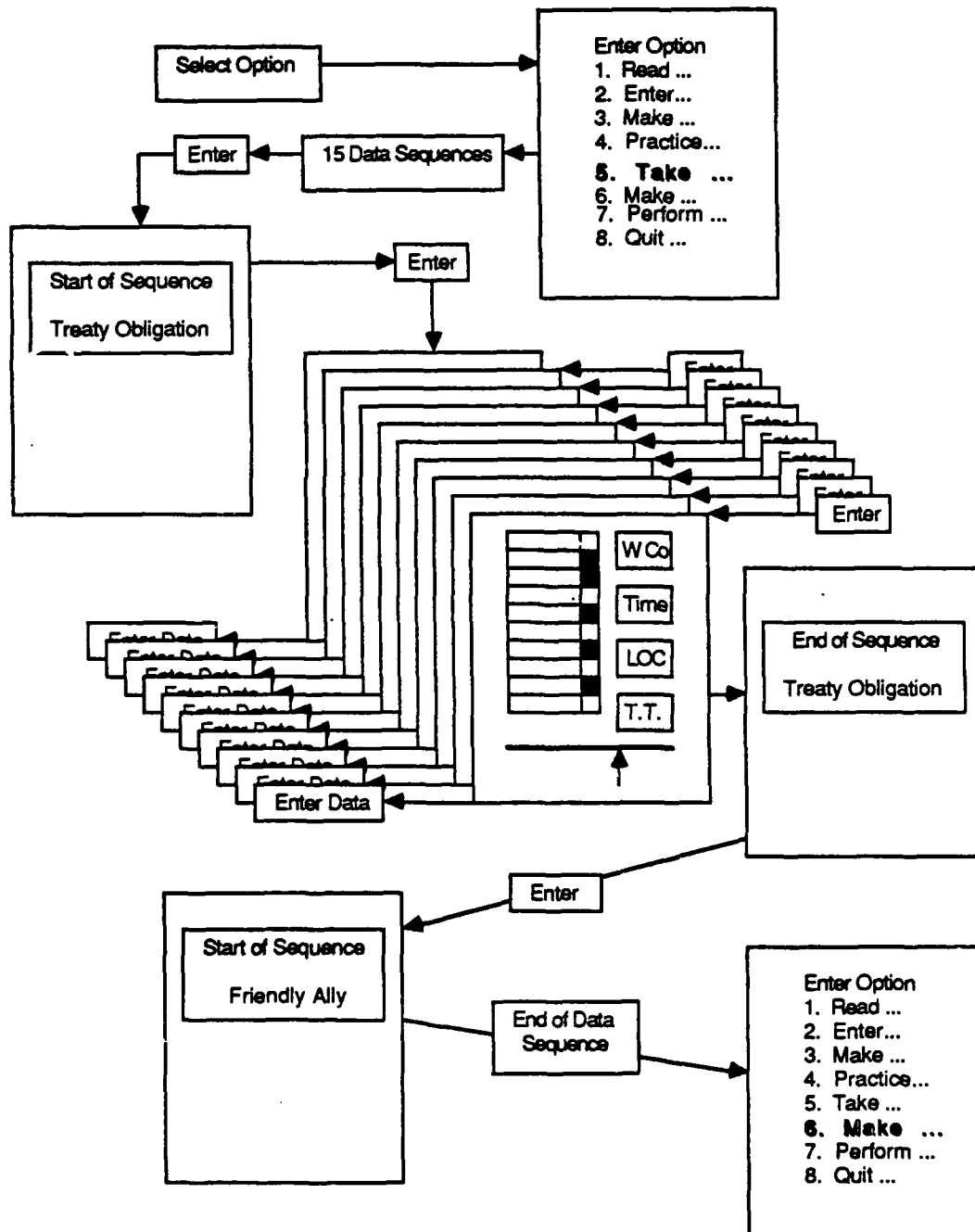
The following series of situations presents a sample of the type of I&W sequence presented to the analyst in a series of "indicator display windows." In each display, only the indicators listed would be "turned on." The confidence level for each display has been provided by the Source Evaluation System described earlier. The type of scenario (Treaty Obligation, Ally Support or Third Party Hostilities), and the attendant weather and day/night conditions are also presented on the warning display screen (Exhibit 3-8).

The analyst forms a judgment as to his or her own relative degree of certainty that the display indicates that an OMG activity is impending, and enters this on a sliding relative scale at the bottom of the display. The display software then captures this registration as a decimal number (0.0 to 1.0), stores it for subsequent statistical analysis, and advances to the next situation to be displayed in the series (Exhibit 3-9).

The sequence below is an annotated sample of the type of indication sequence presented to the analyst. In the actual sequences, the explanatory comments and summary are not presented to the analyst. The actual indicator sequences generally varied randomly between eight and ten situations in length. The type of scenario and weather, day/night conditions were also presented on each display screen (see Exhibits 3-8 and 3-9, for example).

Exhibit 3-9

Analyst OMG Threat Assessment and Data Base Formation Activities



SAMPLE SEQUENCE
Treaty Obligation Scenario

Situation 1

Confidence Level = 30

<u>Indicators</u>	<u>Comments</u>
1	Increase of reconnaissance and intelligence activities may indicate preparation for offensive action. Not clearly OMG associated.
3	Alternative communications indicate some force reconfiguration or unusual operations. Could be OMG associated, or not, under the current circumstances.
4	Increased preparation and positioning for fighter-bomber or attack helicopter support are also indicative of possible offensive preparations.
<hr/>	
Summary:	Confidence level of the information is marginal. The indicators are fairly ambiguous, in that they may signify <i>some</i> type of offensive preparation, not necessarily OMG. However, offensive preparations coupled with alternative communications, which may indicate a "special mission", should raise these early indications above the "noise level" enough to alert the analyst to the possibility of OMG activity.

Situation 2

Confidence Level = 50

<u>Indicators</u>	<u>Comments</u>
1	Reconnaissance activities continue.
2	Concentration of artillery units in the frontal area lends weight to the possibility of offensive preparations.
3	Alternative communications continue.
4	Air support activity continues.
<hr/>	
Summary:	Introduction of the artillery units and the increased confidence level should add some measurable increase to the probability of impending offensive action. Still relatively weak indication of OMG activity.

Situation 3

Confidence Level = 60

<u>Indicators</u>	<u>Comments</u>
1	As above.
2	As above.
3	As above.
8	Dummy units and feinting operations also indicate a significant effort to conceal or cover some other activity.
10	Employment of countermeasures and deception tactics indicate strong and overt efforts to deny information of some movement or activity.
<hr/>	
Summary:	Indicators support the preparation for some unusual activity. Secondary indications for OMG activity, but still no direct evidence uniquely attributable to OMG formation.

Situation 4

Confidence Level = 40

<u>Indicators</u>	<u>Comments</u>
1	As above.
2	As above.
5	As above.
6	Assembly of armor unit(s) close to the FLOT could be an indication that the preceding activity may have been OMG associated.
10	As above.
<hr/>	
Summary:	Detection of the armor assembly area adds a critical element, but the relatively weak confidence level lends an element of complication to the problem. How long has this been going on before detection? What happened to the alternative communications?

Situation 5

Confidence Level = 70

<u>Indicators</u>	<u>Comments</u>
1	As above.
4	Resumption (or rediscovery) of air support activities further supports the possibility of impending offensive operations.
6	As above.
7	Attachment of combat engineers at this point provides a significant indication of possible OMG activity.
9	Electronic silence further indicates efforts to conceal the presence, strength composition and purpose of the unit.
10	As above.
<hr/>	
Summary:	The overall scenario should now represent a fairly high probability that OMG activity may be in progress. The medium-high confidence, coupled with the absence of some key indicators and the disastrous consequences of issuing a false alarm should, however, still give the analyst some significant cause for concern and caution.

Situation 6

Confidence Level = 80

<u>Indicators</u>	<u>Comments</u>
4	As above.
5	Resumption (or redetection of on going) feinting and decoy units continue to indicate efforts to divert attention from some other significant activity.
6	As above.
7	As above.
8	Traffic control activity and the clearing of lanes through the FLOT are indicative of preparation for the difficult maneuver of moving some unit through the established FLOT.
9	As above.
10	As above.

Summary: Together, the indications now point strongly, but perhaps not conclusively, to an impending OMG attack. The absence of indicators 2 and 7 provide fairly weak contra-indications. Still, there is enough information missing to preclude an "easy call." On the other hand, the additional information needed to provide certainty may never come, or may come too late to provide the warning time needed for essential defensive reaction.

3.3.4 THE COLLECTION OF TEST ASSESSMENTS

In order to determine the reaction of I&W analysts to particular types of data, a selection of test data sets, each with different numbers of active indicators and level of confidence information, was presented to the individuals participating in the knowledge development activity. The individuals were involved in an initial period of training and asked to review each element of the data set for a short time. They were then asked to provide an assessment of the posture of an adversary as represented by the data to which they have access. This assessment is made by indicating a position on the following scale which most closely describes the analyst's assessment of the nature of the input I&W-related data sets.



A large number of test data sets were developed by the project staff with the aid of a dedicated scenario generator and stored in the IWCAT computer for assessment by selected I&W analysts. The presentation of such data to the analysts involves a preliminary training and familiarization session in which the analysts are introduced to the type of display to be used in the actual analysis. During the actual assessment activity, the analyst is asked to record his or her perception of the level of threat represented by each set of indicators and associated information and this assessment is stored in the OMG Threat Assessment Data Base (Exhibit 3-10).

3.3.5 THE PROCESSING OF OMG ASSESSMENT DATA

The results for each individual participant are tabulated, recorded, and analyzed with the aid of a statistical catastrophe theory-based computer program developed by Cobb in order to determine whether the data can be described with the aid of a linear model, or whether the data could be described more appropriately with the aid of a model based on the cusp catastrophe manifold. This program, which was initially written in FORTRAN for use on a number of computers by Cobb, has been adapted and rewritten in Turbo Pascal during the IWCAT effort to run on an IBM PC/AT computer. Synectics has been informed by RADC that the IBM PC/AT computer is compatible with the environment provided by the RADC intelligence station (IWS), the eventual host for an implemented IWCAT environment. It is fully expected that the IWCAT software will be capable of transfer from the IBM AT to the IWS facility.

The following is a conceptual outline of processes involved in the analysis of the OMG Threat Assessment Data Base with the aid of procedures based on statistical catastrophe theory. As described above, test data sets constructed to reflect differences in the number of active

indicators and level of confidence will be presented to analysts and will serve as the basis for the formation of the OMG Threat Assessment Data Base. Exhibit 3-10 describes conditions under which four different sets of indicators with their associated level of confidence values (D(a), LOC(B); D(b), LOC(D); D(c), LOC(A); and D(d), LOC(C), respectively) have been presented to an analyst. These four different data sets are considered to have generated four distinct assessments of the likely formation of an OMG and these assessments are assumed to have been recorded in the OMG Threat Assessment Data Base and statistically analyzed (Exhibits 3-10 and 3-11). The number of active indicators and level of confidence parameters have been considered to form the axes of the control space associated with the cusp catastrophe manifold.

The conceptual result of this analysis of the assessment data is illustrated in Exhibit 3-11 and is based on the assumption that the statistical procedures have rejected as unsuitable a linear model for the data and have determined that such data could be more appropriately described with the aid of the cusp catastrophe manifold. Under such circumstances, it appears that presentation of the data set (D(c), LOC(A)) has resulted in the creation of significant levels of ambiguity regarding the formation of an OMG, a circumstance represented by the existence of multiple points of intersection of a line drawn from the control plane of the catastrophe to the catastrophe manifold surface (Exhibit 3-11). Under these conditions, two different analysts might have markedly different perceptions of the meaning of this data set under the same conditions: one analyst might infer that OMG formation was very likely while the other might infer that such an event was very unlikely, for example. By contrast, presentation of the data sets D(b), LOC(D); D(d), LOC(C); and D(a), LOC(B) have produced no such ambiguity.

Exhibit 3-10

Relationship of the OMG Threat Assessment Data Base to the Cusp Catastrophe Control Plane

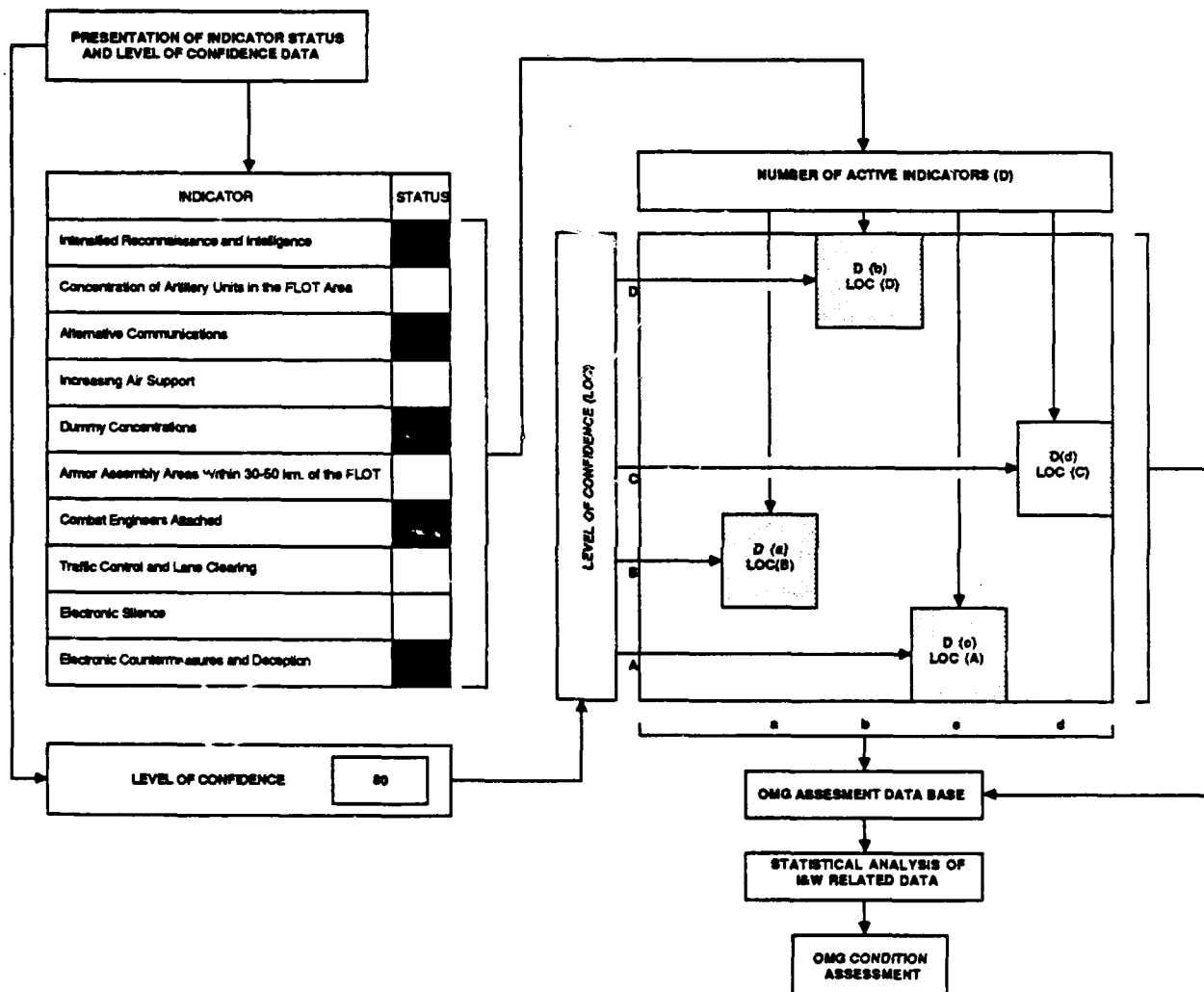
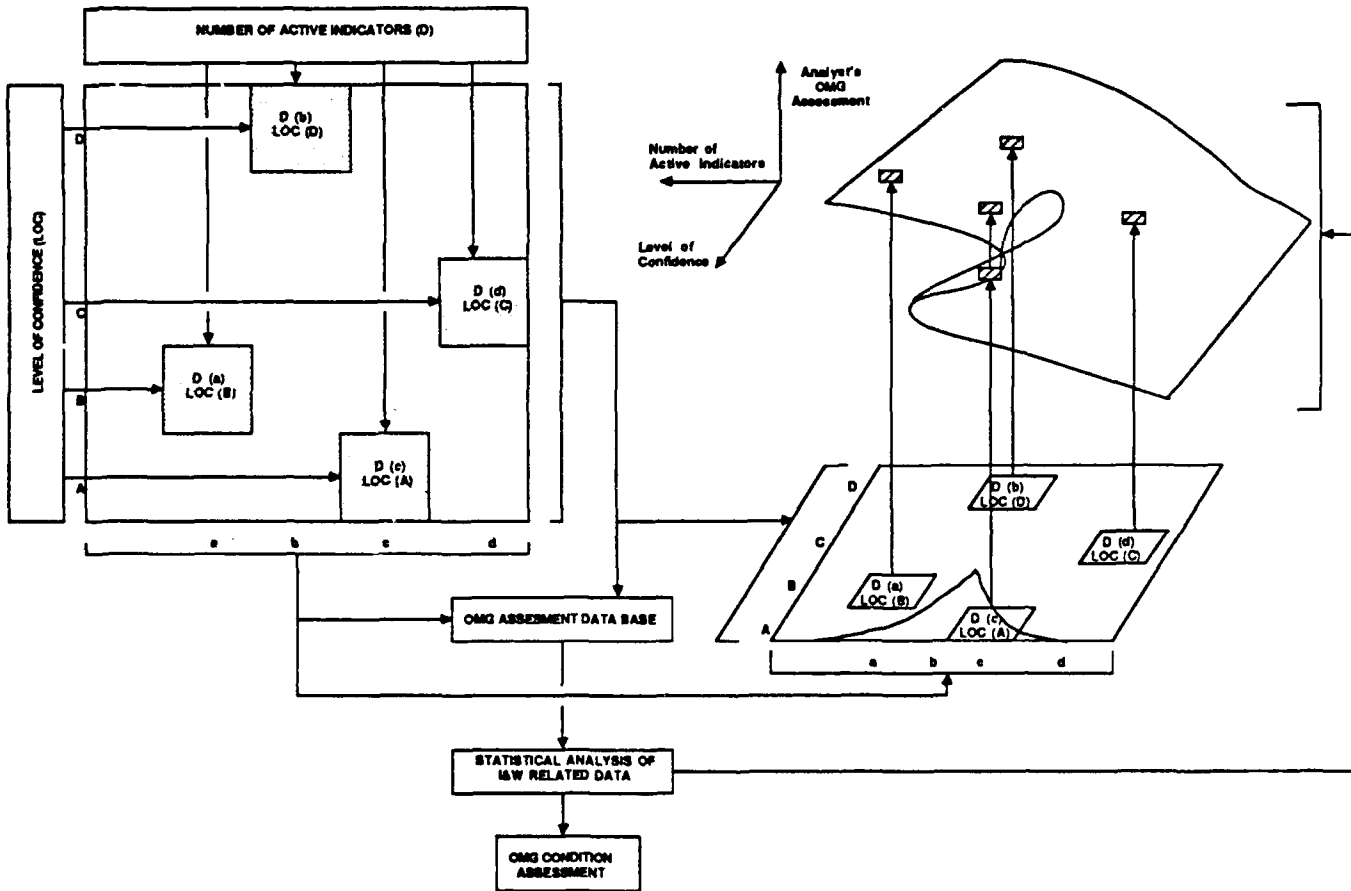


Exhibit 3-11

Fitting the OMG Threat Assessment Data Base to the Cusp Catastrophe Manifold



SECTION 4. CUSP SURFACE ANALYSIS

The analysis of the OMG Threat Assessment Data Base will be performed with the aid of a program based on statistical catastrophe theory. The following section of the report describes some technical aspects of this program and presents sample output derived from its use to analyze a generic test data set.

4.1 PRELIMINARY NOTES ON TERMINOLOGY

The following notes outline some of the major features of the terminology used for the statistical catastrophe theory-based analysis of data.

1. The term "behavioral variable" in the literature on applied catastrophe theory is synonymous with the statistical term "dependent variable."
2. The term "control variable" is similarly synonymous with the statistical term "independent variable."
3. MLE stands for Maximum Likelihood Estimation.
4. NR stands for Newton-Raphson (an optimization method).
5. PDF stands for Probability Density Function.
6. Boldface (\mathbf{X} , \mathbf{Y}) indicates a random variable or function thereof.
7. Underlining ($\underline{\mathbf{X}}$, $\underline{\mathbf{Y}}$) indicates a vector. The dimension of the vector depends on the context.
8. $\mathbf{Y}|\mathbf{X}$ refers to the conditional random variable \mathbf{Y} given \mathbf{X} . The PDF of such a random variable would be written as $f(y|x)$.

4.2 INTRODUCTION

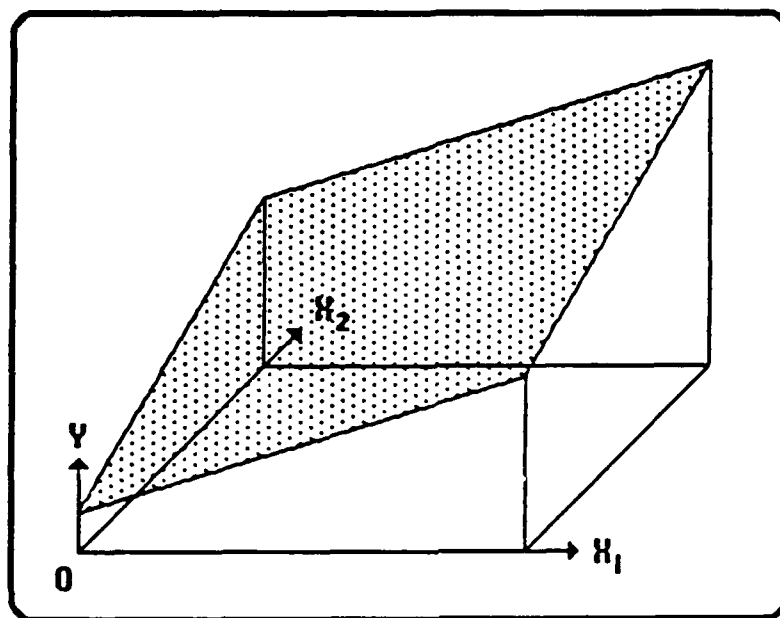
A catastrophe manifold or "cusp surface" is a statistical model derived from catastrophe theory. In this model there is one dependent variable (denoted \mathbf{Y}) and an arbitrary number of independent variables (denoted \mathbf{X}_i , $i=1,\dots,v$). Perhaps the best way to introduce cusp surface analysis is to contrast it with multiple linear regression. The regression model for the effect of the independent variables on \mathbf{Y} is:

$$\mathbf{Y} = \beta_0 + \beta_1\mathbf{X}_1 + \dots + \beta_v\mathbf{X}_v + \mathbf{U}. \quad (1)$$

In this model the random variable \mathbf{U} is assumed to be normally distributed with mean zero, and the independent variables \mathbf{X}_i are fixed (i.e., not random). With this model the response surface of \mathbf{Y} is flat in every direction, and the slope of \mathbf{Y} with respect to \mathbf{X}_i is β_i (Exhibit 4-1).

Exhibit 4-1

The Linear Model Used by Multiple Regression



The regression model described by Equation (1) has $v+2$ degrees of freedom (one for each of the freely varying coefficients, plus one for the variance of U). For our purposes here this model will be referred to as the *linear* model, meaning that Y is a linear function of the independent variables X_i . (This use of the term "linear model" differs from the usual statistical usage, in which it means that Y is a linear function of each of the parameters.) Cusp surface analysis actually begins with the linear model: the first step is the estimation of the coefficients β_i of Equation (1). The linear model is the standard against which the catastrophe model will be compared; thus it will be, in statistical terms, the null hypothesis.

The cusp catastrophe model is a response surface that contains a smooth pleat, as in Exhibit 4-2. To obtain this amount of flexibility it is necessary to introduce $2v+2$ additional degrees of freedom in the model. This is done by defining three "control factors," each a scalar-valued function of the vector \mathbf{X} of independent variables:

$$\begin{aligned} A(\mathbf{X}) &= A_0 + A_1X_1 + \dots + A_vX_v, \\ B(\mathbf{X}) &= B_0 + B_1X_1 + \dots + B_vX_v, \\ C(\mathbf{X}) &= C_0 + C_1X_1 + \dots + C_vX_v. \end{aligned}$$

These factors determine the predicted values of Y in this sense: the predicted values of Y given $\mathbf{X} = (X_1, \dots, X_v)$ are the solutions of

$$0 = A(\mathbf{X}) + B(\mathbf{X})[Y - C(\mathbf{X})] - D[Y - C(\mathbf{X})]^3. \quad (2)$$

This prediction equation is a cubic polynomial in Y , which means that for each value of \mathbf{X} there are either one or three predicted values of Y , as seen in Exhibit 4-2.

The individual effects of the factors A and B on Y can be seen in Exhibits 4-3a and 4-3b. For simplicity, it is assumed in these figures that $C = 0$ and $D = 1$. Note that the panels in these figures represent vertical slices cut through the canonical cusp surface (Exhibit 4-2). Exhibit 4-3a illustrates slices that are parallel to the A axis of Exhibit 4-2, while the slices depicted in Exhibit 4-3b are parallel to the B axis.

The individual effects of the factors A and B are depicted in Exhibits 4-3a and 4-3b as if they were independent, but according to the assumptions of the model both A and B depend on \mathbf{X} . However, a comprehensive discussion of the effect each component of \mathbf{X} on Y will be presented below.

According to the terminology generally used in the literature on catastrophe theory, A and B are closely related to the so called *normal* and *splitting* factors, respectively (the term "normal" is used simply because this factor is perpendicular, i.e., normal to the splitting factor in the canonical representation of the control space). It is recommended that this terminology be avoided for two reasons: (a) to prevent confusion with the normal distribution, and (b) to emphasize the fact that the statistical model is not as flexible as the topological model. Within this report, A , B , and C will be called the *asymmetry*, *bifurcation*, and *linear* factors, respectively.

Those familiar with the literature on applications of catastrophe theory may wonder about the linear factor C and the coefficient D in Equation (2), since they are not ordinarily seen in the equations which define the canonical cusp catastrophe model. The usual equation for the canonical cusp surface, which in terms of our variables would be written as the following:

$$0 = a + by - y^3, \quad (3)$$

Exhibit 4-2
The Cusp Catastrophe Model

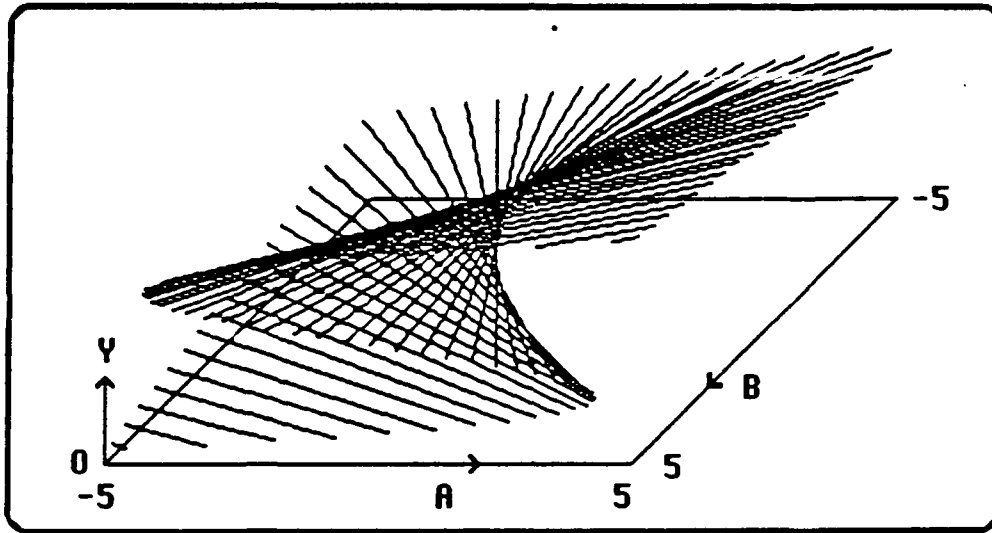
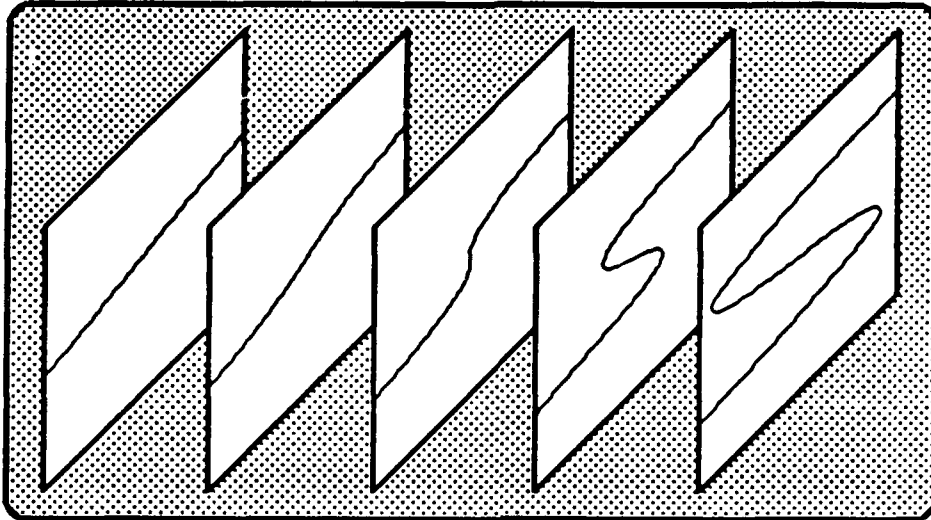
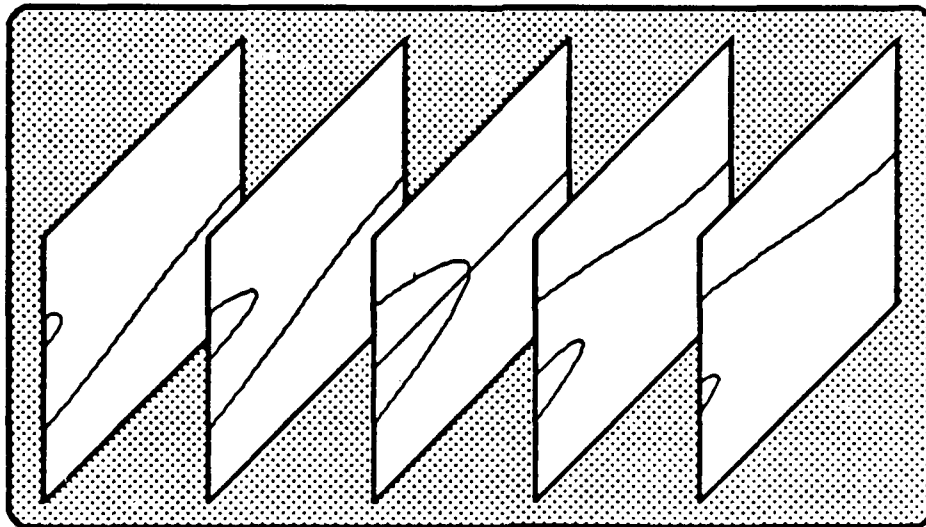


Exhibit 4-3

Sections Through the Cusp Catastrophe Model



3a: The effect of A on Y, with $B = -5, -2.5, 0, 2.5, 5$.



3b: The effect of B on Y, with $A = -3, -1.5, 0, 1.5, 3$.

does *not* represent the relationship between the original control variables and the original behavioral variable, as they might have been measured in the laboratory. Instead, the variables a , b , and y are obtained from the originals by a certain kind of transformation (known as a fiber-preserving diffeomorphism). This nonlinear transformation smoothly and invertibly adjusts the coordinate system so that the shape of the original response surface matches Exhibit 4-2 near the cusp catastrophe point ($a = b = y = 0$). In so doing the original relationship, which may not have been expressible as a polynomial at all, becomes ($a + by - y^3 = 0$) in terms of the transformed variables.

The canonical variable y of Equation (3) is a function of both the original behavioral variable *and the original control variables*, while the canonical control variables a and b are each functions of all of the original control variables. This kind of functional dependence is reflected in Equation (2), with the restriction that these functions are merely affine transformations, not diffeomorphisms. This is the principal difference between the statistical model, which uses Equation (2), and the topological model. A further difference is that the topological model requires that $v = 2$, whereas the statistical model puts no restriction on v .

The catastrophe model defined by Equation (2) can be seen as a generalization of the regression model (Equation (1)): the two models coincide if (1) $A = 0$, (2) $B = \text{constant}$, and (3) $D = 0$. When these conditions are satisfied the coefficients C_i of C are the same as the coefficients B_i of Equation (1). In fact, these are the initial values used by the maximum likelihood method when it begins its iterative search for the best fitting coefficients for the catastrophe model (reciprocal of the estimated variance of U is used as the initial value of B_0).

The statistical model based on Equation (2) is, like the regression model described by Equation (1), a static random model. It is useful to remember that the static catastrophe model is related to, and indeed derived from, a dynamic model. The (deterministic) dynamic cusp catastrophe model is described by a differential equation:

$$dy/dt = a(x) + b(x)[y(t)-c(x)] - d[y(t)-c(x)]^3. \quad (4)$$

In this formulation a , b , and c are scalar-valued functions of the vector x , and d is a scalar. For each value of x this dynamical system has either one or three equilibria. These are the values of y for which $dy/dt = 0$. Thus the equilibria of the dynamical system correspond *exactly* to the predicted values of the static system.

One could use the theory of stochastic differential or difference equations to derive from Equation (4) a statistical model appropriate for nonlinear time series analysis, but that is a topic best left for another monograph. The approach we take here assumes that a time series is not available, and that the data take the form of a random sample of statistically independent replicates of the system, taken at one point in time. Thus we are considering the static random case. To complete the specification of the statistical model, it remains to specify the probability density function for the random variables in Equation (2).

4.3 ESTIMATING THE PARAMETERS

Cusp surface analysis uses the method of maximum likelihood to estimate the parameters in Equation (2). The values of x are assumed to be fixed experimentally or measured without error (thus for estimation purposes, x is not considered a random variable).

The conditional probability density function for Y given X is assumed to be the Type N3 density in the topology given in [Cobb, Koppstein, and Chen, 1983]. This PDF has either one mode or two modes separated by an antimode. These modes and antimodes are precisely the solutions to Equation (2) (see Exhibit 4-4, for example). Therefore the predictions made by the cusp model are the modal values of the conditional density of Y given X . The antimode is an "antiprediction" – a value that is specifically identified as "not likely to be seen."

The differences and similarities between linear regression and cusp surface analysis are worth careful examination. The conditional PDF of $Y|X$ in linear regression is normal, i.e., Type N_1 (an exponential of a quadratic), while the conditional PDF of $Y|X$ in Cusp Surface Analysis is Type N_3 (an exponential of a quartic). The predicted values in linear regression are the means of the conditional densities, which also happen to be modes, while in cusp surface analysis the predicted values are modes (and the densities are frequently bimodal, yielding two predictions). Lastly, in linear regression the formulas for the sampling variance of the estimators are known exactly, while in cusp surface analysis the corresponding formulas are approximations.

The cusp surface analysis program begins with the estimated coefficients of the linear regression model, and iterates towards the parameter vector that maximizes the likelihood of the cusp model given the observed data. The iterative scheme is a modified Newton-Raphson method. If the very first iteration yields a decrease in the the likelihood function, the program immediately halts with a message indicating that the linear model is preferable to any cusp model (this is not a rare occurrence). Upon successful convergence to a parameter vector that maximizes the likelihood function, the estimated coefficients of each factor are printed.

4.4 TESTING THE MODEL

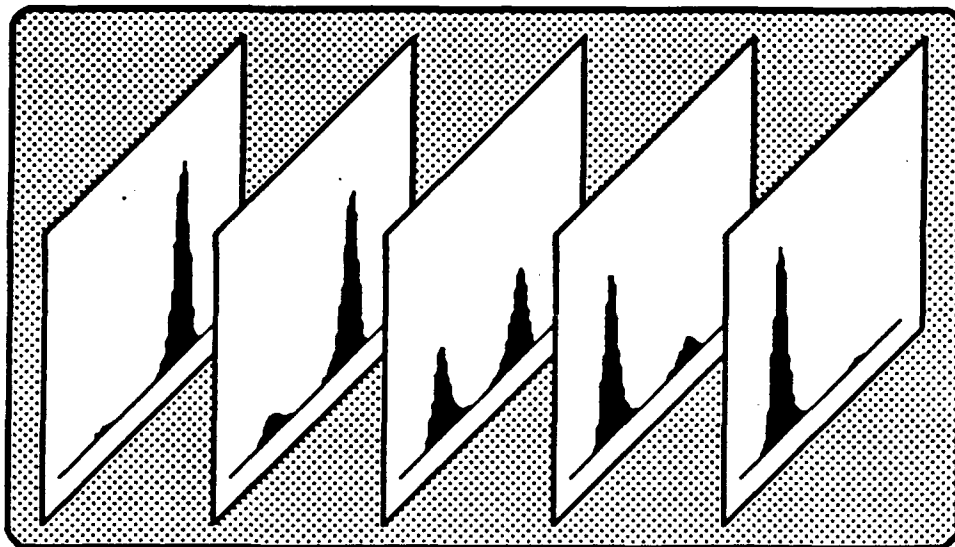
The parameter estimates reported by the cusp surface analysis program are useful for generating predictions, as described in the next section, but their values indicate nothing about their statistical significance. Therefore the program also reports an approximate t-statistic for each coefficient, with $N-3v-3$ degrees of freedom. These can be interpreted in the usual fashion: magnitudes in excess of the critical value indicate that the coefficient is significantly different from zero, at the specified significance level (remember that these t-statistics are only approximate). Of course, these statistics can also be misinterpreted in the usual ways too. For example, it is a mistake to pay attention to any of these values unless the overall model has passed all of its tests for acceptability. We now turn to these more general tests.

There is no single definitive statistical test for the acceptability of a catastrophe model. Part of the difficulty stems from the fact that a catastrophe model generally offers more than one predicted value for Y given X . This makes it difficult to find a tractable definition for prediction error, without which all goodness-of-fit measures that are based on the concept of prediction error (e.g., mean squared error) are nearly useless. Another part of the difficulty arises from the fact that the statistical model is not linear in its parameters. And finally, of course, it is scientifically unsound to base any definitive statement on the analysis of a single data set, no matter what its statistics show. Confirmation must always be sought in the independent replication of results. In spite of these difficulties and caveats however, there are a variety of ways in which a catastrophe model may be tested through statistical means.

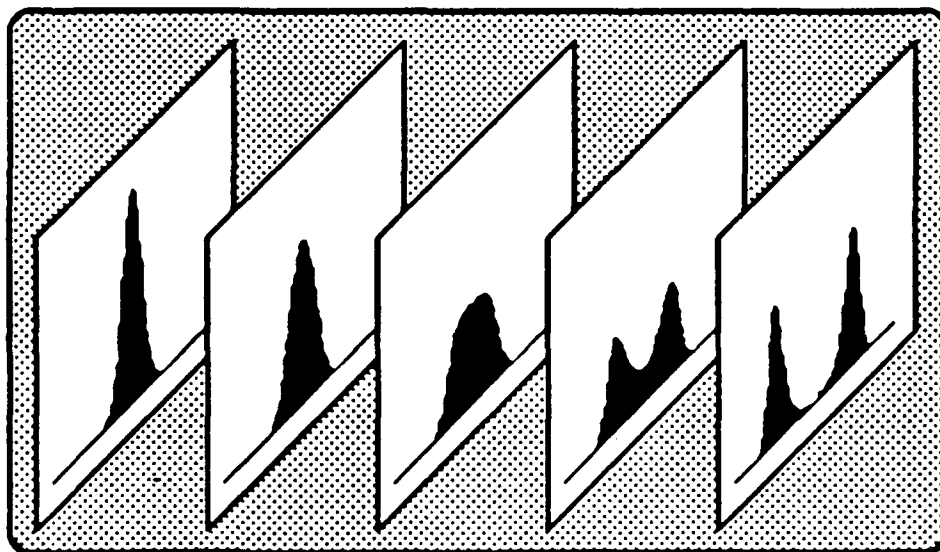
Cusp surface analysis offers three separate tests to assist the user in evaluating the overall acceptability of the cusp catastrophe model (Exhibit 4-5, for example). The first test is based on a comparison of the likelihood of the cusp model with the likelihood of the linear model. The test statistic is an "asymptotic chi-square," which means that as the sample size

Exhibit 4-4

The Type N_3 PDF for the Cusp Catastrophe Model



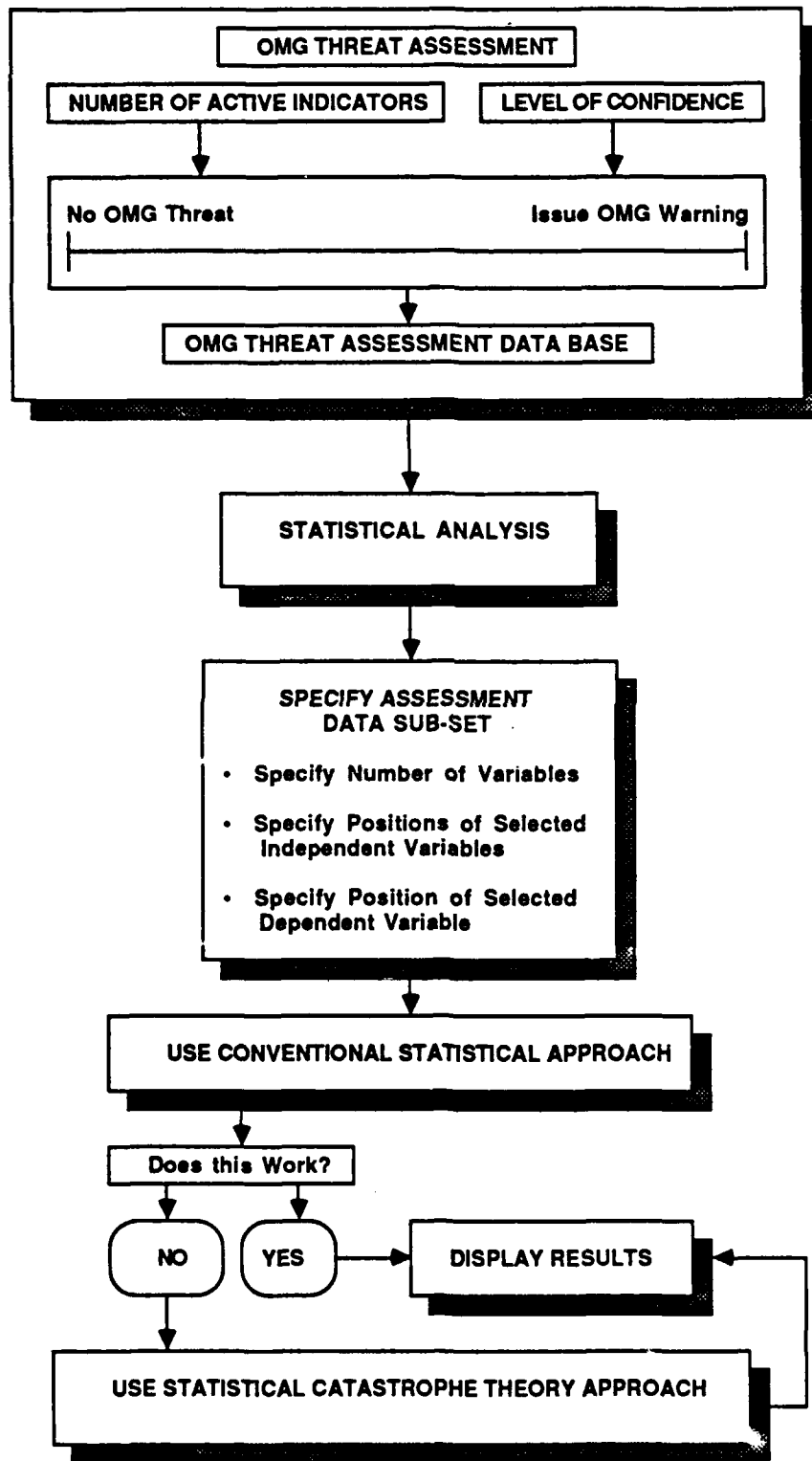
4a: The Type N_3 PDF for the cusp catastrophe model, with $A = (1, .5, 0, -.5, -1)$, $B = 4$, $C = 0$, and $D = 1$.



4b: The Type N_3 PDF for the cusp catastrophe model, with $A = 0$, $B = (-2, -1, 0, 1, 2)$, $C = 0$, and $D = 1$.

Exhibit 4-5

A Conventional or a Catastrophe Theory-Based Approach?



increases the distribution of the test statistic converges to the chi-square distribution. The degrees of freedom for this chi-square statistic is the difference in the degrees of freedom for the two models being compared, i.e., $2v+2$. Sufficiently large values of this statistic indicate that the cusp model has a significantly greater likelihood than the linear model.

Even if the first test shows that Equation (2) is superior to (1), the correct model may still not be the cusp catastrophe model. This can happen if the estimate for D is not significantly different from zero, which it must be for the right hand side of Equation (2) to be a cubic polynomial in Y . If D were zero the prediction equation for Y would read like this:

$$Y = C(X) - A(X)/B(X). \quad (5)$$

This is clearly not a catastrophe model, since for each value of X there is precisely one value of Y (which is infinite whenever $B = 0$). The second test for the adequacy of the cusp model is, therefore, a comparison of D with zero. As it turns out, D must be positive for a PDF of Type N_3 to be defined at all. Thus the appropriate test is a one-tailed t-test using the approximate standard error of D . The degrees of freedom for this test is the sample size less the remaining degrees of freedom of the cusp model after D has been fixed at zero, i.e., $N-3v-3$.

Even if the model passes both of the preceding tests, it can happen that the only coefficients of the factors A and B which are significantly different from zero are the constants A_0 and B_0 . When this happens it is fair to say that the PDF of Y is indeed Type N_3 , but that the variables (if any) which cause this density to shift between bimodal and unimodal have not been included in the given list of independent variables. In this case the factor C determines the slope of the flat response surface. If the PDF is bimodal then there are two parallel response surfaces.

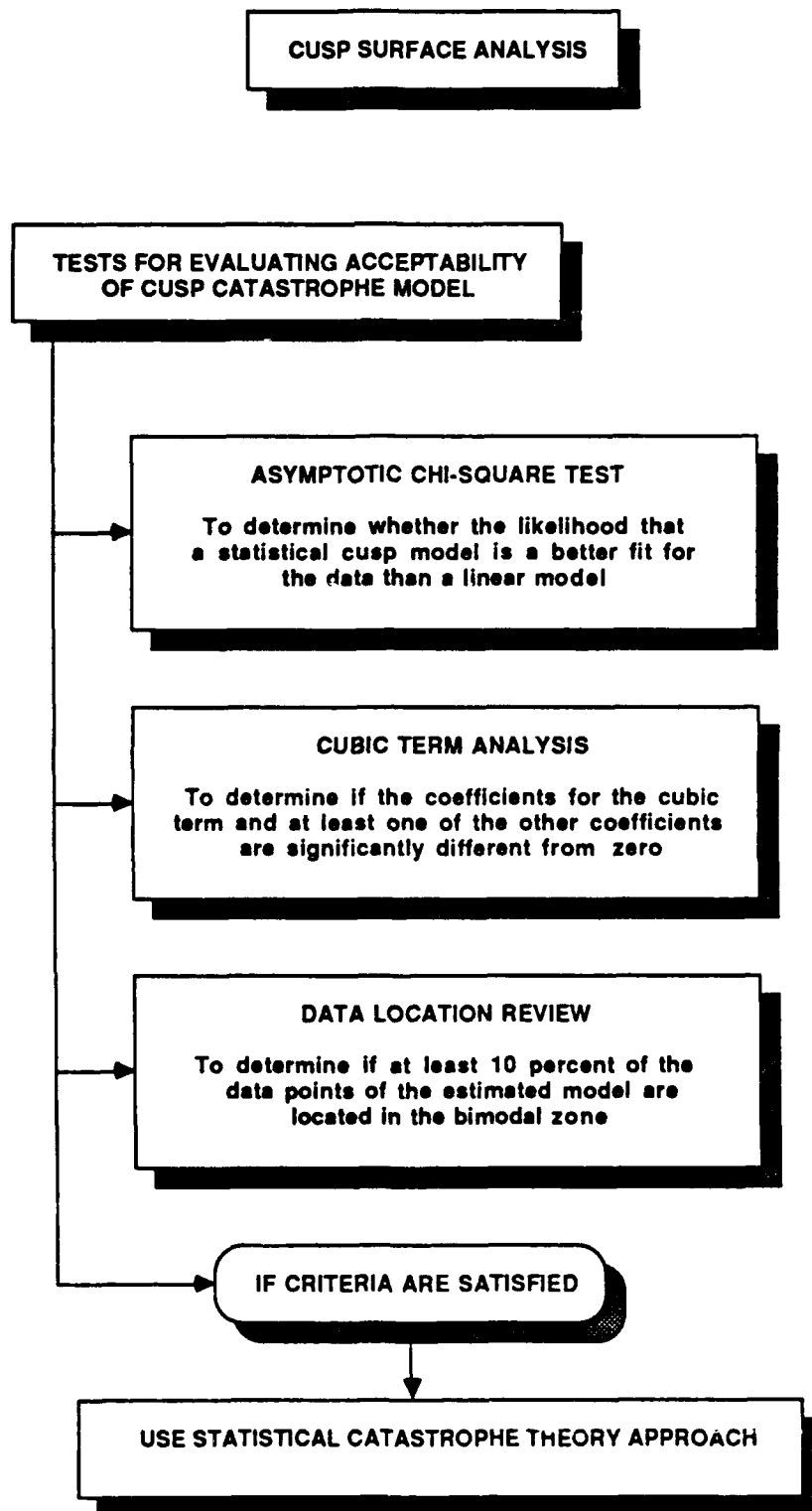
Lastly, it may happen that both of the above tests may yield positive results for the cusp model, implying that the cubic polynomial in Equation (2) is correct, while none of the observed values for X lie in the bimodal zone! In this event the location of the bimodal zone is only known by extrapolation from the given data. Since extrapolation is notoriously unreliable in statistical work, it is reasonable to require that some observations of X lie within the bimodal zone, while others lie outside. To assist the user in perceiving the estimated distribution of the data within the plane spanned by the asymmetry and bifurcation factors, the cusp surface analysis program shows this 2-dimensional distribution, with the bimodal zone highlighted. As a rule of thumb, it is desirable to have at least 10% of the observations fall within the bimodal zone. This constitutes the third test.

To summarize, the cusp catastrophe model may be said to describe the relationship between a dependent variable Y and and vector X of independent variables if all of these three conditions hold (Exhibit 4-6):

1. The chi-square test shows that the likelihood of the cusp model is significantly higher than that of the linear model.
2. The coefficient for the cubic term and at least one of the coefficients of the factors A and B are significantly different from zero.
3. At least 10% of the data points in the estimated model fall in the bimodal zone.

Exhibit 4-6

Criteria for Acceptance of the Cusp Catastrophe-Based Model



4.5 MAKING PREDICTIONS

In the literature on applications of catastrophe theory there are two distinct ways of calculating predicted values from a catastrophe model. In the **Maxwell Convention** the predicted value is the most likely value, i.e., the position of the highest mode of the probability density function. In the **Delay Convention** a mode is also the predicted value, but it is not necessarily the highest one. Instead, the predicted value is the mode that is located on the same side of the antimode as the observed value of the state variable Y . Thus the delay convention uses as its predicted value the equilibrium point towards which the equivalent dynamical system would have moved. This is the convention most commonly adopted in applications of catastrophe theory, but there are circumstances in which the Maxwell convention is the appropriate one (Exhibit 4-7).

The cusp surface analysis program calculates the predictions made under each convention for each datum, and from these it derives a number of statistics and graphs to aid the user in evaluating the quality of the predictions made under each convention, as follows:

Modes and Antimodes: The estimated factors and modes and antimodes of the data are presented to the user in tabular form.

Delay- R^2 : This statistic is simply the estimated value of the quantity $1 - (\text{error variance})/\text{var}[Y]$, in which the errors are based on predictions of the delay rule. Although it is analogous to the multiple- R^2 of regression analysis, there are several important differences which are discussed below.

Maxwell- R^2 : This is the corresponding statistic for the predictions of the Maxwell rule.

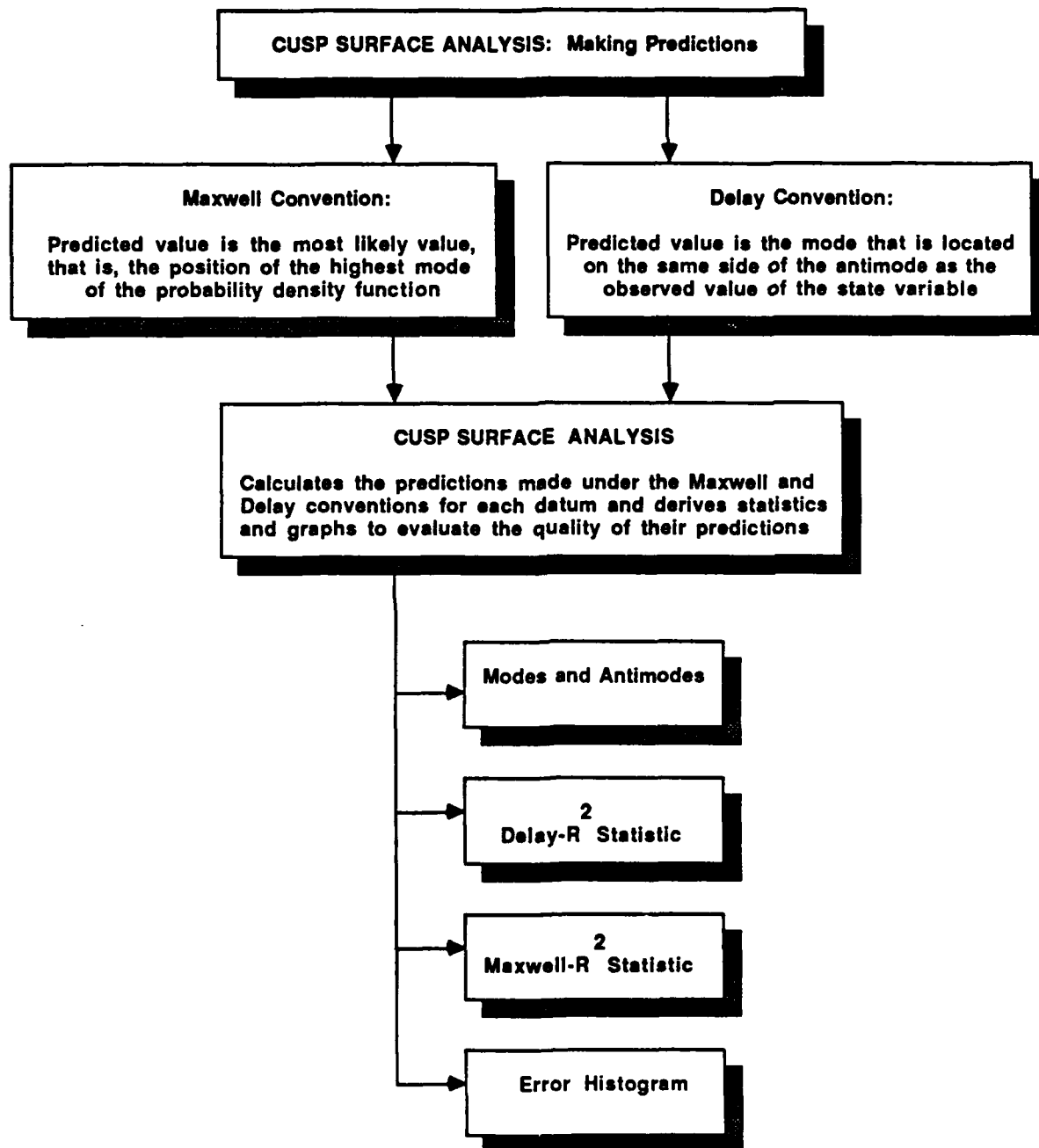
Error Histogram: This graph depicts the distribution of the errors made under the delay rule. Also provided on this page are the mean and standard deviation of this distribution. These statistics are also known as the bias and standard error of estimate, respectively, of the estimates.

In multiple linear regression the estimates obtained by minimizing the mean squared prediction error coincide with those obtained by maximizing the likelihood function. In cusp surface analysis this is not the case, and therefore, the maximum likelihood estimates maximize neither the Delay- R^2 nor the Maxwell- R^2 . Further, neither of these statistics is even guaranteed to be positive! Negative values occur when the cusp model fits so wretchedly that its error variance actually exceeds the variance of Y .

Users of the Cusp Surface Analysis program are urged to try analyzing random data of various types to learn more about how the program works. One such exercise, for example, is to create a set of data that are uniformly distributed about a linear trend. In this case the analysis should yield the correct slopes in the linear factor and a positive bifurcation constant, indicating a model with two parallel prediction lines. Experimentation of this sort will also drive home the importance of having a sufficiently large sample size.

Exhibit 4-7

Cusp Surface Analysis: Making Predictions



A particularly interesting experiment with the cusp surface analysis program can be performed by generating data that fall *exactly* on the canonical cusp surface (i.e., which exactly satisfy Equation (2), with $C = 0$). One would think that the analysis should reproduce the correct coefficients with no error, but this is not what happens. Recall that Equation (2) only describes the deterministic model, and makes no statement about the conditional distribution of the random variable Y . On the other hand, the likelihood method makes the further assumption that this conditional distribution has a particular form, examples of which are depicted in Exhibit 4-4. Notice that in Exhibit 4-4a, for example, the relative height of the two modes changes very rapidly as the asymmetry factor increases. *If the conditional distribution of Y in the manufactured data set does not behave similarly, then the cusp surface program will not yield the correct estimates.* In fact, the maximum likelihood method will find the coefficients which best reproduce the empirical conditional distribution of Y , since this is roughly what it means to maximize the likelihood of a model.

Each of the component variables of \mathbf{X} contributes to each of the factors A , B , and C , and the effect of each factor on Y depends on the values of the other factors. Thus it is hard to visualize and understand the effect of any given variable. For this reason the cusp surface analysis program displays a graph of the effect that each of the independent variables would have on Y *if every other variable were fixed at its mean value.*

Each graph shows the solutions to the cubic polynomial equation:

$$0 = (A_0 + A_i X) + (B_0 + B_i X)[Y - (C_0 + C_i X)] - D[Y - (C_0 + C_i X)]^3, \quad (6)$$

for a given i . The solutions are graphed for $|Y| \leq 2.5$ and $|X| \leq 3.0$, a region which includes all but a tiny fraction of the observed values of X and Y . Thus even when there are three solutions the graph may display only one or two, indicating that the others are far outside the range of commonly observed values.

As an example of such a graph, Exhibit 4-8 shows how the coefficient C in Equation (2) can affect the relationship between X and Y .

$$0 = -X + 3[Y - CX] - [Y - CX]^3. \quad (7)$$

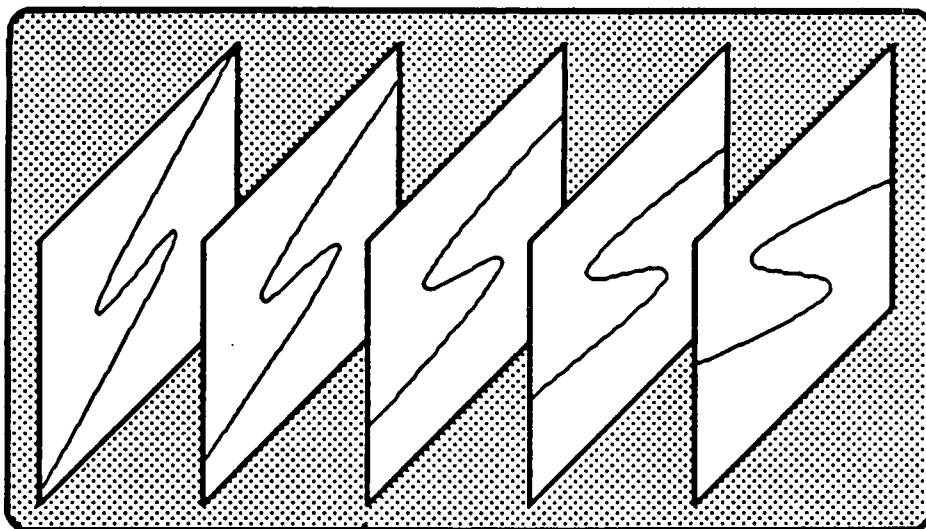
Notice how the *number* and *location* of the catastrophe points does not change as C changes, although the graph itself undergoes considerable deformation. This is characteristic of a "fiber-preserving" transformation (in this case the transformation is $Y \rightarrow Y - CX$).

4.6 A SAMPLE OUTPUT

A sample output from the cusp surface analysis program is presented in the final pages of this report. Exhibit 4-9(a) shows the results of the computation of the mean and standard deviations, correlation matrix, and the computation of log-likelihood values for the linear and cusp models on the basis of the information contained in a generic test data set. Exhibit 4-9(b) presents maximum likelihood estimates for the cusp model, an asymptotic chi-square and t-statistic values. Exhibit 4-9(c) presents the estimated correlations between the Alpha and Gamma estimators and a table of predictions based on the cusp surface analysis. Exhibit 4-9(d) presents a plot of the location of the data on the control space of the cusp catastrophe. Exhibit 4-9(e) presents a histogram plot of the residuals from the predictions of the delay rule. Exhibit 4-9(f) presents a plot showing the effect of variable 2 holding all others fixed at their mean values and represents a "slice" through the cusp catastrophe manifold computed with the aid of the cusp analysis program. Exhibit 4-9(g) shows a similar type of plot for variable 3 with the remaining variables held constant at their mean values.

Exhibit 4-8

The Effect of Linear Factors



The effect of X on Y in Equation 4 for $C = (-.50, -.25, 0, .25, .50)$
Note that the number and location of the catastrophe points does not depend on C .

Exhibit 4-9(a)

Sample Output from the Cusp Surface Analysis Program

Reading 3 variables from testdata

Positions of the variables (dependent variable last):

2 3 1

Var	Mean	St.Dev
2	0.7742	0.5877
3	60.0303	28.2387
1	47.4545	25.7530

Correlation matrix:

	2	3	1
2	1.00	0.55	-0.14
3	0.55	1.00	-0.31
1	-0.14	-0.31	1.00

Log-likelihood of the linear model: -45.1800

Standard linear regression coefficients:

Var	Slope
2	0.042
3	-0.329

Multiple R-squared: 0.095

Newton-Raphson algorithm, version of 1 Oct 1987

Step: 1: Log-likelihood: -45.180

Step: 2: Log-likelihood: -42.235

Step: 3: Log-likelihood: -40.358

Step: 4: Log-likelihood: -35.727

Step: 5: Log-likelihood: -34.197

Step: 6: Log-likelihood: -33.530

Step: 7: Log-likelihood: -33.376

*** NEAR ***

Step: 8: Log-likelihood: -33.354

*** NEAR ***

Step: 9: Log-likelihood: -33.350

*** NEAR ***

Convergence after 10 iterations.

Exhibit 4-9(b)

Sample Output from the Cusp Surface Analysis Program (Continued)

 Cusp Surface Analysis, version of 4 October 1987.
 by Loren Cobb, Department of Biometry
 Medical University of South Carolina.
 Charleston, SC 29425.
 Phone 803-792-7575 for assistance.

Model: $0 = \text{Alpha} + \text{Beta} * (\text{Y} - \text{Gamma}) - \text{Delta} * (\text{Y} - \text{Gamma})^3.$

The conditional density of Y given $X[1], \dots, X[v]$:

$$f(Y|X) = \exp[\text{Psi} + \text{Alpha} * Z + \text{Beta} * Z^2/2 - \text{Delta} * Z^4/4],$$

where $Z = Y - \text{gamma},$

Psi = constant (with respect to Y),

Alpha = $A[0] + A[1]*X[1] + \dots + A[v]*X[v],$

Beta = $B[0] + B[1]*X[1] + \dots + B[v]*X[v],$

Gamma = $C[0] + C[1]*X[1] + \dots + C[v]*X[v],$

and $v = 2$ (in this analysis).

Maximum Likelihood Estimation for the Cusp Model:

Cases = 33

Log-Likelihood = -33.3502

Standard coefficients, with t-statistics in parentheses:

Var	Alpha	Beta	Gamma	Delta
Const	0.131 (0.5)	3.326 (2.2)	-0.146 (-1.5)	3.071 (2.7)
2	0.438 (1.5)	-0.318 (-0.4)	-0.304 (-2.4)	
3	-0.313 (-1.3)	-1.829 (-2.1)	0.036 (0.4)	

(Each t-statistic has 24 degrees of freedom)

Raw coefficients:

Var	Alpha	Beta	Gamma	Delta
Const	8.517e-3	1.151e-2	5.202e+1	6.981e-6
2	2.897e-2	-8.159e-4	-1.331e+1	
3	-4.306e-4	-9.768e-5	3.308e-2	

Test for H0: Conditional densities are Type N2 (linear regression)

versus H1: Conditional densities are Type N4 (cusp regression)

>>>>> Asymptotic Chi-square = 23.66 (df = 6) <<<<<

Test for H0: Delta = 0 (i.e. no cubic term)

versus H1: Delta > 0 (a one-tailed test)

>>>>> t = 2.65 (df = 24) <<<<<

Exhibit 4-9(c)

Sample Output from the Cusp Surface Analysis Program (Continued)

Estimated correlations between Alpha and Gamma estimators:

	G0	G1	G2
A0	-0.48	0.27	-0.07
A1	0.19	-0.39	-0.28
A2	-0.20	-0.19	-0.53

Predictions based on this analysis:

Case	Asymm	Bifur	Mode	Antimode	Mode	Y(# 1)	X(# 2)	X(# 3)
1	0.68	0.96			49.41	35.00	1.91	87.00
2	-0.73	3.81	27.85	63.83	84.51	38.00	-0.35	62.00
3	0.30	4.23	13.76	41.20	74.11	74.00	0.79	46.00
4	0.26	1.05			54.06	53.00	1.39	90.00
5	-0.15	2.22	23.06	47.60	66.78	41.00	0.66	78.00
6	-0.20	1.38	25.74	48.47	59.63	35.00	0.78	90.00
7	0.11	4.36	15.99	45.69	77.37	53.00	0.54	46.00
8	0.43	5.30	10.57	41.19	78.12	6.00	0.73	30.00
9	0.24	4.27	14.47	42.64	75.15	74.00	0.71	46.00
10	0.32	1.01			53.23	65.00	1.47	90.00
11	-0.36	2.96	24.14	54.08	74.39	62.00	0.26	70.00
12	-0.89	2.54	31.39	71.76	72.12	26.00	-0.28	81.00
13	0.12	7.20	13.44	52.22	92.29	22.00	-0.03	7.00
14	0.54	0.85			50.00	10.00	1.77	90.00
15	0.05	1.42	23.67	39.96	58.72	74.00	1.06	87.00
16	0.44	0.92			51.53	10.00	1.63	90.00
17	0.34	4.19	13.24	40.10	73.32	10.00	0.85	46.00
18	0.58	2.71	14.28	29.06	61.56	53.00	1.44	64.00
19	-0.19	1.59	25.02	48.01	61.71	35.00	0.75	87.00
20	0.08	7.22	13.96	53.02	92.96	86.00	-0.08	7.00
21	0.34	4.20	13.33	40.29	73.46	62.00	0.84	46.00
22	0.22	4.29	14.73	43.18	75.54	62.00	0.68	46.00
23	-0.23	2.28	23.77	49.76	67.90	74.00	0.56	78.00
24	0.21	6.91	12.25	49.69	89.51	94.00	0.14	10.00
25	0.57	6.65	7.35	41.88	83.06	94.00	0.62	10.00
26	-0.38	2.39	25.37	54.23	70.21	26.00	0.35	78.00
27	0.40	3.57	13.66	36.88	68.94	10.00	1.05	54.00
28	0.37	2.87	15.80	35.46	65.21	35.00	1.15	64.00
29	0.51	6.91	7.96	43.67	85.15	74.00	0.50	7.00
30	-0.34	5.27	21.14	57.32	88.56	14.00	-0.18	38.00
31	0.03	2.23	21.04	42.44	64.97	71.00	0.88	76.00
32	0.41	0.94			51.96	53.00	1.59	90.00
33	0.24	1.06			54.27	35.00	1.37	90.00

Exhibit 4-9(d)

Sample Output from the Cusp Surface Analysis Program (Continued)

Location of data in the control space:

Vertical axis: Bifurcation (splitting) factor

Horizontal axis: Asymmetry (normal) factor

Asterisks: Bimodal zone

	-5	-4	-3	-2	-1	0	1	2	3	4	5	

5.0	0	0	0	0	0	0	0	0	0	0	0	5.0

4.5	0	0	0	0	0	0	0	0	3	0	0	4.5

4.0	0	0	0	0	0	0	0	0	1	0	3	4.0

3.5	0	0	0	0	0	0	0	0	0	1	0	3.5

3.0	0	0	0	0	0	0	0	0	1	0	1	3.0

2.5	0	0	0	0	0	0	0	1	1	1	1	2.5

2.0	0	0	0	0	0	0	0	0	2	0	0	2.0

1.5	0	0	0	0	0	0	0	0	3	0	0	1.5

1.0	0	0	0	0	0	0	0	0	1	6	0	1.0

0.5	0	0	0	0	0	0	0	0	0	0	0	0.5
	*											
0.0	0	0	0	0	0	0	0	0	0	0	0	0.0
-0.5	0	0	0	0	0	0	0	0	0	0	0	-0.5
-1.0	0	0	0	0	0	0	0	0	0	0	0	-1.0
-1.5	0	0	0	0	0	0	0	0	0	0	0	-1.5
-2.0	0	0	0	0	0	0	0	0	0	0	0	-2.0
-2.5	0	0	0	0	0	0	0	0	0	0	0	-2.5
-3.0	0	0	0	0	0	0	0	0	0	0	0	-3.0
-3.5	0	0	0	0	0	0	0	0	0	0	0	-3.5
-4.0	0	0	0	0	0	0	0	0	0	0	0	-4.0
-4.5	0	0	0	0	0	0	0	0	0	0	0	-4.5
-5.0	0	0	0	0	0	0	0	0	0	0	0	-5.0
	-5	-4	-3	-2	-1	0	1	2	3	4	5	

7 cases did not fit in the above figure.

>>>>> Fraction of cases in bimodal zone: 0.788 <<<<<

Linear R^2 = 0.095 (Multiple regression)

Delay R^2 = 0.695 (Attracting-mode convention)

Maxwell R^2 = -0.178 (Most-likely-mode convention)

* Negative R^2 values occur when the cusp model is worse than a constant.

Exhibit 4-9(e)

Sample Output from the Cusp Surface Analysis Program (Continued)

Histogram of residuals from predictions of the delay rule:
(Units are standard deviations of the dependent variable.)

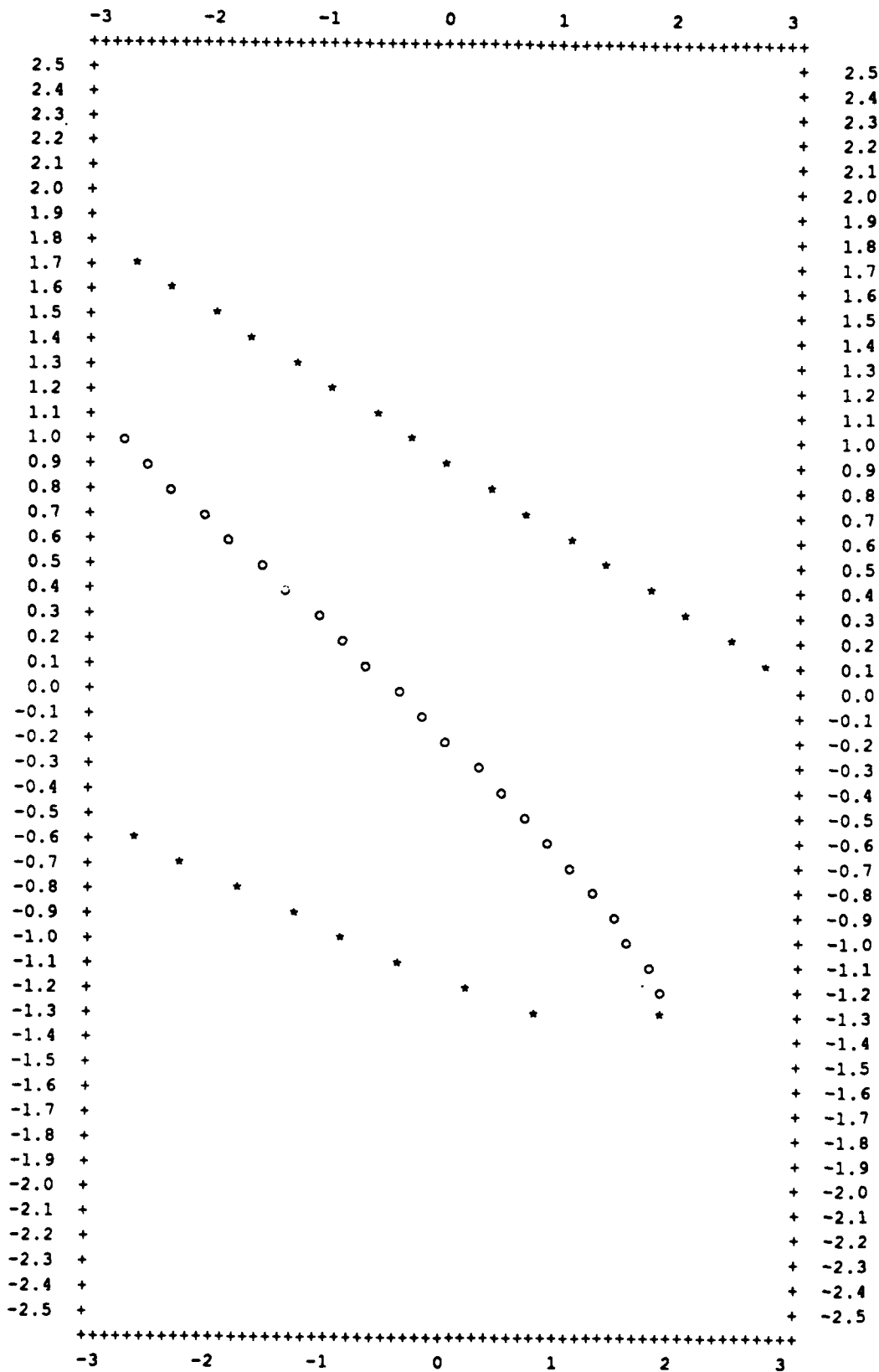
Y	N
-3.00	0
-2.75	0
-2.50	0
-2.25	0
-2.00	0
-1.75	0
-1.50	2 ##
-1.25	0
-1.00	1 #
-0.75	1 #
-0.50	5 #####
-0.25	7 #####
0.00	5 #####
0.25	5 #####
0.50	5 #####
0.75	2 ##
1.00	0
1.25	0
1.50	0
1.75	0
2.00	0
2.25	0
2.50	0
2.75	0
3.00	0

Error Mean = -0.116
Error St.Dev = 0.553

Exhibit 4-9(f)

Sample Output from the Cusp Surface Analysis Program (Continued)

Effect of variable 2, holding all others constant at their mean values.

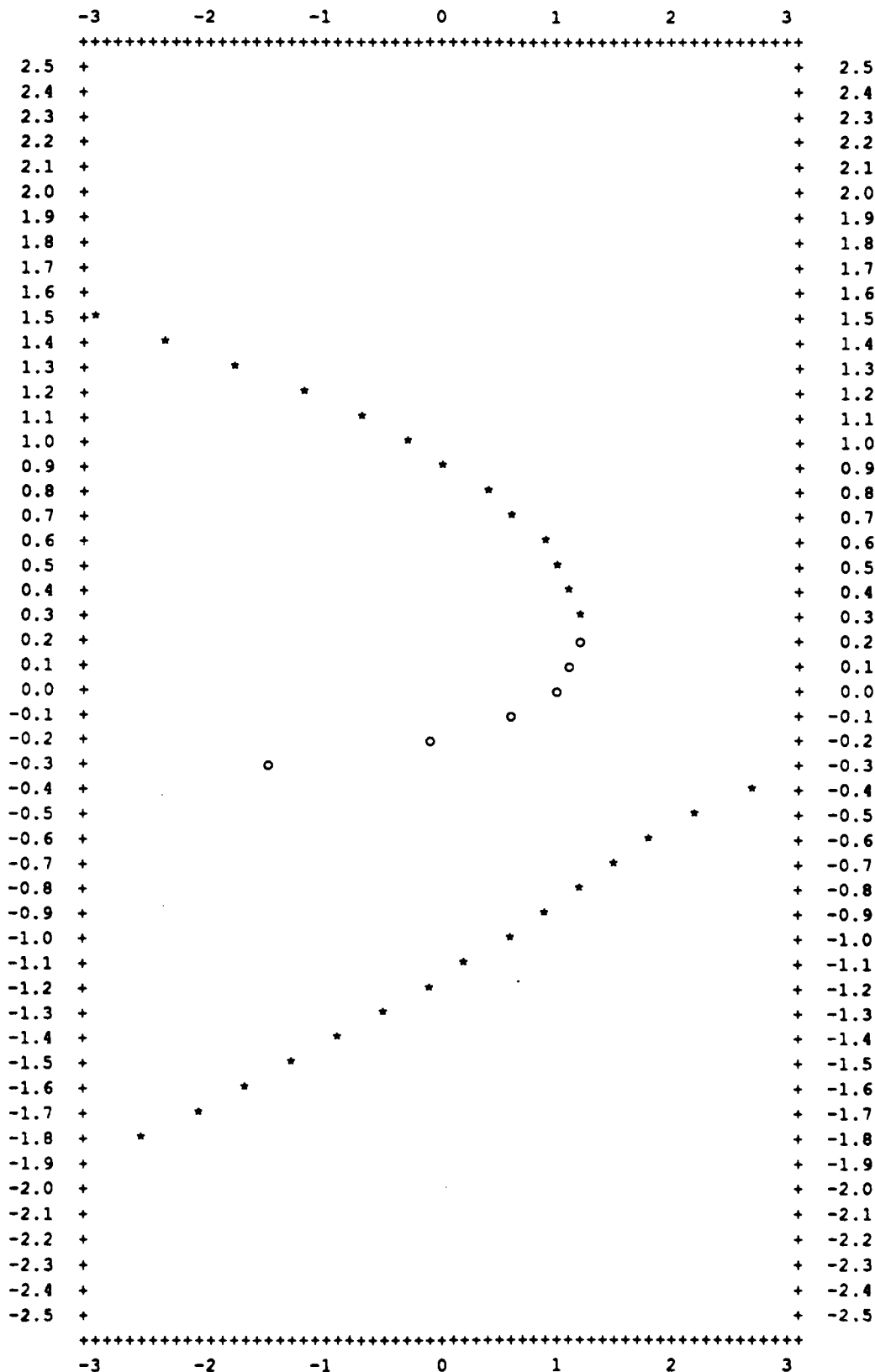


* Mode symbol
o Antimode symbol

Exhibit 4-9(g)

Sample Output from the Cusp Surface Analysis Program (Continued)

Effect of variable 3, holding all others constant at their mean values.



* Mode symbol
o Antimode symbol

SECTION 5. USING THE IWCAT SYSTEM

The IWCAT effort has investigated the use of catastrophe theory and related mathematical techniques as a methodology to support the activities of indications and warning (I&W) analysts. Prototype software, that implements these techniques in a manner that recognizes that I&W analysts are not mathematicians and therefore require appropriate non-mathematical user interfaces so that mathematical operations are performed automatically, has been produced. The following section describes how the IWCAT system has been used with Operational Maneuver Group (OMG)-related indicators to determine the likelihood that particular combinations of these indicators reflect the formation of an OMG.

The IWCAT software runs on an IBM-AT class computer and permits the analysis of the response of I&W analysts and others to indicators associated with a specific I&W-related situation. Synectics, in collaboration with the government, has determined that the conditions under which an OMG is formed from an otherwise "normal" pattern of military advance are of sufficient interest to the government to warrant its selection as the appropriate "I&W situation" as specified in the IWCAT statement of work.

5.1 STATISTICAL ANALYSIS AND CATASTROPHE THEORY

The IWCAT system presents its users with sets of OMG-related indicators and asks them to assess whether or not the information that they contain reflects the formation of an OMG. These assessments are analyzed with a statistical procedure that performs both a standard linear type of analysis and a nonlinear, cusp catastrophe based, analysis in order to determine which technique provides the best "fit" for the data.

The cusp catastrophe model provides a description of systems which exhibit nonlinear and ambiguous behavior and of situations in which changes in the values of system variables can give rise to either gradual or sudden changes in behavior under different circumstances. Such behavior appears to be associated with indications and warning and its nature can only be examined with great difficulty, if at all, with the aid of the more classical statistical approaches involving normal distributions and with linear regression techniques.

5.1.1 KEY VARIABLES

The catastrophe analysis involves the identification of key system variables and the use of these as factors in the statistical procedures. Review and analysis of the I&W environment has led the IWCAT project team to the identification of the following two major key input variables (called control factors) and one major output variable (called a behavior variable) which can describe at least some of the major features of the activities performed by I&W analysts.

1. The control factors represent the inputs to the process of I&W analysis. At this time, the IWCAT project team has selected two control variables which they have described as the number of active indicators and the level of confidence to represent these inputs. The "number of active indicators" variable represents the number of indicators which are activated when the analyst makes a judgment. Additional factors

including weather, time of day, sequence type, and scenario type have also been included as potential control factors, as described below. The "level of confidence" variable represents a measure of the degree to which the set of indicators presented to the analyst is considered to reflect the actual conditions of a situation of interest.

2. The behavior variable represents the result of the I&W process. The IWCAT project team has tentatively named this variable the analyst's perception of OMG threat since it represents the perception of the I&W analyst of the likelihood that a particular military situation reflected in the set of indicators presented to the analyst represents the formation of an OMG from an apparently "normal" pattern of military advance, for example.

During each assessment activity, a selection of test data sets, each with different numbers of active indicators and level of confidence and related information, are presented to the users of the IWCAT system and they are asked to assess the level of OMG threat as reflected in these data by indicating a position on a scale line. These assessments and the input information form the basis of the OMG Threat Assessment data base. The results for each individual participant are tabulated, recorded, and analyzed as described in below, in order to determine whether the data can be described with the aid of a linear model, or whether the data could be described more appropriately with the aid of a nonlinear model based on the cusp catastrophe.

5.2. SYSTEM REQUIREMENTS

5.2.1 HARDWARE

The IWCAT system is designed to run on any IBM-AT compatible computer equipped with a mathematics coprocessor and a color monitor. The program can also print out the results of the statistical analysis onto printers of the IBM Proprinter class.

5.2.2 SOFTWARE

The IWCAT software consists of a compiled computer program IWCAT.COM contained on a 5.25 inch floppy disk which contains all the special utilities required for running the IWCAT system.

5.2.3 GETTING STARTED

Insert the IWCAT disk into the disk drive and turn the power on. When prompted by the computer with the prompt symbol, >, enter the symbol A (thus: >A) and then enter when prompted: > IWCAT. The IWCAT system will then be ready for use and the main menu can be accessed by pressing the "enter" key and will be available for user selection.

5.3. THE IWCAT SYSTEM

The IWCAT system provides menu-driven access to the series of facilities displayed in Exhibit 5-1 and outlined below.

5.3.1 MENU DISPLAY OVERVIEW

The IWCAT Menu provides the user with access to the various components of the IWCAT system (Exhibit 5-1). Access to these components is obtained by typing the required option code number when instructed by the program.

Personal information has to be entered under menu item "2" before proceeding to the remainder of the system since this information is used to identify the data files created during use of the IWCAT system. The following selections are available to the user:

- > 1 Provides a description of the IWCAT system and presents the text of the three scenarios which provide the context for the OMG threat assessment activities.
- > 2 Allows entry of the necessary identifying personal and/or code information.
- > 3 Makes a test file of indicator data for user assessment.
- > 4 Permits the user to practice indicator assessment and data entry.
- > 5 Enables the user to perform assessments of the test data set.
- > 6 Creates a file for analysis with the identification of primary and secondary indicators.
- > 7 Performs the cusp analysis.
- > 8 Terminates the use of the IWCAT system.

5.3.2 READ ABOUT THE PROGRAM

Menu option "1" ("Read About the Program") (Exhibit 5-2) provides the user with access to a text file that outlines the properties of catastrophe theory, some of the properties of OMGs and the OMG indicators, describes the nature of the IWCAT program and the tasks that the user will be asked to perform and other details of the analytic environment. The I&W indicator sets will be presented against the background provided by one of three scenarios, reflecting conditions characterized as:

1. Treaty Obligation
2. Friendly Ally

Exhibit 5-1

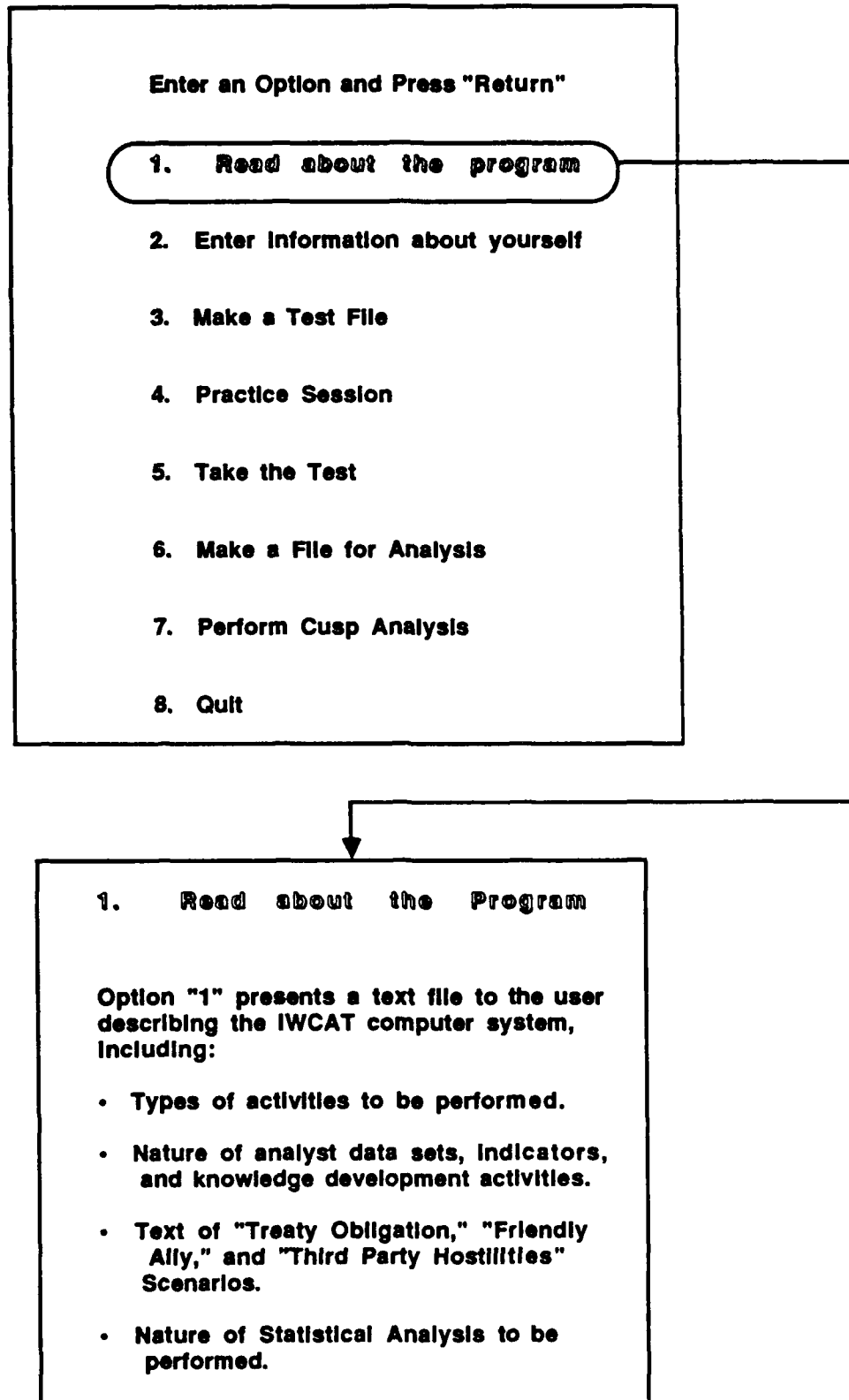
IWCAT System Overview Menu Display

Enter an Option and Press "Return"

- 1. Read about the program**
- 2. Enter Information about yourself**
- 3. Make a Test File**
- 4. Practice Session**
- 5. Take the Test**
- 6. Make a File for Analysis**
- 7. Perform Cusp Analysis**
- 8. Quit**

Exhibit 5-2

IWCAT System Overview Menu Display, Option 1: Read About the Program



3. Third Party Hostilities

The material contained in the text files accessed under option "1" is self explanatory.

5.3.3 ENTER INFORMATION ABOUT YOURSELF

Menu option "2" ("Enter Information About Yourself") (Exhibit 5-3) asks the user to enter items of personal information which serves as the basis for creating an identification code for the user-generated data sets.

Under option "2," the user enters the following items of information:

1. First Name (Amnopq, for example).
2. Family Name (BCDEFGHijk, for example).
3. Number of years as an analyst (XX, for example).

The first letter of the first name and the first seven letters of the family name are used to create the data file for cusp analysis after the primary and secondary indicators have been identified at the completion of the testing sequence. Thus the above entries would create a data file named: ABCDEFGH.CUS, for example.

In the case of multiple uses of the IWCAT system by a single user, the user should be aware that entry of the same items of personal information for a subsequent use will cause the creation of a data file which will obliterate a previously created file with the same identifier. Under such circumstances, some form of personal coding scheme should be adopted for each IWCAT system use.

Completion of these data entry activities is signaled by pressing the "Esc" key once.

5.3.4 MAKE A TEST FILE

Menu option "3" ("Make a Test File") (Exhibit 5-4) permits the user to generate a new test data file consisting of fifteen sub-files each of which is associated with one of the three types of scenario (Treaty Obligation, Friendly Ally, or Third Party Hostilities) and one of five different patterns of indicator sequences. The data file is given a label based on the personal information entered under option "2." When prompted, the user enters an appropriate identifier label, the user's initial and the first seven characters of the family name, or a user-selected code is recommended for this purpose (Exhibit 5-5). Entry of this information triggers the creation of a test data file and the creation of such a test data file is indicated by a self explanatory sequence of displays.

Exhibit 5-3

IWCAT System Overview Menu Display, Option 2: Enter Information About Yourself

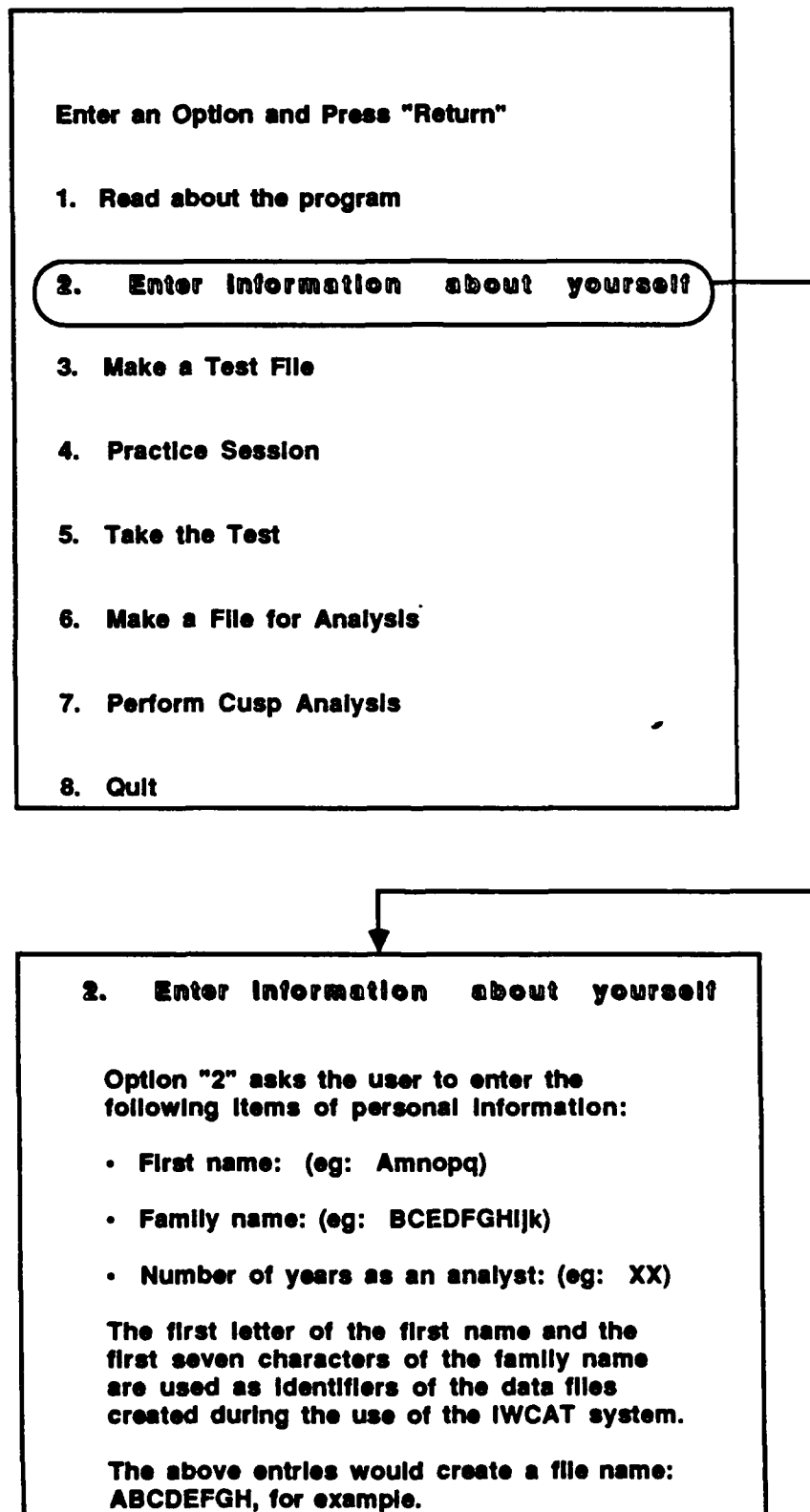


Exhibit 5-4

IWCAT System Overview Menu Display, Option 3: Make a Test File

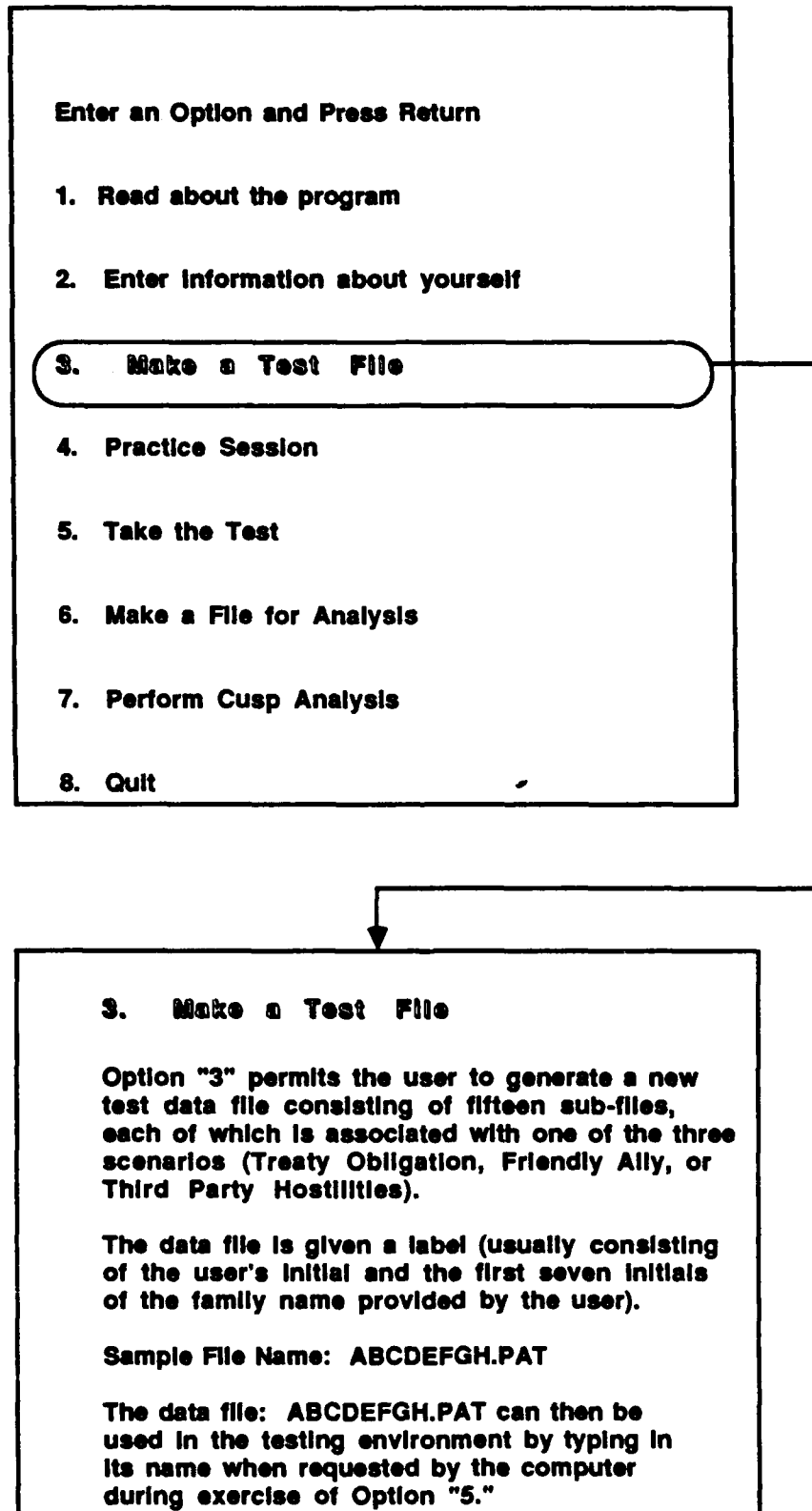
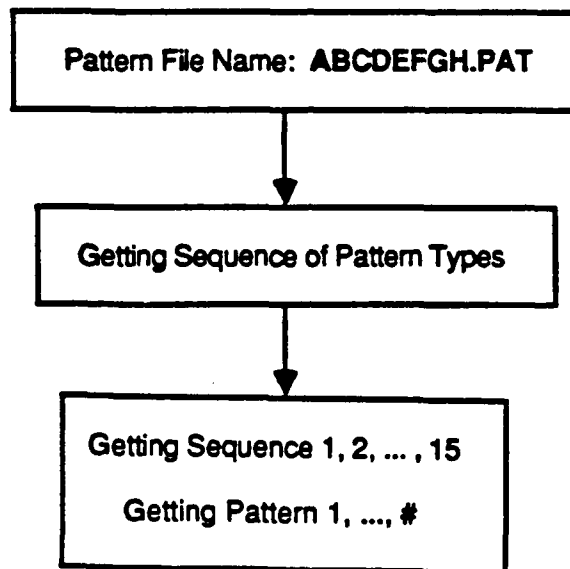


Exhibit 5-5
Making a Test File

OPTION 3: MAKE A TEST FILE



5.3.5 PRACTICE SESSION

Option "4" ("Practice Session") (Exhibit 5-7) permits the user to gain experience in the processes of assessing the levels of OMG threat represented in the IWCAT data sets and records this information within a test data file in a manner that is identical to the way in which OMG threat data will be assessed and this information is entered into the computer during the actual testing process.

The system will ask the user to specify the practice data set. A practice data set labeled **PRACTICE.PAT** has been provided for practice purposes and the user should enter this identifier when prompted by the computer. The system asks the user to specify the number of scales to be placed on the OMG Threat Assessment Indicator line (Exhibit 5-7). The following prompt message appears:

number of scale divisions? 2/4

The user will select one of these options by typing in either 2 or 4, producing the display:

number of scale divisions? 2/4 2, for example.

5.3.5.1 Practice Data Presentation

The practice data set (**PRACTICE.PAT**) provides a series of data screens each of which contain the following data elements (Exhibit 5-8):

1. An array of 10 OMG-related indicators (Exhibit 5-9). Active indicators are indicated by the presence of a white colored rectangle.
2. Type of weather information is presented in text format.
3. Time of day information is presented in text format.
4. Level of confidence information is presented in numerical format with numbers ranging from 0 (zero level of confidence in the reliability of the data) to 100 (absolute confidence in the reliability of the data).
5. Scenario type information (Treaty Obligation, Friendly Ally, or Third Party Hostilities) presented in text format.

The data is presented in a series of blocks with each block of data corresponding to one type of scenario. Thus, the practice sequence can begin with the presentation of a screen identifying a series of data sets relating to a "Treaty Obligation" scenario. These data are assessed for OMG threat level (see section 5.3.5.2) and this assessment recorded by pushing the "Enter" key.

Exhibit 5-6

IWCAT System Overview Menu Display, Option 4: Practice Session

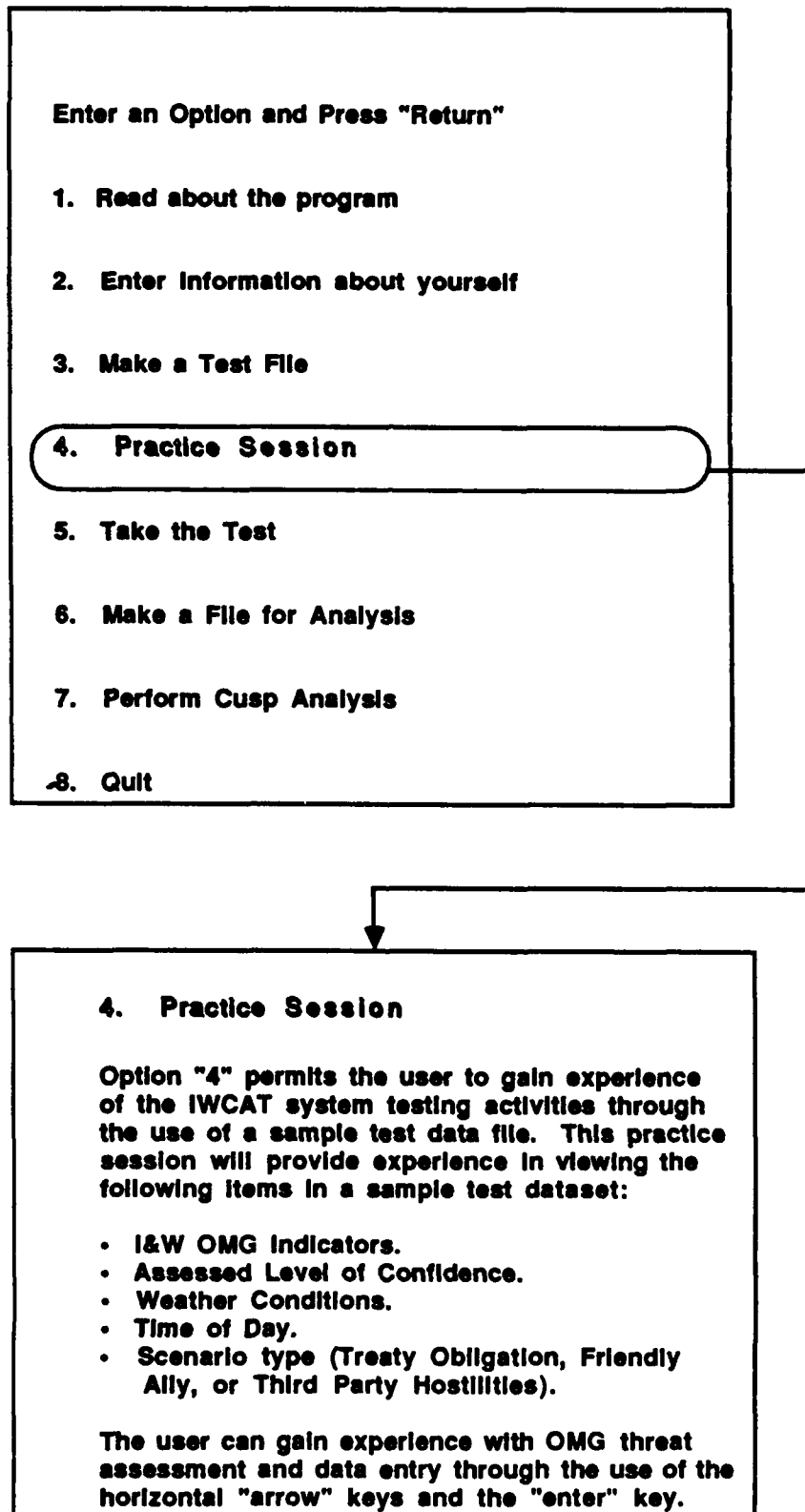


Exhibit 5-7

Entering OMG Assessment Practice Data

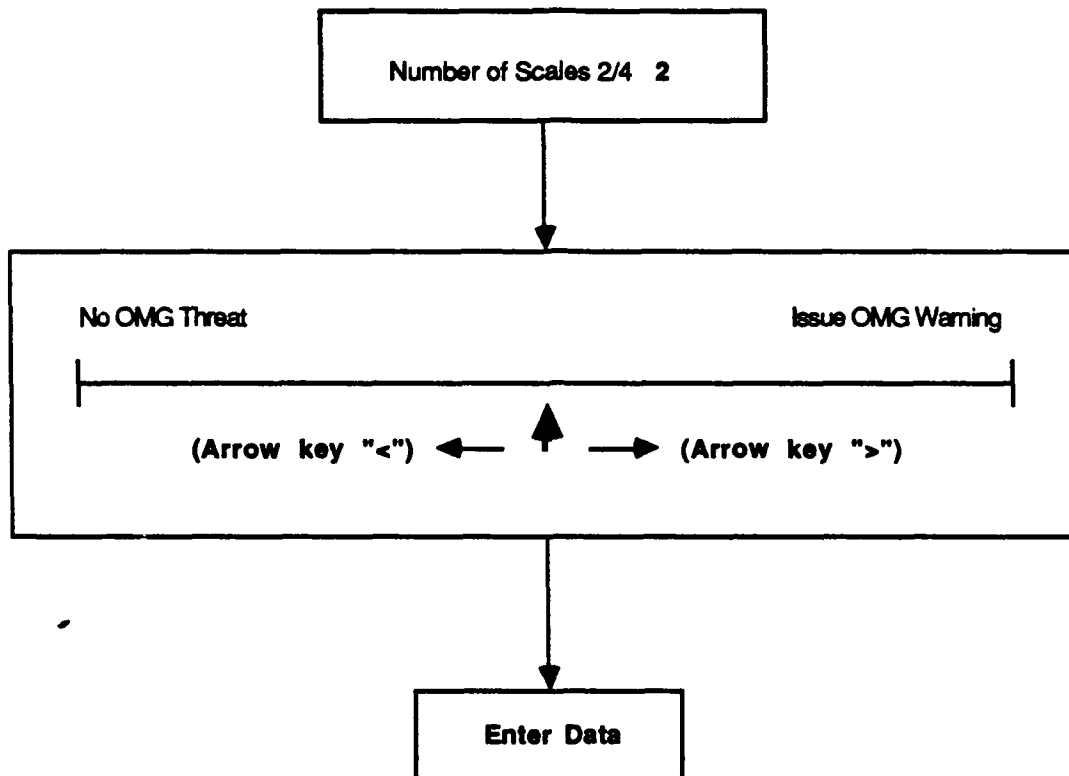


Exhibit 5-8

Practice Session Activities

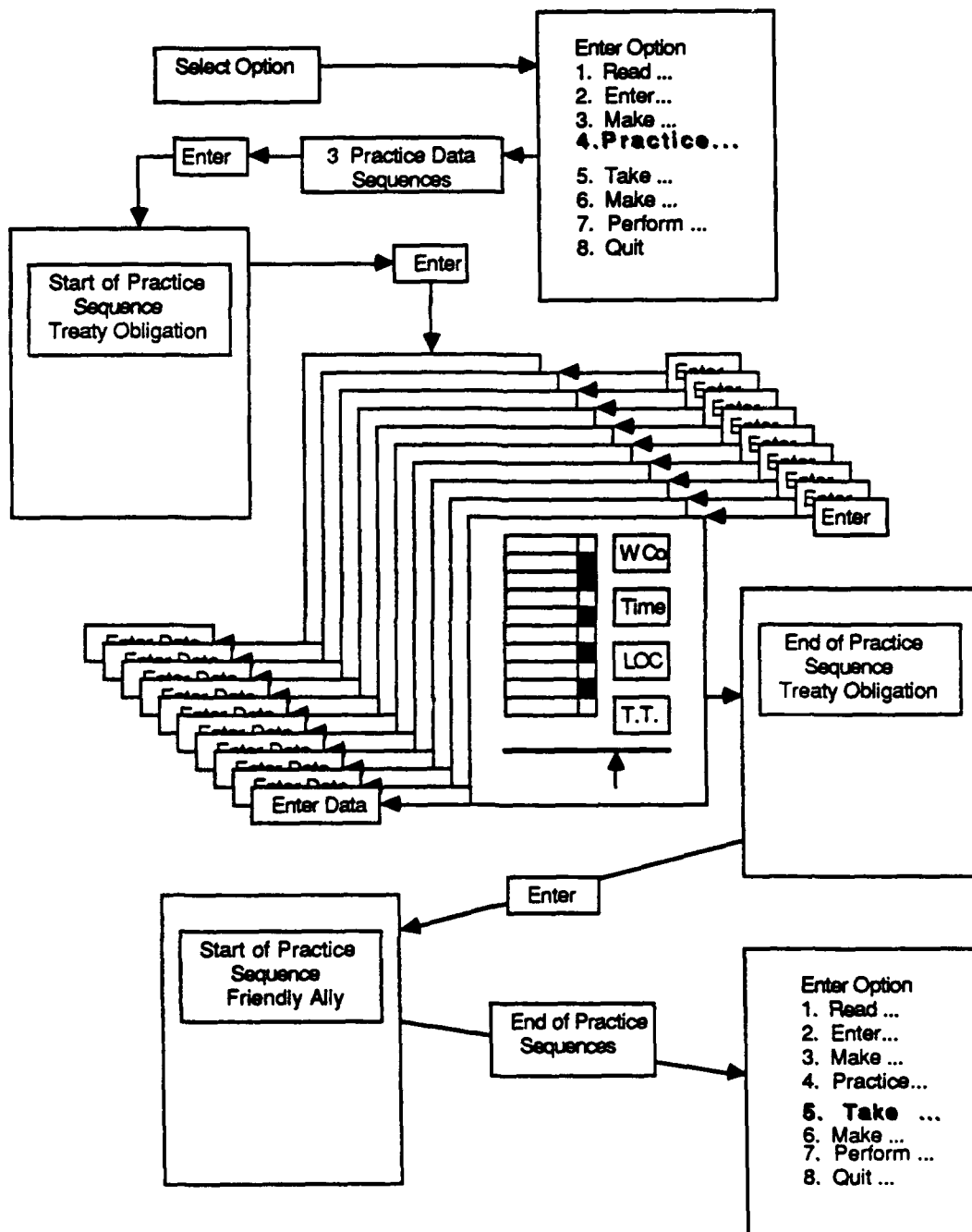


Exhibit 5-9
Practice Data Display

Intensified Reconnaissance and Intelligence	<input type="checkbox"/>	Weather Overcast
Concentration of Artillery Units In the FLOT Area	<input type="checkbox"/>	
Alternative Communications	<input type="checkbox"/>	Time Night
Increasing Air Support	<input type="checkbox"/>	
Dummy Concentrations	<input type="checkbox"/>	Level of Confidence 80
Armor Assembly Areas Within 30-50 km. of the FLOT	<input type="checkbox"/>	
Combat Engineers Attached	<input type="checkbox"/>	Treaty Obligation
Traffic Control and Lane Clearing	<input type="checkbox"/>	
Electronic Silence	<input type="checkbox"/>	
Electronic Countermeasures and Deception	<input type="checkbox"/>	

No OMG Threat

|-----|

↑

Issue OMG Warning

|-----|

5.3.5.2 Practice OMG Threat Assessment

The user is asked to assess the level of OMG threat reflected in the OMG-related indicators and enter this information into the computer with the aid of the indicator scale positioned in the lower portion of the computer screen (see Exhibits 5-7, 5-8, and 5-9).

This task is performed with the aid of the horizontal arrow keys "<" and ">" which permit the user to position the blinking indicator arrow at a position of the OMG Threat Assessment line that corresponds to the level of OMG Threat that the analyst perceived as being reflected in the data presented on the screen (Exhibit 5-7).

Completion of each assessment is signaled by pushing the "Enter" key which enters the results of the assessment into the OMG Threat Assessment data file. The end of each block of data is indicated by a self-explanatory display (Exhibit 5-8). The next block of data is accessed by pressing the "Enter" key. Completion of the complete practice sequence is indicated by the return to the menu display for further option selections.

5.3.6 TAKE THE TEST

Option "5" ("Take the Test") (Exhibit 5-10) permits the user to assess the levels of OMG threat represented in the IWCAT data sets and records this information within a test data file.

Option 5 ("take the test") should always be accessed before options 6 ("Make a File for Analysis") and 7 ("Perform Cusp Analysis"), although these activities can be performed during different sessions of activity.

On selection of option 5, the system asks the user to specify the number of scales to be placed on the OMG Threat Assessment Indicator line (Exhibit 5-11). The following prompt message appears:

number of scale divisions? 2/4

The user will select one of these options by typing in either 2 or 4, producing the display:

number of scale divisions? 2/4 2, for example.

The system automatically presents the first test data set for analyst assessment.

5.3.6.1 Test Data Presentation

The test data set provides a series of data screens each of which contain the following data elements (Exhibit 5-11):

1. An array of OMG indicators (Exhibit 5-12), with white squares in the array signifying which of these indicators is active.
2. Type of weather information.
3. Time of day information.

Exhibit 5-10

IWCAT System Overview Menu Display, Option 5: Take the Test

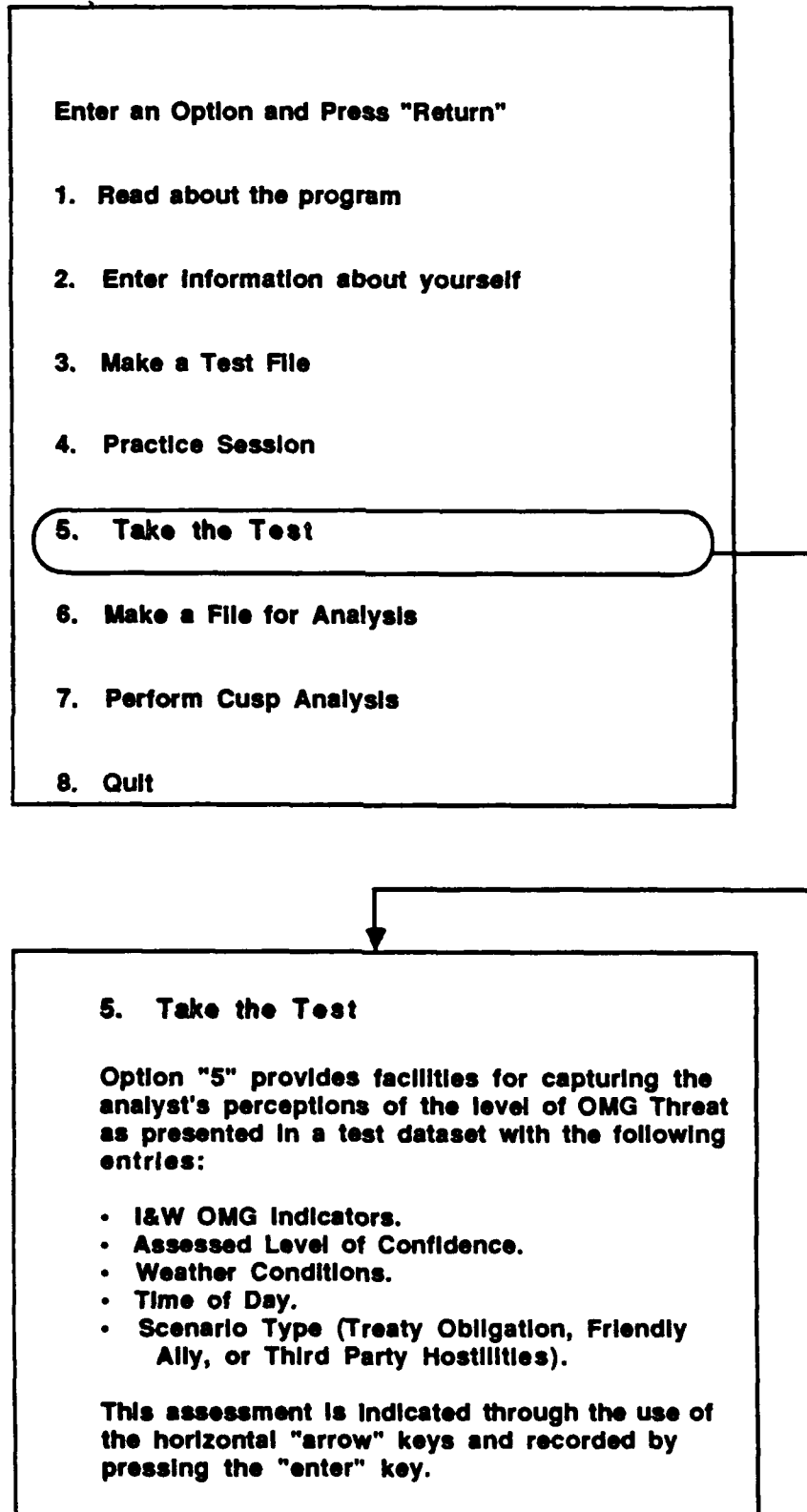


Exhibit 5-11

Enter OMG Test Data Assessments

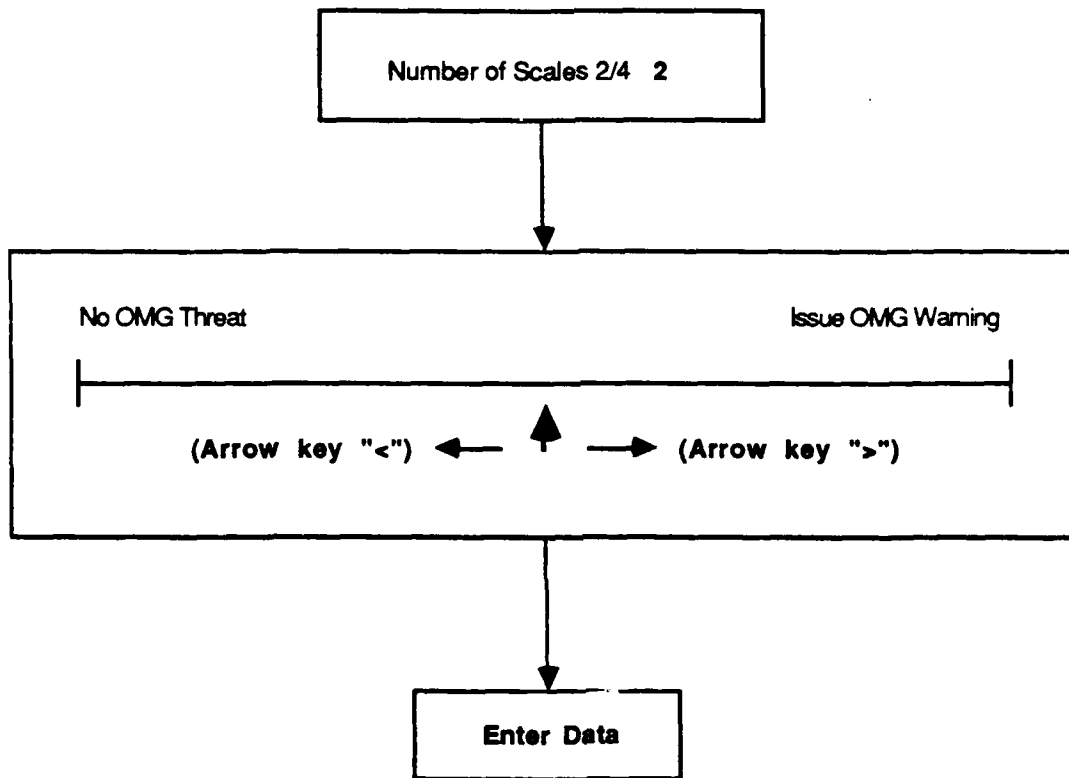
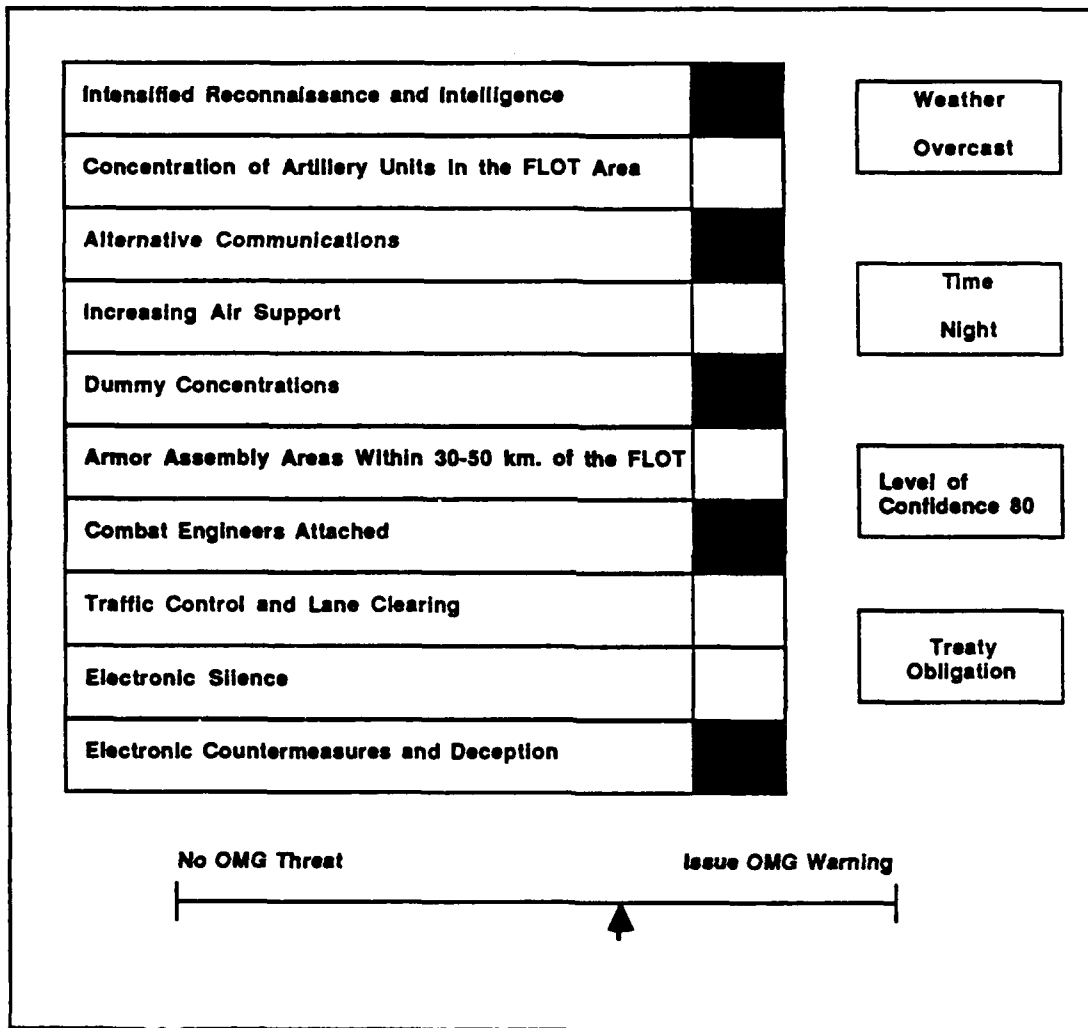


Exhibit 5-12

OMG Test Data Presentation



4. Level of confidence information.
5. Scenario type information (Treaty Obligation, Friendly Ally, or Third Party Hostilities).

As in the case of the practice session, the data is presented in a series of blocks with each block of data corresponding to one type of scenario. Thus, the practice sequence can begin with the presentation of a screen identifying a series of data sets relating to a "Treaty Obligation" scenario. These data are assessed for OMG threat level (see Section 5.3.6.2) and this assessment recorded by pushing the "Enter" key on the keyboard.

5.3.6.2 OMG Threat Assessment

The user is asked to assess the level of OMG threat reflected in the OMG-related indicators and enter this information into the computer with the aid of the indicator scale positioned in the lower portion of the computer screen (see Exhibits 5-11, 5-12, and 5-13).

This task is performed with the aid of the horizontal arrow keys "<" and ">" which permits the user to position the blinking indicator arrow at a position on the OMG Threat Assessment line that corresponds to the level of OMG Threat that the analyst perceived as being reflected in the data presented on the screen (Exhibit 5-12).

Completion of each assessment is signaled by pushing the "Enter" key which enters the results of the assessment into the OMG Threat Assessment data file (Exhibit 5-13). The end of each block of data is indicated by a self-explanatory display. The next block of data is accessed by pressing the "Enter" key. Completion of the complete test sequence is indicated by the return to the menu display for further option selections.

5.3.7 MAKE A FILE FOR ANALYSIS

Option "6" ("Make a File for Analysis") (Exhibit 5-14) permits the user to separate the I&W indicators into "primary" and "secondary" categories to facilitate the analysis of the user generated OMG threat assessment data.

Option 6 ("Make a File for Analysis") should always be accessed after option 5 ("Take the Test") and before option 7 ("Perform Cusp Analysis") although these activities can be performed during different sessions of activity.

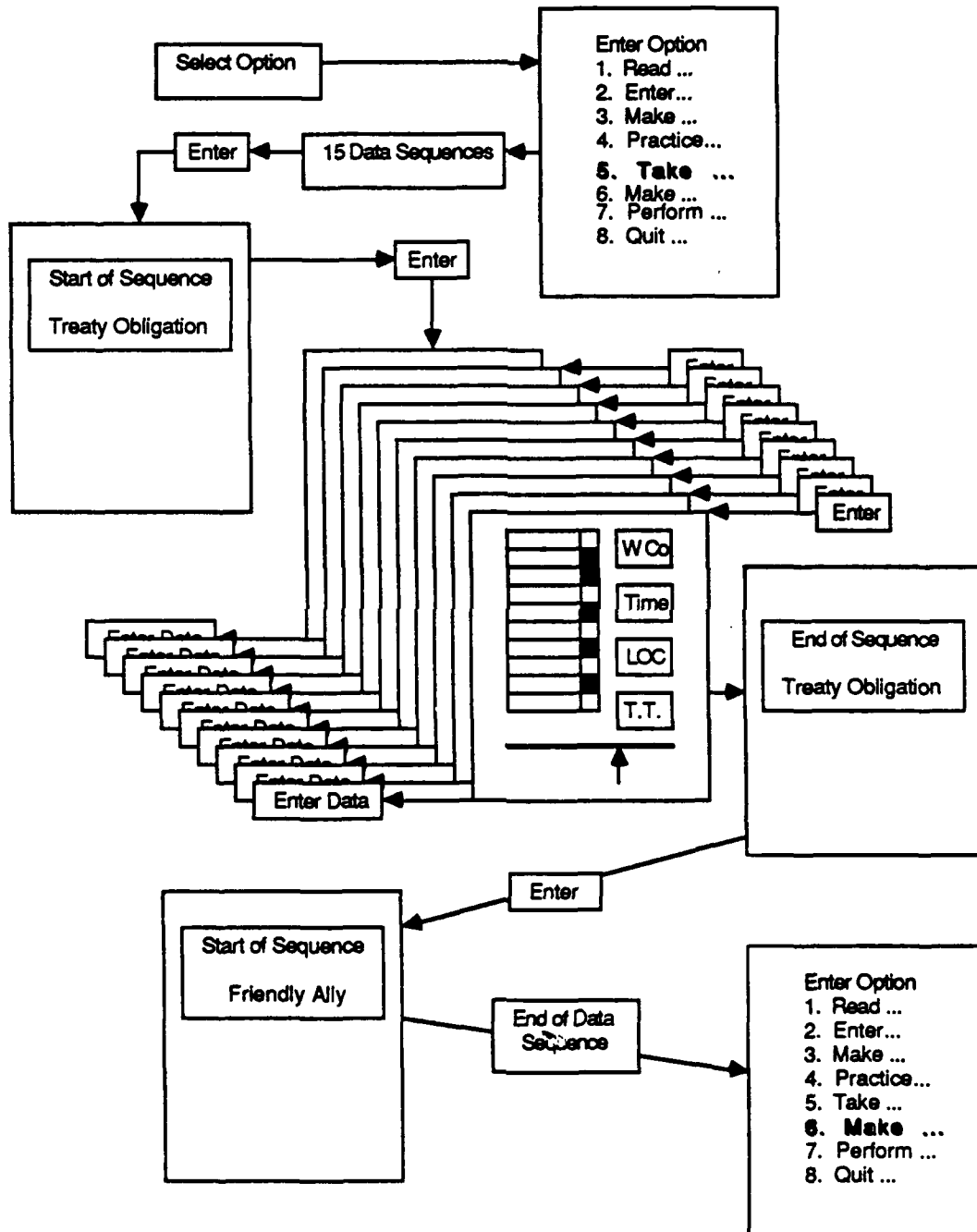
When option 6 is selected, the user is presented with a screen display (Exhibit 5-15) and asked to enter data indicating choice of the primary indicators by typing a "1" into the appropriate positions. Completion of the task is indicated by pressing the "Esc" key once. This task results in the automatic creation of a data file with the label:

ABCDEFGH.CUS

This file is the one that the user will select during the cusp analysis activity (option 7) described below.

Exhibit 5-13

OMG Threat Assessment Activities



IWCAT System Overview Menu Display, Option 6: Make a File For Analysis

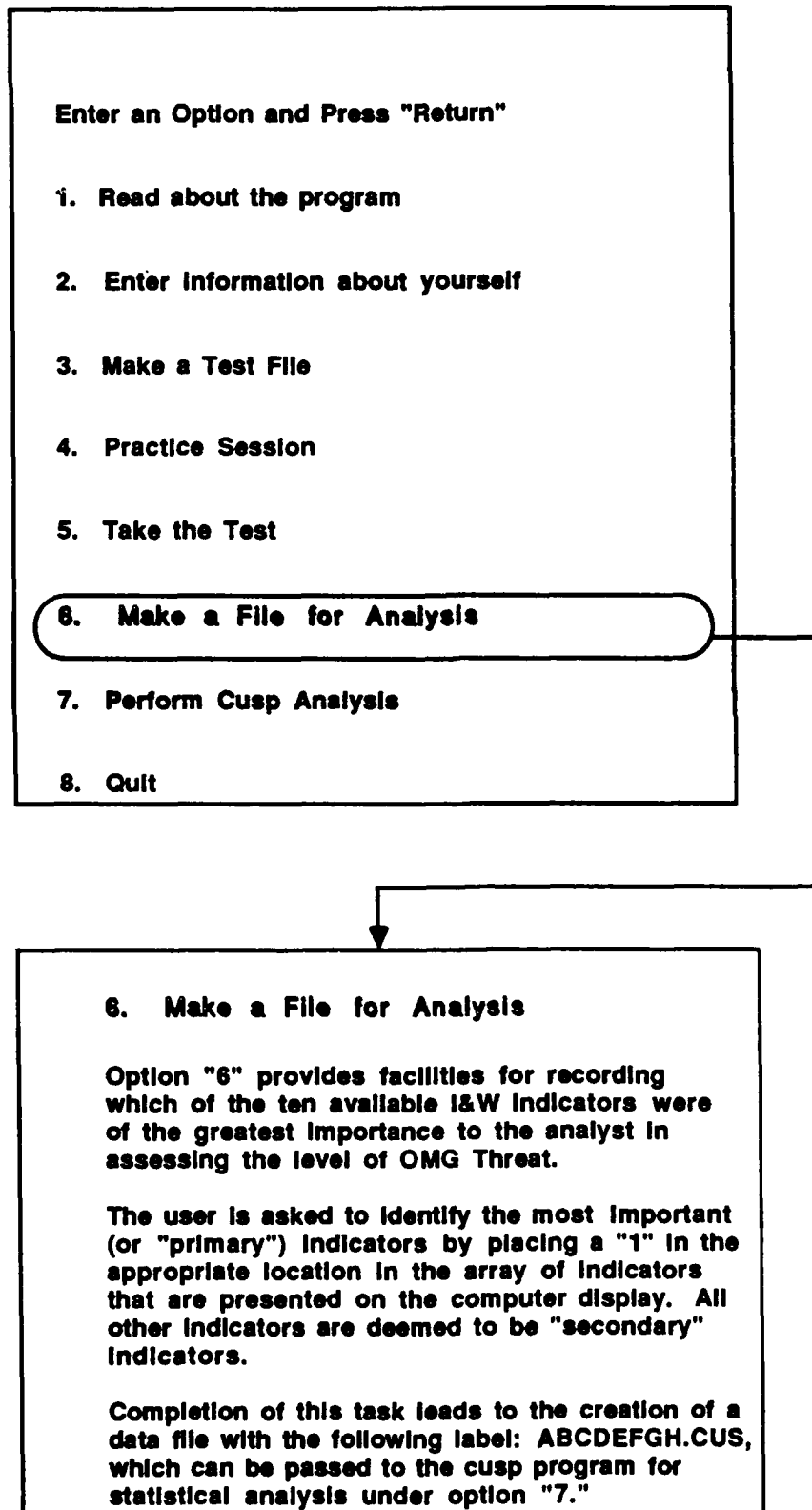


Exhibit 5-15

Making a File for Statistical Analysis

OPTION 6: MAKE A FILE FOR ANALYSIS

Intensified Reconnaissance and Intelligence	
Concentration of Artillery Units in the FLOT Area	
Alternative Communications	
Increasing Air Support	
Dummy Concentrations	
Armor Assembly Areas Within 30-50 km. of the FLOT	
Combat Engineers Attached	
Traffic Control and Lane Clearing	
Electronic Silence	
Electronic Countermeasures and Deception	

5.3.8 PERFORM CUSP ANALYSIS

Option "7" ("Perform Cusp Analysis") (Exhibits 5-16 and 5-17) provides access to facilities for performing a statistical analysis of analyst OMG threat assessment.

Option 7 ("Perform Cusp Analysis") should always be accessed after options 5 ("Take the Test") and 6 ("Make a File for Analysis"), although these activities can be performed during different sessions of activity.

Selection of option 7 causes the computer to display the request for a data file name (see Exhibit 5-15):

Data File Name:

Following the examples presented earlier, the user should type in the following:

Data File Name: ABCDEFGH.CUS

where ABCDEFGH is the user's name or code identifier (see Exhibit 5-3). The program will print the following request:

Total Number of Variables in Input List:

The user should enter the number 11:

Total Number of Variables in Input List: 11

The program then asks for the number of variables to be used in the analysis. Here the user can select which of the eleven variables should be passed to the cusp program for analysis.

Number of Variables to be used in this analysis:

The user now enters a number, which is "3" in the example illustrated in Exhibit 5-17 but more than three variables can be used in the analysis, with the restrictions mentioned below.

Number of Variables to be used in this analysis: 3

The program then asks the user to identify the positions of the variables in the data file ABCDEFGH.CUS that will be used in the analysis.

Position of these dependent variables within the input list, with dependent variable LAST:

Here the user should enter the selected list. In the example shown in Exhibit 5-17, the following entries are made:

1: 4 (enter)
2: 10 (enter)
3: 11 (enter)

The pattern number (position 1) or BOTH the number of primary and secondary indicators (positions 2 and 3) AND the total number of indicators (position 4) should not be used together since this will produce irrelevant correlations, but BOTH "2" and "3" CAN be entered WITHOUT entering "4."

IWCAT System Overview Menu Display, Option 7: Perform Cusp Analysis

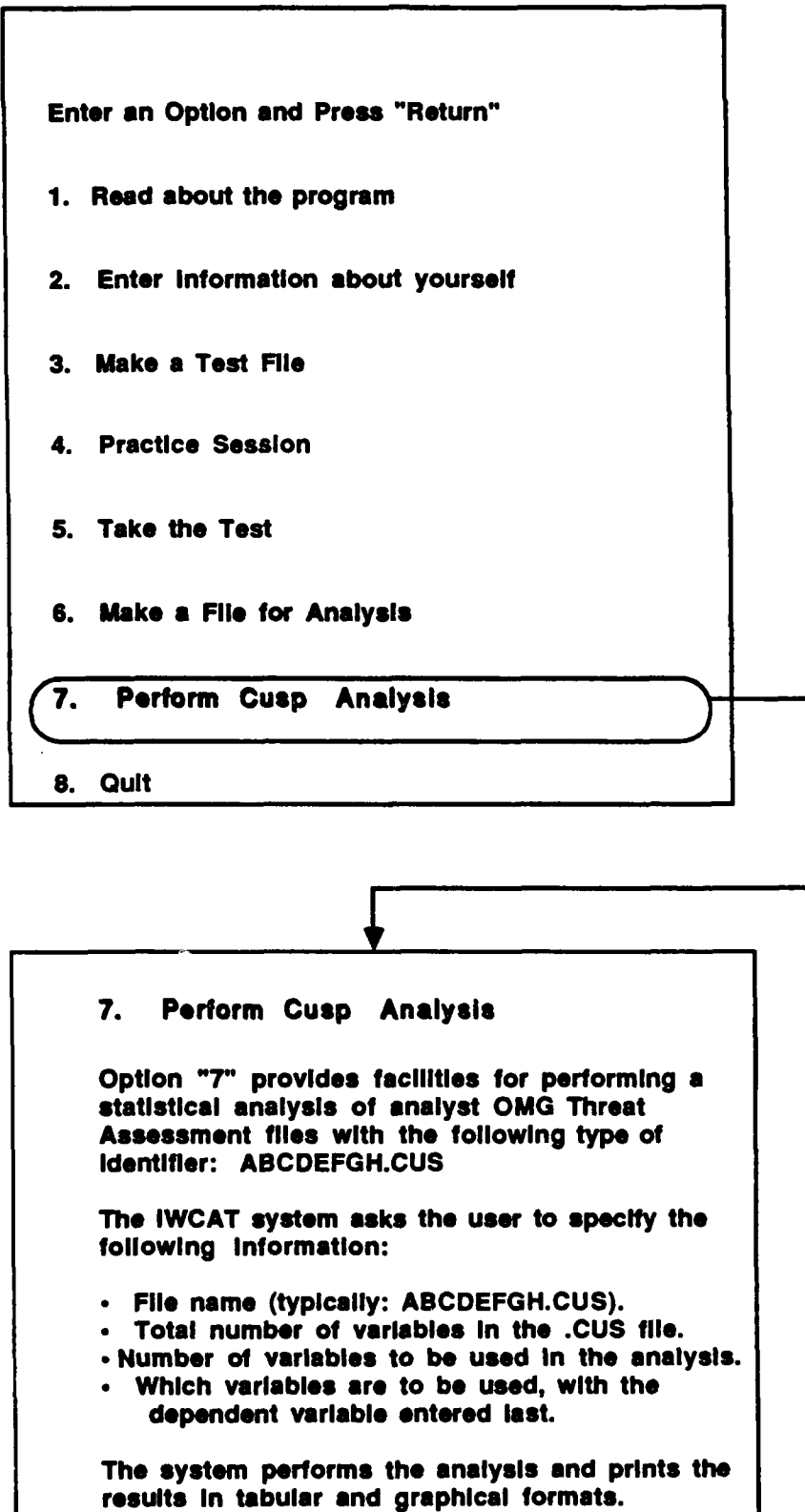
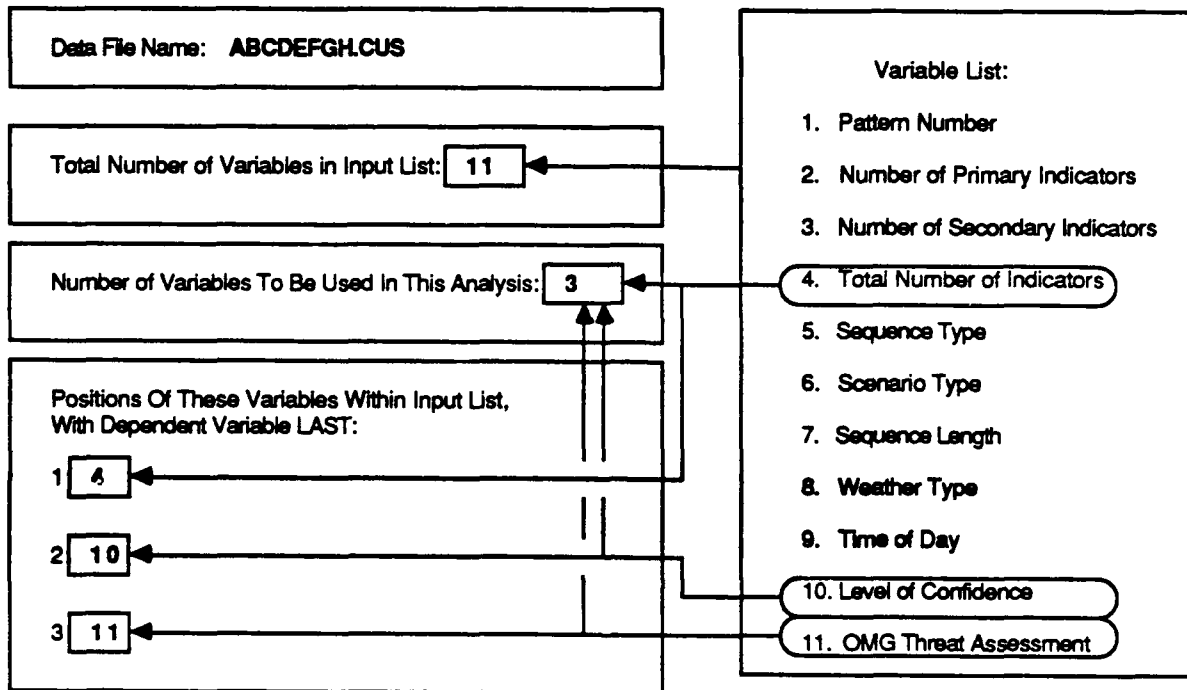


Exhibit 5-17

Cusp Analysis Activities

OPTION 7: CUSP ANALYSIS



Messages:

- Iteration halted, cubic coefficient vanishing
output has been printed to a file named "Cuspout"
- det(Hessian) negative trying again.
- Log likelihood not increasing, iteration terminated.
- *** NEAR ***; *** NEAR ***; *** NEAR ***
Convergence after # iterations, output has been
printed to a file named "Cuspout"

➤ **Print Cuspout**
Name of List Device [PRN]: prn
Resident part of PRINT installed
C:\WCAT\CUSPOUT is currently being printed

The statistical analysis program first fits a linear model to the user-generated data file and then determines whether or not the cusp model provides a better fit to these data. The cusp analysis program ceases when it is determined that the linear model is better than the cusp model.

The sample information presented in Exhibits 5-18 to 5-24 are the result of the use of the statistical program to analyze test data contained in a file named: "testdata." The cusp surface analysis program begins by displaying the mean and standard deviations of the input variables and the correlation matrix for these variables and also calculates the log-likelihood statistic and the standard linear regression coefficients for a linear model (Exhibit 5-18).

The statistical program then iterates toward conditions that maximize the likelihood of the cusp model based on the observed data. The iterative scheme is based on a modified Newton-Raphson method. If the very first iteration yields a decrease in the likelihood function, the program immediately halts with a message indicating that the linear model is preferable to any cusp model (this is not a rare occurrence). Upon successful convergence to a condition that maximizes the likelihood function, the estimated coefficients of each factor are stored in a file labeled CUSPOUT, for printing or review.

As iteration proceeds, the completion of each step in the process is signaled by the printing of a hyphen ("-"). When the iterative process requires the use of special numerical integration procedures, the following symbol ("+") is printed. Use of these special numerical procedures signal that the convergence process is being undertaken with difficulty, and might fail. One of several different messages can be printed out during the process of iteration toward convergence (see Exhibit 5-15, for example).

1. Iteration can be halted if the cubic coefficient vanishes and it is not possible to form a cusp-based model with the available data.
2. During the convergence processing, it may happen that the determination of a Hessian matrix used in the computation becomes negative. This suggests the program is having trouble with achieving a suitable convergence, and that the attempt might fail.
3. Convergence may also fail if the process does not lead to a reduction of the log-likelihood value.
4. As convergence is approached, the program prints out the message "**** NEAR ****" and when convergence is achieved, it prints out the following message:

Convergence after # iterations, please wait, output has been printed to a file named "CUSPOUT."

The user can now display the contents of the file labeled "cuspout" on the screen by responding to the instruction:

> Type cuspout

A hardcopy of the file cuspout can be generated for review and analysis by responding to the instruction (Exhibit 5-15):

> Print cuspout

Exhibit 5-18

Sample Output from the Cusp Surface Analysis Program

Reading 3 variables from testdata
Positions of the variables (dependent variable last):
2 3 1

Three variables entered from
the data file "testdata" with
positions of variables as specified

Var	Mean	St.Dev
2	0.7742	0.5877
3	60.0303	28.2387
1	47.4545	25.7530

Mean and Standard Deviation of
the values of the variables

Correlation matrix:

	2	3	1
2	1.00	0.55	-0.14
3	0.55	1.00	-0.31
1	-0.14	-0.31	1.00

Correlation matrix shows degree
of relationship between variables

Value of the log-likelihood for a linear
model

Log-likelihood of the linear model: -45.1800

Standard linear regression coefficients:

Var	Slope
2	0.042
3	-0.329

Coefficients obtained from a linear
regression analysis

Multiple R-squared: 0.095

Low multiple R-squared value
suggests that a linear fit of the
data is not satisfactory

Newton-Raphson algorithm, version of 1 Oct 1987

Step: 1: Log-likelihood: -45.180

Step: 2: Log-likelihood: -42.235

Step: 3: Log-likelihood: -40.358

Step: 4: Log-likelihood: -35.727

Step: 5: Log-likelihood: -34.197

Step: 6: Log-likelihood: -33.530

Step: 7: Log-likelihood: -33.376

*** NEAR ***

Step: 8: Log-likelihood: -33.354

*** NEAR ***

Step: 9: Log-likelihood: -33.350

*** NEAR ***

Convergence after 10 iterations.

Use of Newton-Raphson algorithm to
fit data to non-linear (cusp) model

Convergence after 10 iterations to
a lower log-likelihood value than for
a linear model shows the cusp model
to be a better fit for the data

5.3.8.1 Testing the Model

The parameter estimates reported by the cusp surface analysis program are useful for generating predictions, but their values indicate nothing about their statistical significance. Therefore the program also reports an approximate t-statistic for each coefficient, with $N-3v-3$ degrees of freedom. These can be interpreted in the usual fashion: magnitudes in excess of the critical value indicate that the coefficient is significantly different from zero, at the specified significance level (remember that these t-statistics are only approximate). Of course, these statistics can also be misinterpreted in the usual ways too. For example, it is a mistake to pay attention to any of these values unless the overall model has passed all of its tests for acceptability.

Cusp surface analysis offers three separate tests to assist the user in evaluating the overall acceptability of the cusp catastrophe model. The first test is based on a comparison of the likelihood of the cusp model with the likelihood of the linear model. The test statistic is an "asymptotic chi-square," which means that as the sample size increases the distribution of the test statistic converges to the chi-square distribution. The degrees of freedom for this chi-square statistic is the difference in the degrees of freedom for the two models being compared, i.e., $2v+2$. Sufficiently large values of this statistic indicate that the cusp model has a significantly greater likelihood than the linear model. The cusp catastrophe model may be said to describe the relationship between a dependent variable Y and vector X of independent variables if all of these three conditions hold:

1. The chi-square test shows that the likelihood of the cusp model is significantly higher than that of the linear model.
2. The coefficient for the cubic term and at least one of the coefficients of the factors A and B are significantly different from zero.
3. At least 10% of the data points in the estimated model fall in the bimodal zone.

The results of performing these tests are transferred to the "cuspout" file and are displayed for the test data file "testdata" in Exhibit 5-19, for example.

The data used in the cusp analysis, including the dependent and independent variables, the cusp estimation of the values of the modes and antimodes predicted by the cusp analysis program, and the position of each data point with respect to the axes of the cusp control space are displayed in Exhibit 5-20.

The computed positions of the data points from the "testdata" file are displayed in Exhibit 5-21. The analysis reveals that 78.8 % of the test data cases fall within the bimodal zone and that 7 of the 33 cases are located in coordinate positions outside the boundary of the figure. These latter 7 cases are, however, still part of the cusp analysis. The R^2 statistics suggest that the data is best fitted by a delay (or attracting-mode) convention transition rule (Exhibit 5-21).

The histogram of residuals (Exhibit 5-22) provides an indication of the degree to which the test data can be fitted to the cusp model. Exhibit 5-23 presents a plot showing the effect of variable 2 holding all others fixed at their mean values and represents a "slice" through the cusp catastrophe manifold computed with the aid of the cusp analysis program. Exhibit 5-24 shows a similar type of plot for variable 3 with the remaining variables held constant at their mean values.

Exhibit 5-19

Sample Output from the Cusp Surface Analysis Program

Cusp Surface Analysis, version of 4 October 1987.
 by Loren Cobb, Department of Biometry
 Medical University of South Carolina.
 Charleston, SC 29425.
 Phone 803-792-7575 for assistance.

Model: $0 = \text{Alpha} + \text{Beta} \cdot (\text{Y-Gamma}) - \text{Delta} \cdot (\text{Y-Gamma})^3.$

The conditional density of Y given X(1),...,X(v):

$f(Y|X) = \exp[\text{Psi} + \text{Alpha} \cdot Z + \text{Beta} \cdot Z^2/2 - \text{Delta} \cdot Z^4/4].$

where $Z = Y - \text{gamma},$

Psi = constant (with respect to Y),

Alpha = $A(0) + A(1) \cdot X(1) + \dots + A(v) \cdot X(v),$

Beta = $B(0) + B(1) \cdot X(1) + \dots + B(v) \cdot X(v),$

Gamma = $C(0) + C(1) \cdot X(1) + \dots + C(v) \cdot X(v),$

and $v = 2$ (in this analysis).

Maximum Likelihood Estimation for the Cusp Model:

Cases = 33

Log-Likelihood = -33.3502

Standard coefficients, with t-statistics in parentheses:

Var	Alpha	Beta	Gamma	Delta
Const	0.131 (0.5)	3.326 (2.2)	-0.146 (-1.5)	3.071 (2.7)
2	0.438 (1.5)	-0.318 (-0.4)	-0.314 (-2.4)	
3	-0.313 (-1.3)	-1.829 (-2.1)	0.036 (0.4)	

(Each t-statistic has 24 degrees of freedom)

Raw coefficients:

Var	Alpha	Beta	Gamma	Delta
Const	8.517e-3	1.151e-2	5.202e-1	5.981e-6
2	2.697e-2	-8.159e-4	-1.331e-1	
3	-4.306e-4	-9.769e-5	3.308e-2	

Test for H0: Conditional densities are Type N2 (linear regression)
 versus H1: Conditional densities are Type N4 (cusp regression)

>>>>> Asymptotic Chi-square = 23.66 (df = 6) <<<<<

Test for H0: Delta = 0 (i.e. no cubic term)
 versus H1: Delta > 0 (a one-tailed test)

>>>>> t = 2.65 (df = 24) <<<<<

This is the printing of the file:
 "Cuspsoul"

The Alpha Variable is the Normal
 Factor (at "right-angles" to the cusp)

The Beta Variable is the Splitting
 Factor ("opens up the cusp")

The Gamma variable is a linear
 factor that can linearly displace the
 cusp surface

The Delta Variable represents the
 coefficient multiplying the cubic or
 "cusp" term

Chi-square test shows cusp-like
 properties are significant

the "t" test shows cusp-like
 properties are significant

Exhibit 5-20

Sample Output from the Cusp Surface Analysis Program

Estimated correlations between Alpha and Gamma estimators:

	G0	G1	G2
A0	-0.48	0.27	-0.07
A1	0.19	-0.39	-0.70
A2	-0.20	-0.19	-0.33

Predictions based on this analysis:

Case	Asym	Bifur	Mode	Antimode	Mode	Y(0 1)	X(0 1)	X(0 2)
1	0.64	0.94			49.41	25.00	1.01	87.00
2	-0.73	3.01	27.05	63.03	64.51	30.00	-0.35	62.00
3	0.30	4.23	13.76	41.20	74.11	74.00	0.79	46.00
4	0.26	1.05			54.06	53.00	1.39	90.00
5	-0.15	2.22	23.04	47.60	64.70	41.00	0.64	70.00
6	-0.20	1.30	25.74	48.47	59.63	35.00	0.70	90.00
7	0.11	4.36	15.09	45.69	77.37	53.00	0.54	46.00
8	0.43	5.30	10.57	41.10	70.12	6.00	0.73	30.00
9	0.24	4.27	24.47	42.64	75.15	74.00	0.71	46.00
10	0.32	1.01			53.23	65.00	1.47	92.00
11	-0.36	2.96	24.11	54.00	74.39	62.00	0.26	70.00
12	-0.89	2.54	31.39	71.76	72.12	26.00	-0.20	81.00
13	0.12	7.20	13.44	52.22	92.29	22.00	-0.03	7.00
14	0.54	0.85			50.00	10.00	1.77	92.00
15	0.05	1.42	23.67	39.96	50.72	74.00	1.06	87.00
16	0.44	0.02			51.53	10.00	1.43	92.00
17	0.34	4.19	13.24	40.10	73.32	10.00	0.85	46.00
18	0.50	2.71	14.20	29.06	61.56	53.00	1.44	64.00
19	-0.19	1.59	25.02	48.01	61.71	35.00	0.75	87.00
20	0.00	7.22	13.06	53.02	92.94	66.00	-0.00	7.00
21	0.34	4.20	13.33	40.29	73.46	62.00	0.64	46.00
22	0.22	4.20	14.73	43.10	75.54	62.00	0.60	46.00
23	-0.73	2.30	23.77	40.76	67.90	74.00	0.56	70.00
24	0.21	6.91	12.25	49.69	89.51	94.00	0.14	10.00
25	0.57	6.65	7.35	41.88	83.06	94.00	0.62	12.00
26	-0.30	2.39	25.37	54.23	70.21	26.00	0.35	70.00
27	0.40	3.57	13.44	36.00	68.94	10.00	1.05	54.00
28	0.37	2.07	15.00	35.44	65.21	35.00	1.15	64.00
29	0.51	6.01	7.04	43.67	85.15	74.00	0.50	7.00
30	-0.34	5.27	21.14	57.32	60.56	14.00	-0.10	30.00
31	0.03	2.23	21.04	42.44	64.97	71.00	0.00	76.00
32	0.41	0.94			51.96	53.00	1.59	92.00
33	0.24	1.06			54.27	35.00	1.37	92.00

For the 33 sets of data elements in the "testdata" file, the cusp analysis program estimates the following properties:

Position with respect to the Normal and Splitting Factor Axes

Positions of the mode(s) and antimode of the property distribution function with regard to the behavior variable axis

The program also prints the values of the following variables:

a. The dependent variable Y (#1)

b. The first independent variable X (#2)

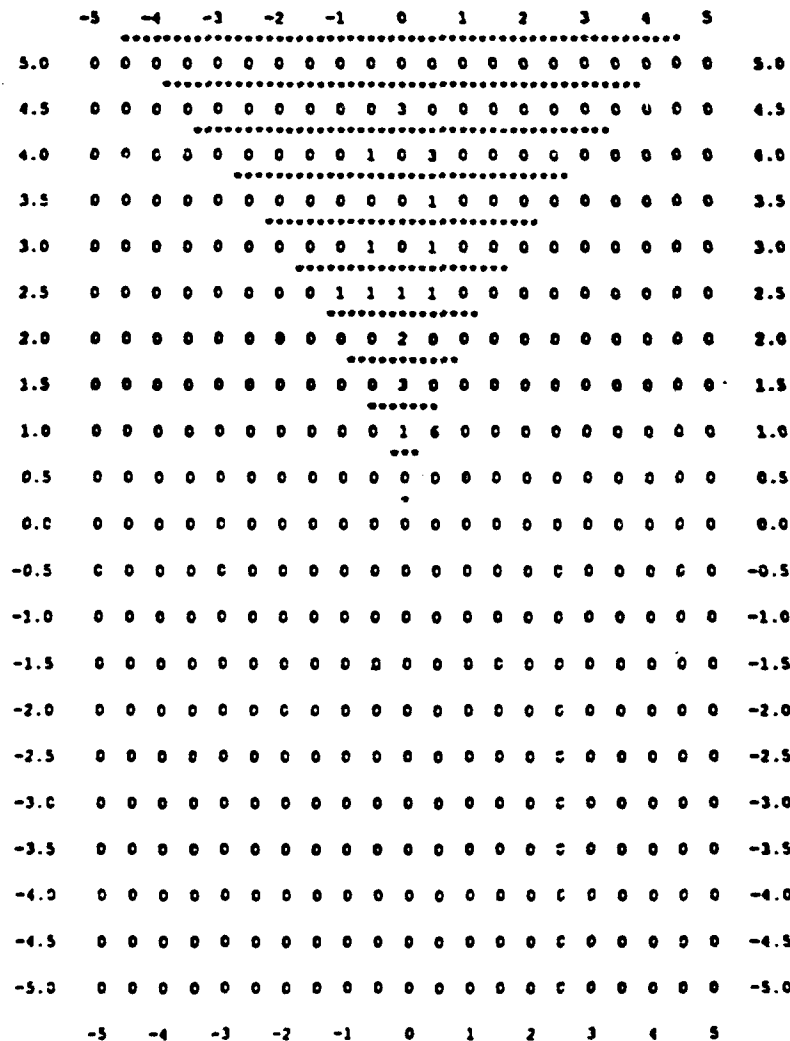
c. The second independent variable X (#3)

Exhibit 5-21

Sample Output from the Cusp Surface Analysis Program

Location of data in the control space:

Vertical axis: Bifurcation (splitting) factor
Horizontal axis: Asymmetry (normal) factor
Asterisks: Bimodal zone



7 cases did not fit in the above figure.
 >>>>> Fraction of cases in bimodal zone: 0.788 <<<<<
 Linear R² = 0.095 (Multiple regression)
 Delay R² = 0.695 (Attracting-mode convention)
 Maxwell R² = -0.178 (Most-likely-mode convention)

* Negative R² values occur when the cusp model is worse than a constant.

The computed positions of the data from the file "testdata" have been plotted with respect to the control plane of the cusp catastrophe model

The asterisks (*) represent the region of bimodality or ambiguity

The zeros (0) represent the remaining region of the control space

The numbers 1, ..., 6 in this chart represent the number of times that data elements occurred at a particular location on the control space

7 of the 33 cases possess estimated values that place them outside the range +/- 5.0 for the two independent variables

78.8% of the cases fell within the bimodal zone

R-squared values for three cases indicate that the data is best fitted by a delay (or attracting-mode) convention transition rule

Exhibit 5-22

Sample Output from the Cusp Surface Analysis Program

Histogram of residuals from predictions of the delay rule:
(Units are standard deviations of the dependent variable.)

Y	N
-3.00	0
-2.75	0
-2.50	0
-2.25	0
-2.00	0
-1.75	0
-1.50	2
-1.25	0
-1.00	1
-0.75	1
-0.50	5 + +
-0.25	7 + + +
0.00	5 + +
0.25	5 + +
0.50	5 + +
0.75	2
1.00	0
1.25	0
1.50	0
1.75	0
2.00	0
2.25	0
2.50	0
2.75	0
3.00	0

Histogram of residuals represents the degree to which the data used in the statistical analysis differs from data associated with a "perfect" cusp model. Such perfect data would have zero mean error and zero standard deviation.

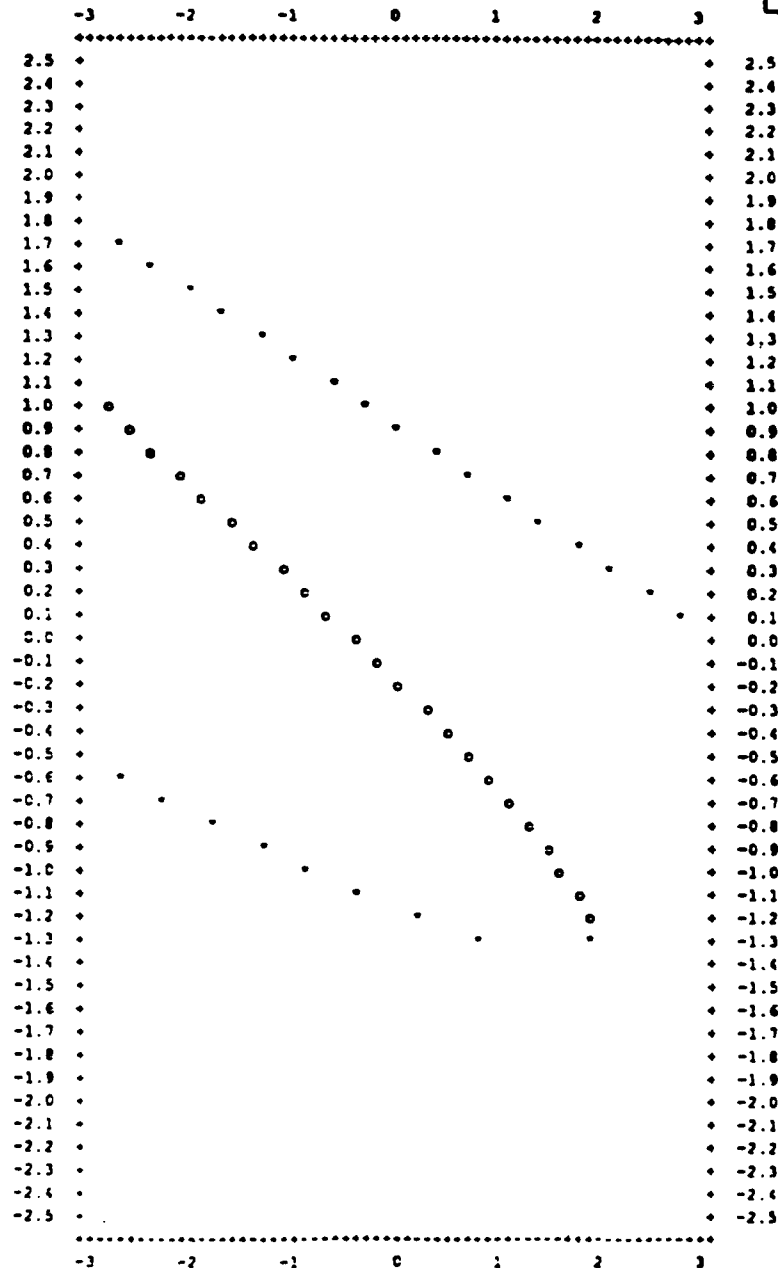
Error Mean = -0.116
Error St.Dev = 0.553

Exhibit 5-23

Sample Output from the Cusp Surface Analysis Program

Plot of a "slice" of the cusp catastrophe manifold constructed with the data derived from the file "testdata" drawn for a range of values of independent variable 2 shown on the horizontal scale with the value of the independent variable 3 fixed at its mean value. The vertical scale is the value of the dependent variable.

Effect of variable 2, holding all others constant at their mean values.

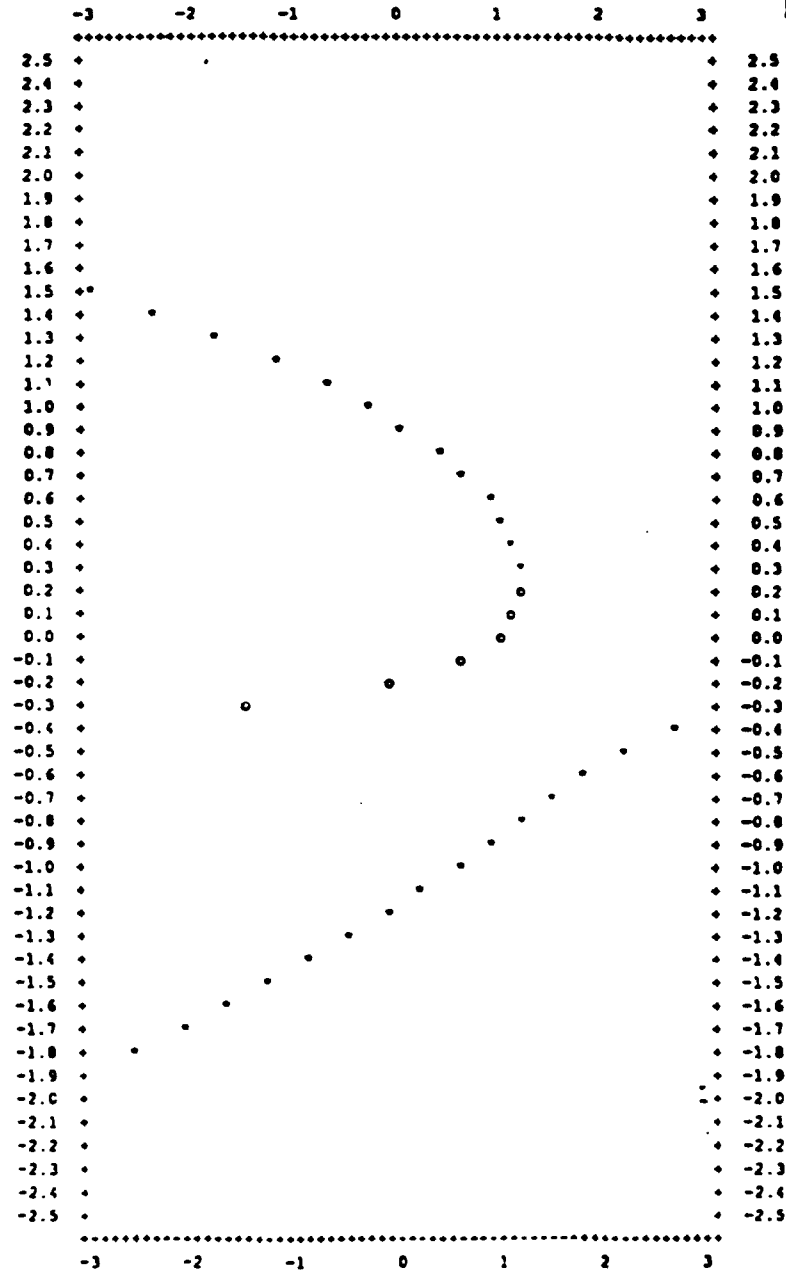


* Mode symbol
o Antimode symbol

Exhibit 5-24

Sample Output from the Cusp Surface Analysis Program

Effect of variable 3, holding all others constant at their mean values.



* Base symbol
o Anti-base symbol

5.3.8.2 Making Predictions

In the literature on applications of catastrophe theory there are two distinct ways of calculating predicted values from a catastrophe model. In the **Maxwell Convention** the predicted value is the most likely value, i.e., the position of the highest mode of the probability density function. In the **Delay Convention** a mode is also the predicted value, but it is not necessarily the highest one. Instead, the predicted value is the mode that is located on the same side of the antimode as the observed value of the state variable Y . Thus the delay convention uses as its predicted value the equilibrium point towards which the equivalent dynamical system would have moved. This is the convention most commonly adopted in applications of catastrophe theory, but there are circumstances in which the Maxwell convention is the appropriate convention.

The cusp surface analysis program calculates the predictions made under each convention for each datum, and from these it derives a number of statistics and graphs to aid the user in evaluating the quality of the predictions made under each convention, as follows:

Modes and Antimodes: The estimated factors and modes and anti-modes of the data are presented to the user in tabular form.

Delay- R^2 : This statistic is simply the estimated value of the quantity $1 - (\text{error variance}) / \text{var}[Y]$, in which the errors are based on predictions of the delay rule. Although it is analogous to the multiple- R^2 of regression analysis, there are several important differences which are discussed below.

Maxwell- R^2 : This is the corresponding statistic for the predictions of the Maxwell rule.

Error Histogram: This graph depicts the distribution of the errors made under the delay rule. Also provided on this page are the mean and standard deviation of this distribution. These statistics are also known as the bias and standard error of estimate, respectively, of the estimates.

In multiple linear regression the estimates obtained by minimizing the mean squared prediction error coincide with those obtained by maximizing the likelihood function. In Cusp Surface Analysis this is not the case, and therefore the maximum likelihood estimates maximize neither the Delay- R^2 nor the Maxwell- R^2 . Further, neither of these statistics is even guaranteed to be positive! Negative values occur when the cusp model fits so wretchedly that its error variance actually exceeds the variance of Y .

5.3.9 TERMINATION OF ANALYTIC SESSION

Selecting option 8 ("Quit") terminates the analytic session.

SECTION 6. OMG THREAT ASSESSMENT ANALYSIS

The IWCAT system software was used in a series of tests. During these tests, individuals with experience in the indications and warning (I&W) and intelligence analysis areas were asked to assess the perceived level of Operational Maneuver Group (OMG) threat associated with a series of sets of OMG-related indicators (Exhibit 6-1). The IWCAT system permits the creation of an OMG threat assessment data base and its subsequent analysis with a nonlinear statistical program based on catastrophe theory in order to construct a mathematical model of the data. Such a model could be used as the basis for further analysis of the responses of I&W analysts to situations of interest.

In this model-making process, a linear regression model is constructed first. Then an attempt is then made to construct a nonlinear model of the data that provides a better fit for the data than does this linear model. Under some circumstances a nonlinear model cannot be constructed for a number of reasons and the linear regression model is the model of choice. However, when a nonlinear model can be constructed, it is possible to describe a range of different I&W analyst response behaviors such as sudden and gradual perceptual changes, divergence, ambiguity, hysteresis, perceptual "trapping," and counter-intuitive or paradoxical behavior that are characteristic of the analyst responses. One particularly interesting discovery provides statistical evidence that suggests that I&W analysts with different types of training and previous mission responsibilities appear to respond to different features of the OMG threat assessment data set. Details and interpretations of the models constructed with the aid of the IWCAT system are presented below.

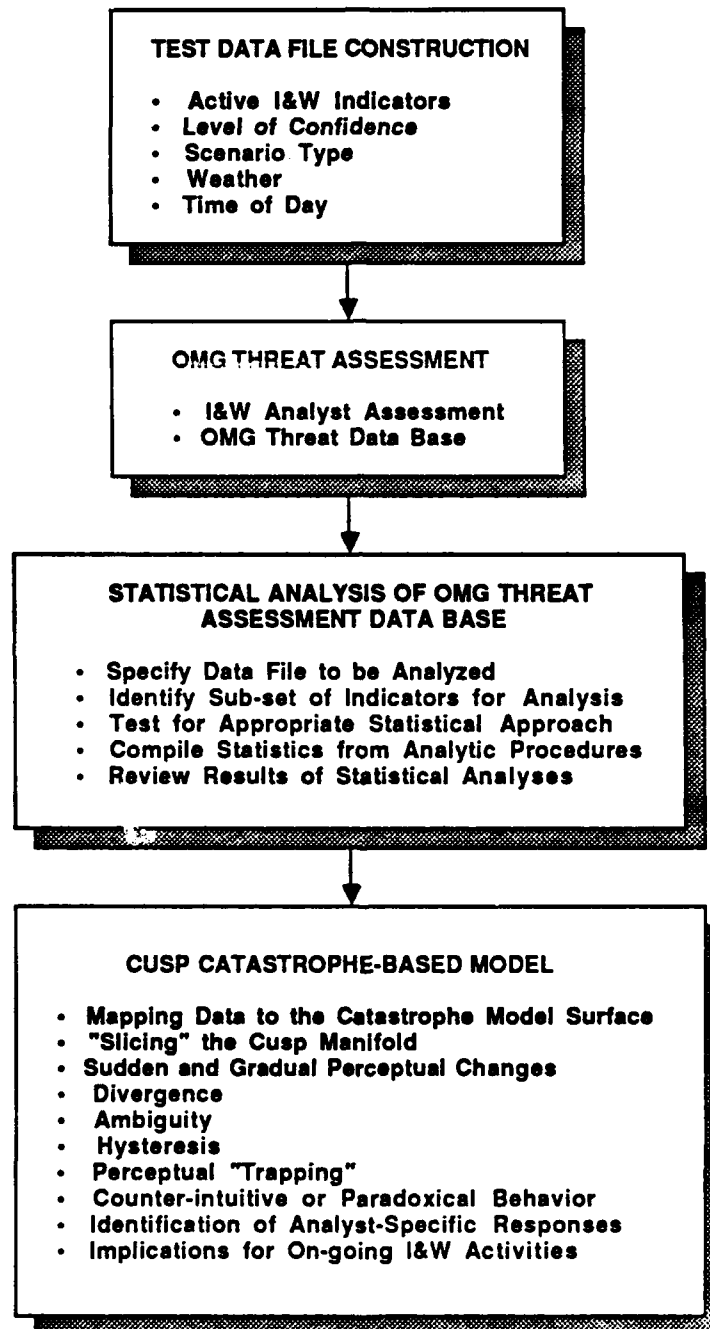
6.1 STATISTICAL ANALYSIS OF THE OMG THREAT ASSESSMENT DATA BASE

The IWCAT system described in this report provides methods for capturing analyst's assessments of the level of OMG threat. These assessments are derived from analyst inspection of a series of OMG-related indicators and other information and stored in a data base for subsequent statistical analysis. This statistical analysis is performed with the aid of procedures based on statistical catastrophe theory and results in the construction of mathematical models of the analyst's OMG threat assessment data. These models can be used to reveal potential or actual occurrences of ambiguous or conflicting perceptions and to identify situations where sudden perceptual changes may take place.

As a first step in the process of model formation, the program constructs a linear regression model of the data. Then a nonlinear model based on the cusp catastrophe is constructed if possible. Computations are performed to determine the degree of nonlinearity of the data and comparisons are made between the relative successes of the linear and nonlinear models in capturing the nature of these data. A series of statistical measures, including log-likelihood values, R^2 -, t- and chi-squared statistics are computed and provide methods for assessing the relative validities of the linear and nonlinear models. Review of these statistics and inspection of control plane plots and "slices" of the catastrophe surface model based on analyst-derived data provides a comprehensive understanding of the results of OMG-threat perception.

Exhibit 6-1

IWCAT System Activities



6.1.1 MAPPING DATA TO THE CATASTROPHE MODEL SURFACE

The IWCAT system uses the method of maximum likelihood to estimate the parameters of a model based on the observed data. When a cusp-based model can be constructed from these data, the system performs statistical tests to determine whether a linear or the cusp-based model is the most appropriate. A cusp-based model of the data is accepted when the chi-square test shows the likelihood of the cusp model to be significantly higher than that of the linear model; the coefficient of the cubic term and one of the other coefficients of the cusp model are significantly different from zero; and at least 10% of the data points in the estimated model fall in the bimodal region (see Section 4., for example).

In the process of constructing the cusp model, the system transforms the input data to fit a cusp catastrophe surface. This surface is an inherently three-dimensional object which can be drawn as a structure with three axes, each of which represents a component of the cusp model. Two of these axes represent the control factors or input variables and the third axis represents the behavior or output variable of the system of interest. Positions on the surface can be located with respect to the values of these three axes. The two control factors, which may themselves be a function of other variables, define a plane called the control plane.

The IWCAT system provides the user with a series of diagrams which display the features of the cusp model. In the control plane display (see Exhibit 6-2, for example), the transformed data are presented as locations on the control plane formed from (transformed) versions of the control factors called the bifurcation (or splitting) and asymmetry (or normal) factors. In this figure the asterisks (*) represent the region of bimodality and the control factor axes are scaled to ranges of values ± 5.0 . Exhibit 6-2 represents a case where some 41.5% of the data are located within the bimodal region. This fact and the display of the R-square statistical values in the exhibit indicate that a cusp model with a delay (or attracting mode) transition convention is a much better "fit" for the analyst-derived data than is a linear model. Under such circumstances, the predictions based on a linear model would have been misleading in at least four out of ten cases.

Based on this analysis, it is possible to construct a cusp catastrophe model that captures the properties of the I&W analyst-derived OMG-threat assessment data (see Exhibit 6-3, for example). This exhibit shows a representation of a cusp model that might be constructed from such data. The (transformed) actual data is located within the circle drawn on the control plane formed from transformations of the number of active indicators and level of confidence control factors as described in Section 4. Some of these data lie inside, and the remainder lie outside, the region of bimodality or ambiguity on the control plane. Consideration of the third (behavioral or output) variable value for each of the data points permits the construction of the cusp model surface and the resolution of the condition of perceptual ambiguity, for example (Exhibit 6-3).

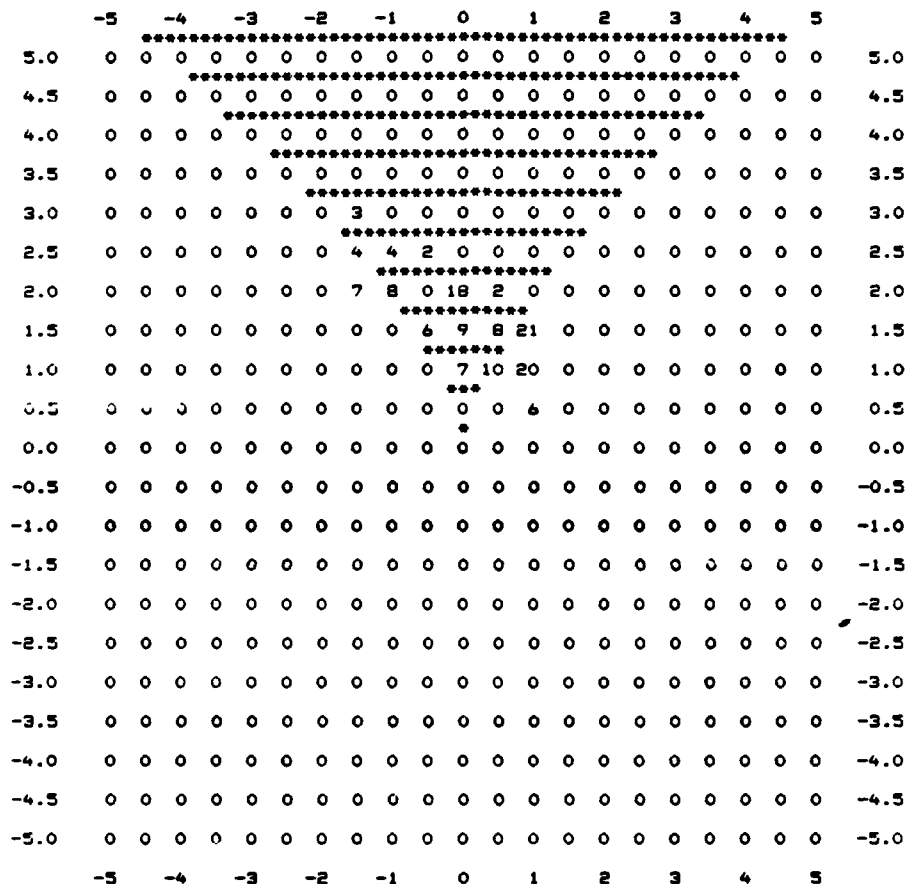
This process, which results in the mapping of the data to the cusp surface, provides a model which is the best available "fit" for the input data, and provides prediction of the behavior to be expected for conditions that are outside the range of these data. Thus, while the actual data might be located in a proscribed region of the surface, other locations on the surface, representing other conditions, are also available and might be accessed under appropriately changed conditions. The remainder of this section discusses some of the general properties of cusp catastrophe-based models of I&W analyst OMG threat assessment and then relates these general properties to the results of specific analyst assessments.

Exhibit 6-2

I&W Analyst-Derived Data Plotted on the Control Plane of the Cusp Model

Location of data in the control space:

Vertical axis: Bifurcation (splitting) factor
 Horizontal axis: Asymmetry (normal) factor
 Asterisks: Bimodal zone



0 cases did not fit in the above figure.

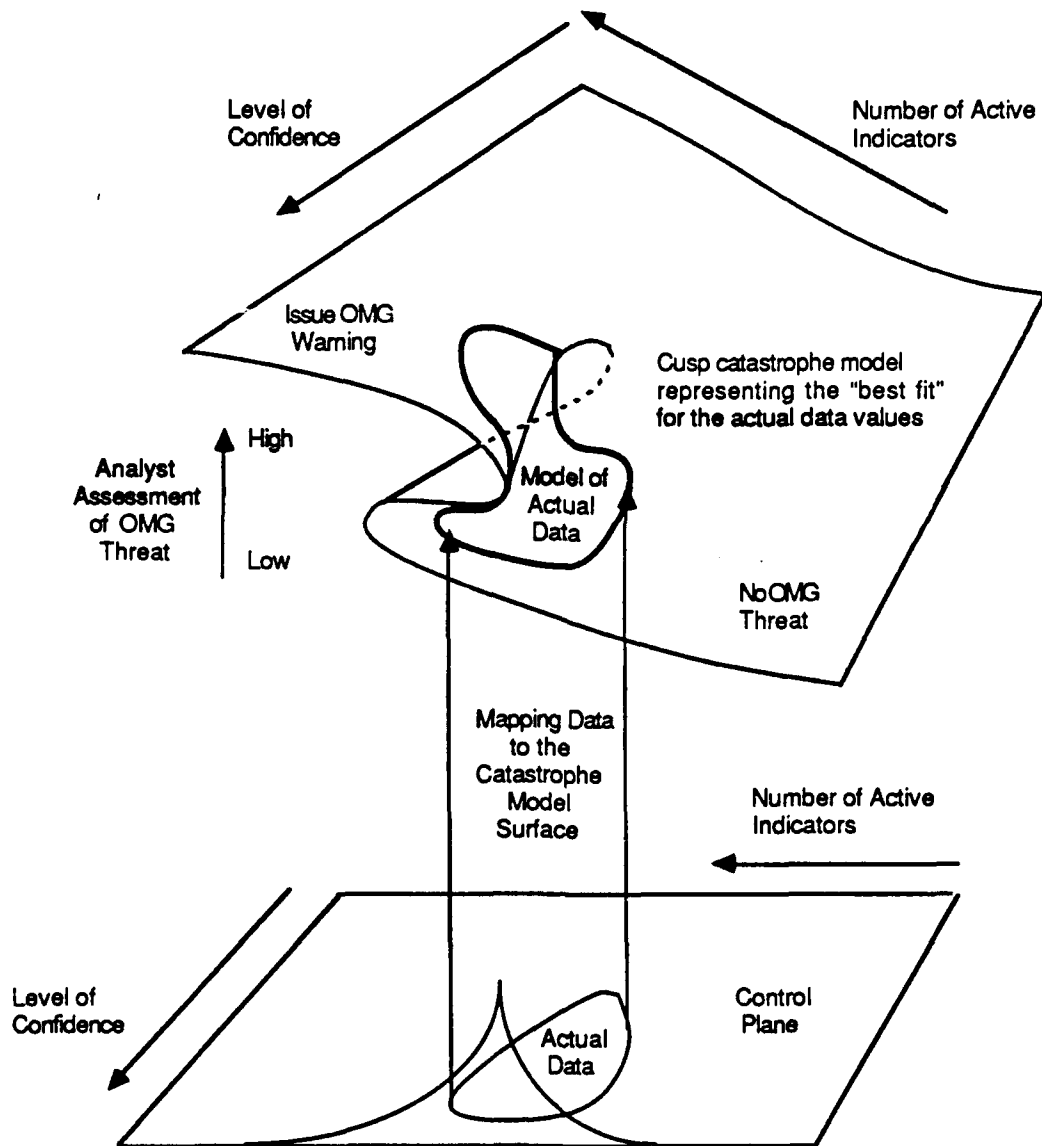
Linear $R^2 = 0.285$ (Multiple regression)
 Delay $R^2 = 0.472$ (Attracting-mode convention)
 Maxwell $R^2 = -0.299$ (Most-likely-mode convention)

* Negative R^2 values occur when the cusp model is worse than a constant.

>>>>> Fraction of cases in bimodal zone: 0.415 <<<<<

Exhibit 6-3

Mapping Data to the Cusp Model Surface



6.1.2 SUDDEN AND GRADUAL PERCEPTUAL CHANGES

The cusp catastrophe model constructed from I&W analyst-derived data describes conditions under which sudden and gradual changes in analyst perceptions can take place. As an example, the model permits the following types of observations to be made. An increase in the number of active indicators with high confidence values presented to the analyst can cause a sudden increase in the perceived level of OMG threat and may lead to the issuing of an OMG warning (Exhibit 6-4, path (a-b-c), for example). By contrast, a similar increase in the number of active indicators with low confidence value may lead to a slight increase in perceived OMG threat and may not lead to the issuing of an OMG warning (Exhibit 6-4, path (d-e), for example).

6.1.3 DIVERGENCE

The catastrophe model can also illustrate the property of perceptual divergence, as shown in Exhibit 6-5, where relatively small differences in the initial number of active indicators presented to an I&W analyst can have a profound impact on the nature of the analyst's OMG threat assessment as the level of confidence in these data is increased. Thus, the model suggests that presenting the analyst with a number of active indicators and level of confidence data sets represented by positions (a) and (c) on the cusp surface and an intermediate OMG threat would lead to a small difference in perceived threat. Increasing the level of confidence in these data, without changing the number of active indicators, can lead to dramatic changes in threat assessment. Path (a-b) represents changing conditions which would lead to the issuing of an OMG warning, while path (c-d) would lead to the perception that no OMG threat existed.

6.1.4 AMBIGUITY

Preconditioning can lead to perceptual ambiguity, a phenomena which is illustrated with the aid of the catastrophe model shown in Exhibit 6-6. Presenting an I&W analyst with the sequence of data sets represented by positions (a), (b), (c), and (d), for example, would lead an analyst to the perception of a low level of OMG threat. By contrast, the data sequence (e), (f), (g), and (h), for example, suggest the perception of a high level of OMG threat and the possible issuing of an OMG warning by the analyst. Thus, while positions (d) (no perceived OMG threat) and (h) (issue an OMG warning) can represent exactly similar levels of confidence and number of active indicators, these inputs produce manifestly different, ambiguous, results (Exhibit 6-6). Under these circumstances, the catastrophe model provides a method for understanding the causes of perceptual ambiguity and the basis for techniques that could be used to alert analysts to the existence of such ambiguities.

6.1.5 "SLICING" THE CUSP SURFACE

The IWCAT system provides the user with a series of diagrams representing "slices" of the catastrophe model surface in which all but one of the control factors are held fixed at their mean values and the effect of changes in the remaining factor on the shape of the surface is displayed. A slice formed by maintaining a fixed number of active indicators and varying the level of confidence values and another slice formed by maintaining a fixed level of confidence and varying the number of active indicators are illustrated in Exhibits 6-7a and 6-7b, respectively. When the catastrophe model has been derived from I&W analyst data, the shapes of these

Exhibit 6-4

Cusp Model of Sudden and Gradual Changes in Analyst Perceptions

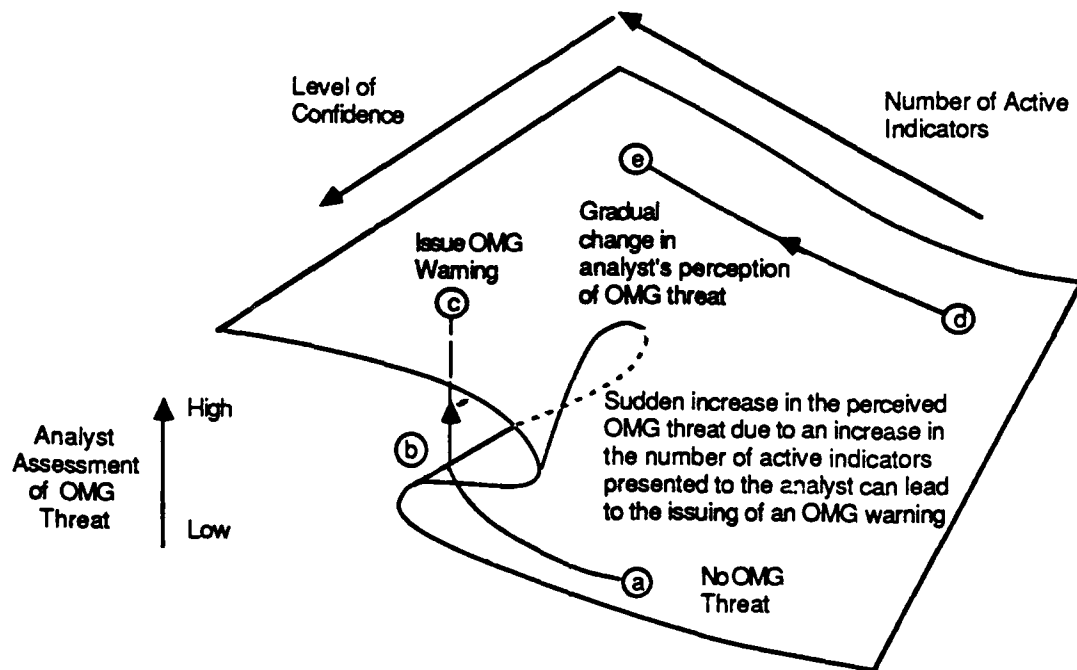


Exhibit 6-5

Cusp Model of Divergent Perceptions

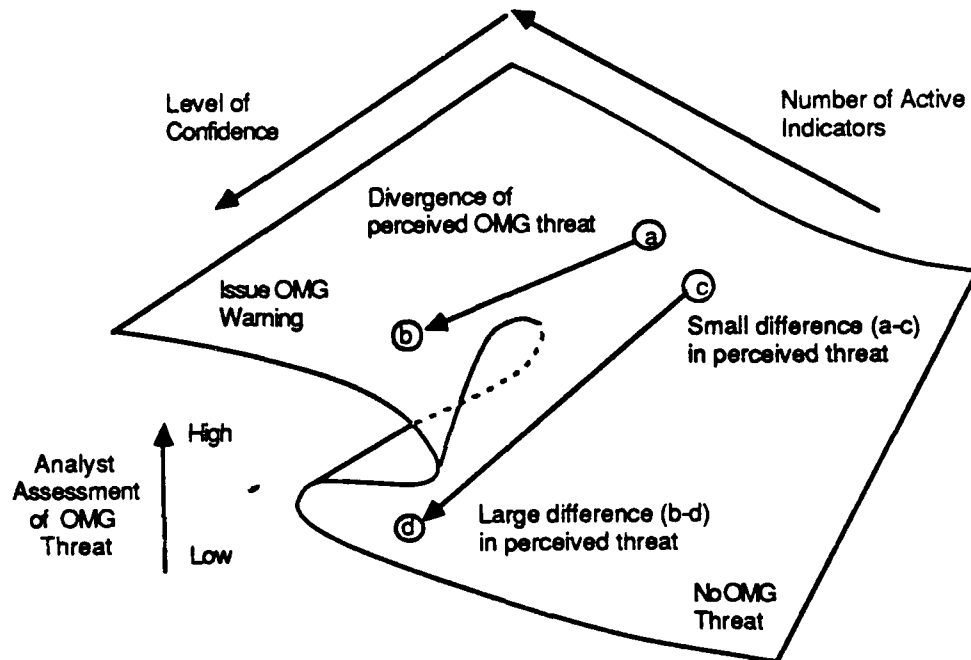


Exhibit 6-6

Cusp Model Can Provide a New Understanding of the Causes of Perceptual Ambiguity

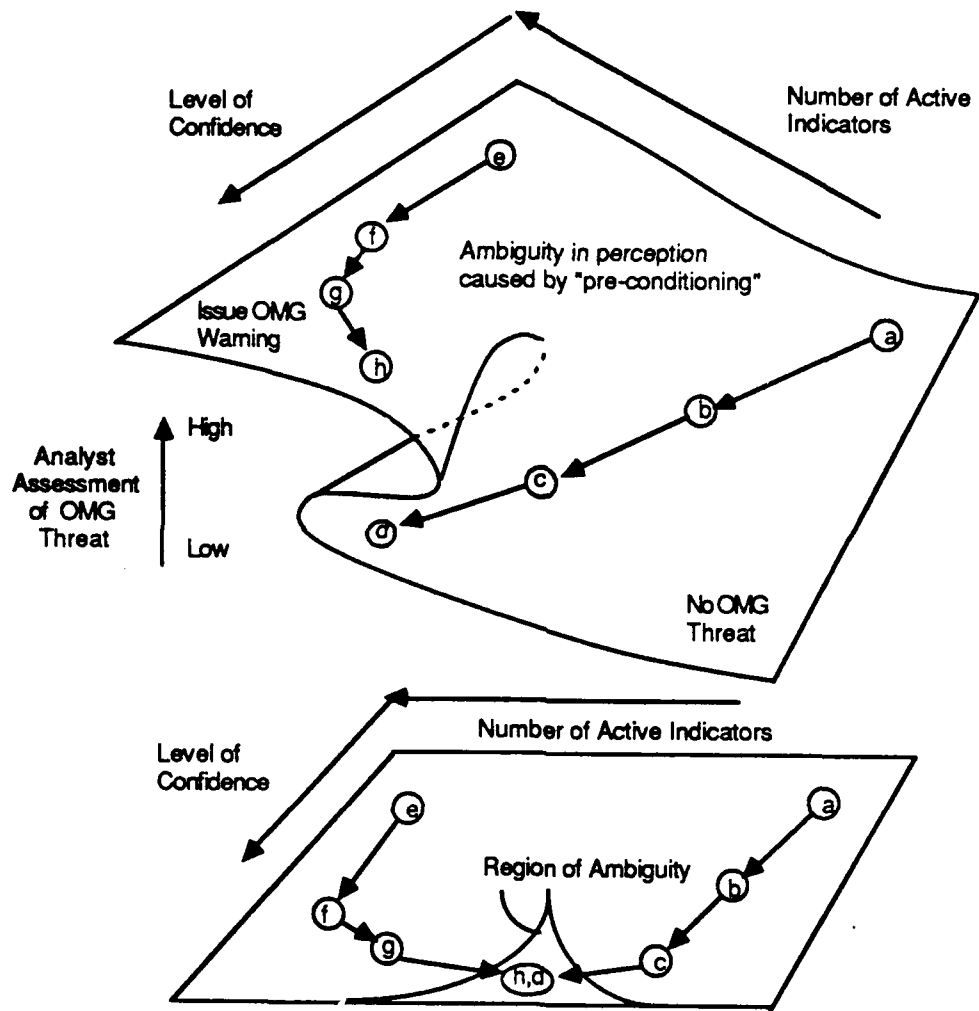
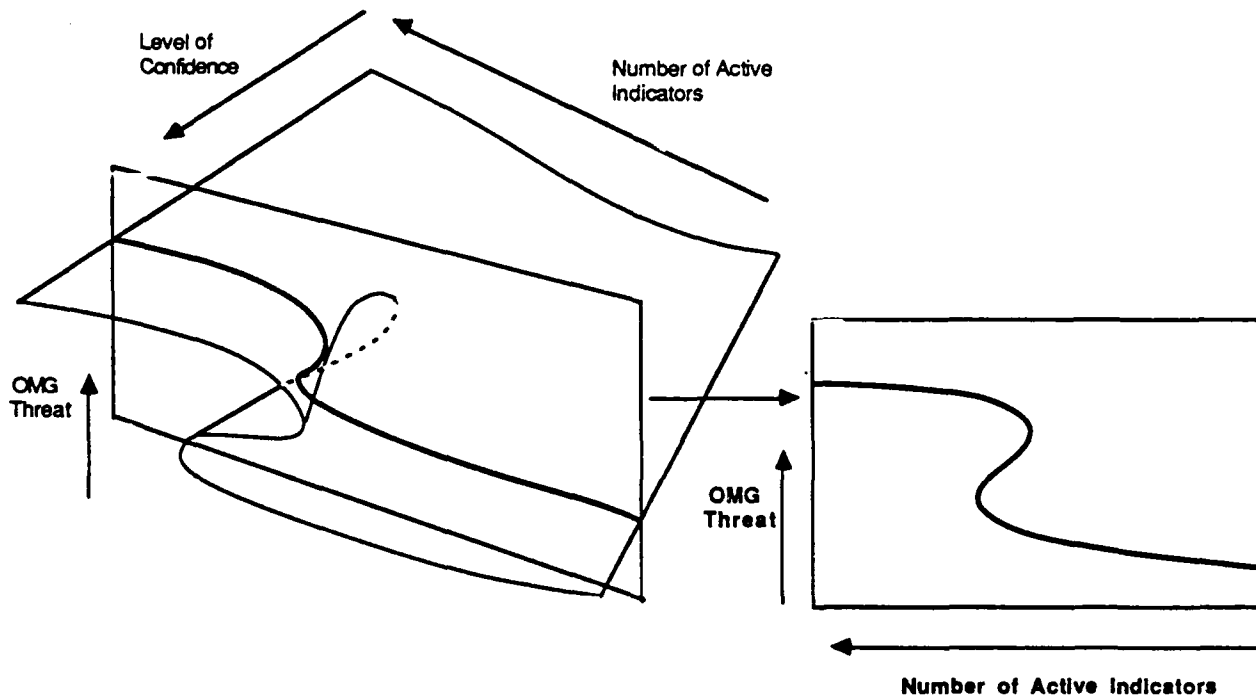
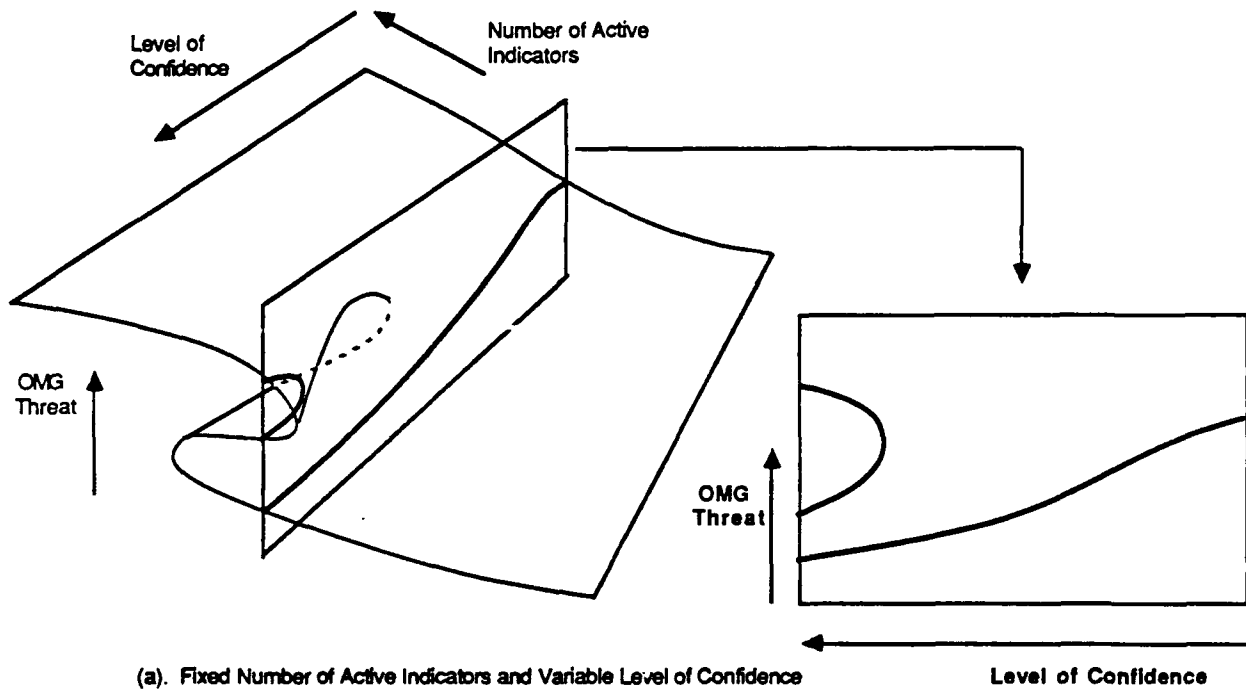


Exhibit 6-7
"Slicing" the Cusp Surface



sections of the model surface have profound implications for the range of possible analyst behaviors and the nature of analyst assessments.

Thus, with a fixed number of active indicators, the "slice" of the catastrophe surface consists of two apparently unconnected portions (Exhibit 6-7a) while that for fixed level of confidence is a continuous curve resembling an overfolded "S." Such shapes can illustrate the phenomenon of perceptual hysteresis (Exhibit 6-8), a phenomena that Woodcock has called partial and complete perceptual "trapping," (Exhibits 6-9 and 6-10) and counter-intuitive or paradoxical behavior (Exhibit 6-11), as described below.

6.1.6 PERCEPTUAL HYSTERESIS

Small changes in the number of active indicators (or for that matter in the level of confidence on the data) can lead to sudden changes in the perceived level of OMG threat and simply returning to this initial number of active indicators will not lead to an immediate restoration in the level of perceived threat to its existing level. This phenomena, which can be identified as a form of "perceptual hysteresis," may be illustrated with the aid of the catastrophe surface model (Exhibit 6-8). Beginning with a low number of active indicators and low level of perceived OMG threat, an increase in the number of active indicators presented to the analyst can lead to a sudden change in perceived threat. By contrast, beginning with large number of active indicators and a high OMG threat level and then decreasing the number of active indicators can lead to conditions under which a sudden decrease in perceived OMG threat can take place. These sudden changes in perceived OMG threat generally take place at different numbers of active indicators. Thus, a sudden increase in perceived threat could occur when the number of indicators presented to the analyst is increased from seven to eight while a decrease in threat perception might only occur when the number of active indicators is decreased from eight to two indicators, for example.

6.1.7 PERCEPTUAL "TRAPPING"

Use of the IWCAT system by a series of intelligence analysts has led to the characterization of a phenomenon which Woodcock has called "perceptual trapping." Under such circumstances, the analyst's perceptions would be restricted to a particular state (such as the perception of no OMG threat or the issuing of an OMG warning). The analyst would be unable to change this perception based on available data. Thus, such an analyst might continue to issue an OMG warning despite the fact that another analyst who was not similarly perceptually trapped, would have withdrawn the warning, for example.

Exhibit 6-9 illustrates the phenomena of partial perceptual trapping while Exhibit 6-10 illustrates complete trapping. Perceptual trapping occurs because an individual's perceptions appear to be restricted to a particular portion of the catastrophe surface and no "return path" exists to permit a return to the initial state after a sudden change in perception has occurred. This restriction is caused by the shape of the catastrophe surface and the "slices" that are constructed from this surface. However, such a return path may be created as the result of the consideration of an extended set of I&W indicators since the assessments of these new data items would lead to a change in the shape of the catastrophe model derived from these data, for example.

Exhibit 6-8
Perceptual Hysteresis

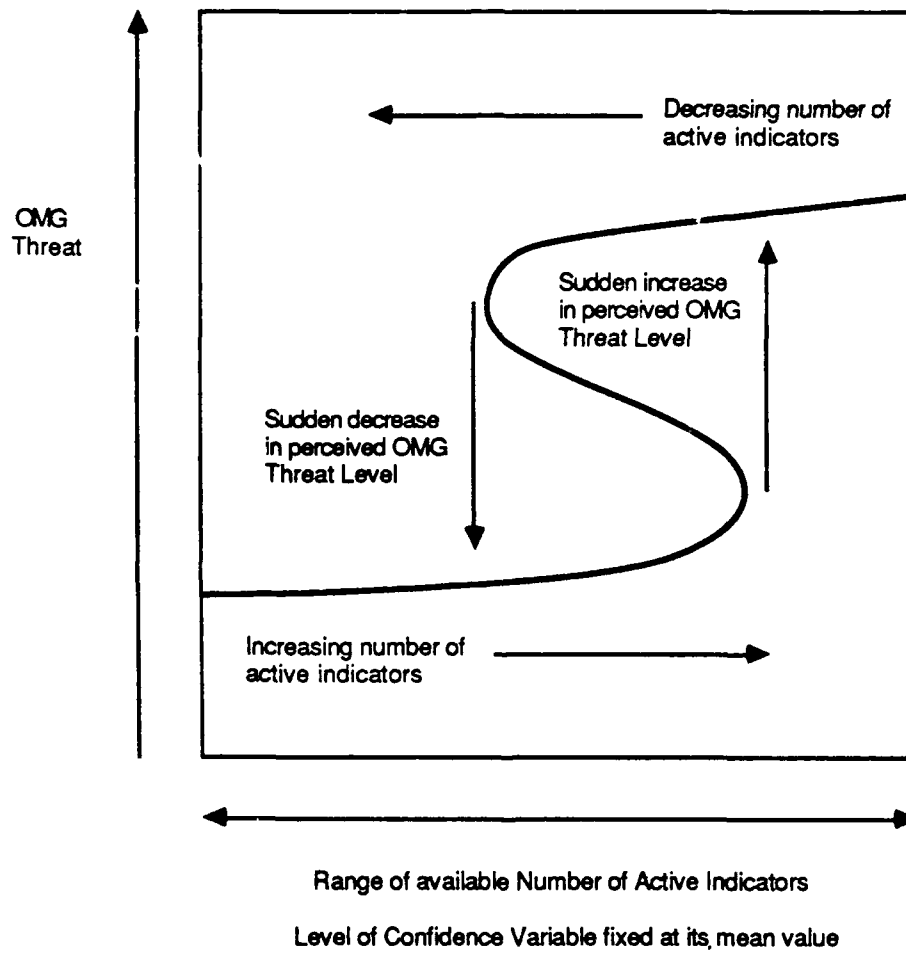


Exhibit 6-9
Partial Perceptual "Trapping"

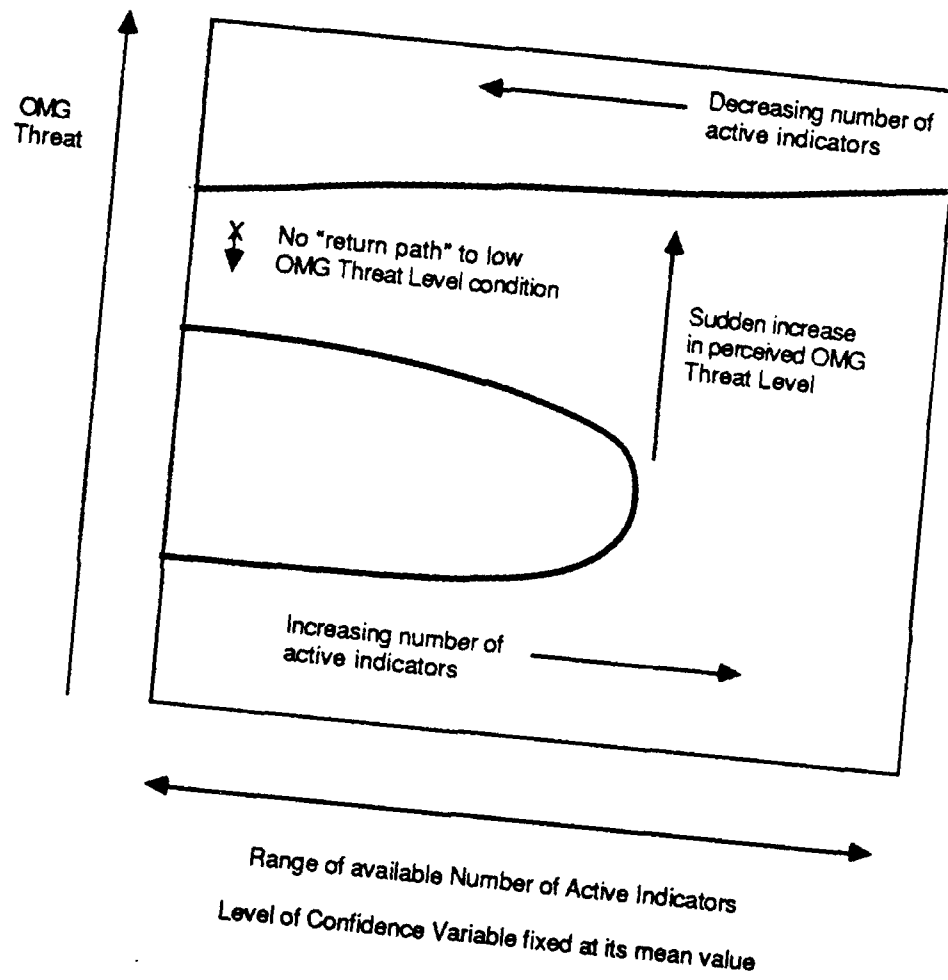
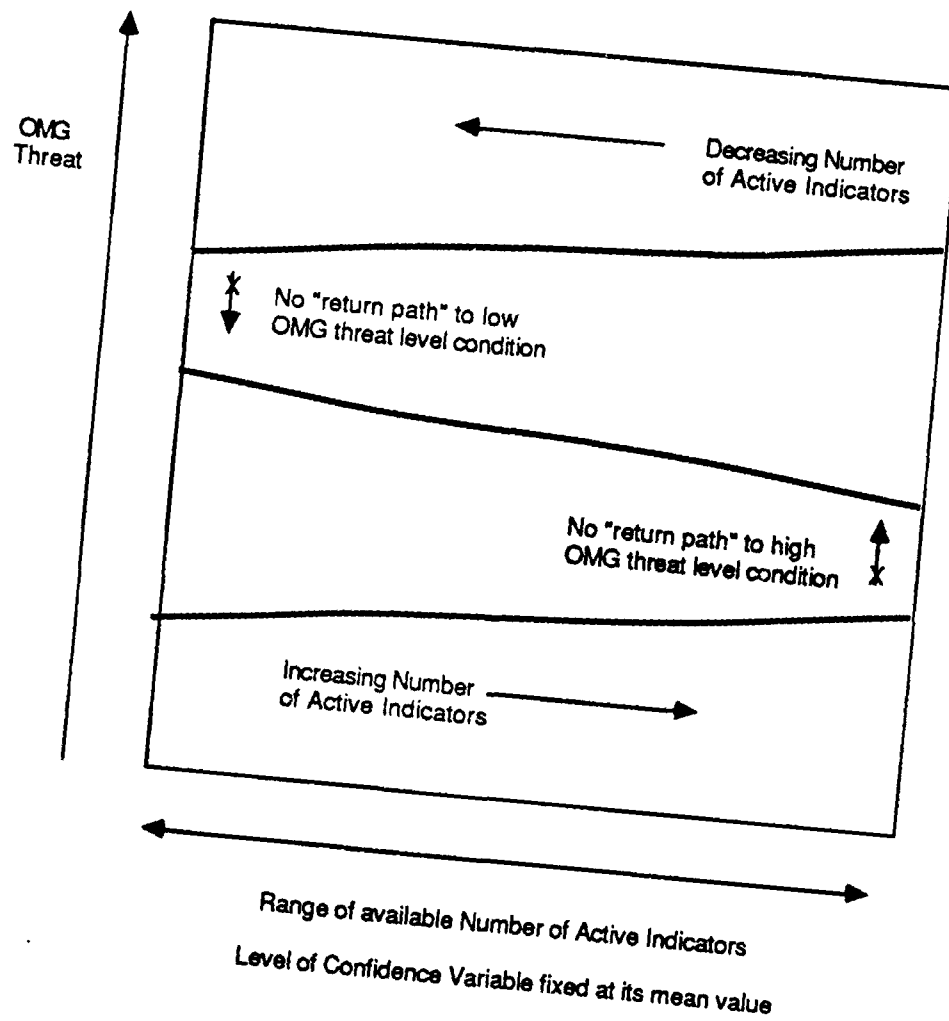


Exhibit 6-10
Complete Perceptual "Trapping"



1. Partial Trapping:

Partial trapping conditions occur when one type of path exists for transitions between low and high OMG threat conditions. With the level of confidence variable fixed at its mean value, beginning with low number of active indicators and perceived OMG threat can lead to an increase in the number of active indicators presented to the analyst and this can set the scene for a sudden increase in perceived OMG threat (Exhibit 6-9). Subsequent decreases in the number of active indicators cannot cause a subsequent rapid decline in the perceived level of OMG threat because no return path exists for such a transition within the limits of available number of active indicators supplied to the analyst. This observation, which will be substantiated below by presenting and discussing the statistical analysis of actual analyst OMG threat assessments, reflects statements by some of the analysts that, having made a commitment to issuing an OMG warning, a subsequent reduction in the level of information, even to a very low level, would not cause these analysts to withdraw the warning.

2. Complete Trapping:

Complete perceptual trapping can occur when no path exists for a transition between the low OMG and high OMG threat perception states (Exhibit 6-10) for a cusp surface model constructed on the basis of the data set presented to the analyst. Under these circumstances, the analyst's perception of OMG threat would remain at either a low or a high level throughout a complete analytic session for the complete set of number of active indicator data elements and with the level of confidence variable fixed at its mean value, for example.

6.1.8 COUNTER-INTUITIVE OR PARADOXICAL BEHAVIOR

Cusp surface models based on analyst's perceptions of OMG threat suggest that these perceptions may exhibit patterns of counter-intuitive or paradoxical behavior, as shown in Exhibit 6-11. Beginning with a high level of perceived OMG threat and with a low number of active indicators and the level of confidence maintained at its mean value, a subsequent increase in the number of active indicators presented to the analyst can lead to a sudden decrease in the level of perceived threat. Additional increases in the number of active indicators can now cause a gradual increase in the level of perceived threat, but the analyst's perceptions now appear to be "trapped" to a part of the particular S-shaped slice of the cusp surface. Under these conditions, a return to the initial conditions would appear to be impossible without changes in the level of confidence or the presentation of additional sets of data elements to the analyst and the subsequent creation of a new cusp surface model based on such an extended data set.

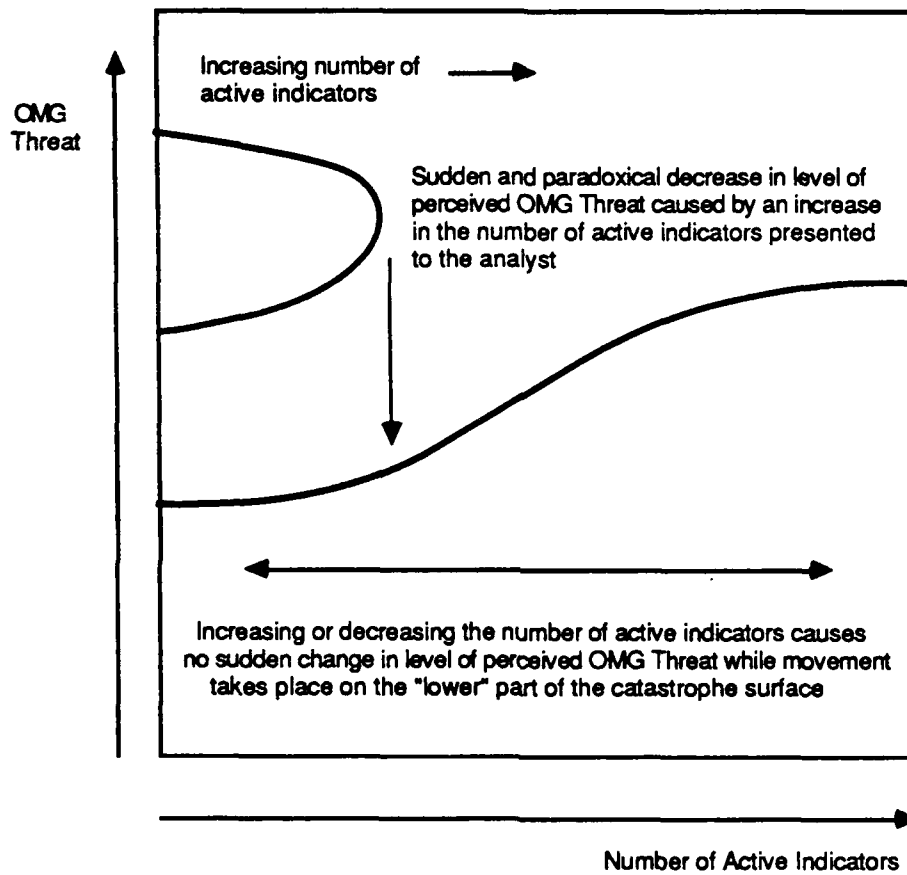
6.2 SPECIFIC ANALYST ASSESSMENTS

As mentioned above, several members of Synectics staff who have been involved in various forms of I&W and intelligence analysis activity participated in testing the IWCAT system. The following constitutes a summary discussion of the results of these different tests and the detailed analysis of these results that is presented can serve as a starting point for further research investigations and for the development of an operational IWCAT facility.

In each case the analyst was presented with a test data set of indicators and other information described in Section 5 and asked to assess the level of OMG threat that they appear to reflected to the analyst. Following this task, the analyst was asked to designate which of the indicators were of primary importance and which were of secondary importance to them in

Exhibit 6-11

Counter-Intuitive or Paradoxical Behavior



Level of Confidence fixed at its mean value

determining the level of perceived OMG threat. Information generated by this process was then subjected to analysis using the cusp surface analysis program.

It is a suggestive finding of the statistical analyses performed during this investigation that the nature of the response of the different analysts to the OMG threat test data appears to depend upon their background and experience. Thus, analysts with extensive active duty tactical-level military experience (Analysts "B" and "C," below) appeared to pay almost exclusive attention to the number of active indicators, while another analyst (Analyst "D," below) with much more national strategic-level intelligence experience appeared to pay almost exclusive attention to the patterns (or sequence type) of the displayed indicators. Analyst "A," with extensive military and experience and involvement in more national level intelligence analytic activity, appeared to pay attention to both number of active indicators and their pattern. Analyst "E," with national level weapons targeting experience, performed the test and the data collected in this process appeared to form a linear model since the cusp analysis program terminated its activities because no cubic term was detected in the analyst-derived data.

While the observation that the nature of analyst perceptions of OMG threat is predicated by the nature of the experience and training of the analysts, is a tentative finding due to the small sample size of analysts that were used in the experiment, such a suggestion can have profound implications on the way that I&W and other forms of intelligence analyses are performed. Results of the IWCAT project suggest that analysts (who could be referred to as "front-line" analysts) closely associated with the more immediate or tactical aspects of the combat environment concentrate on the number of active indicators while those analysts (who could be referred to as "headquarters" analysts) who are involved in the analysis of the overall aspects of combat, and who may receive most of their intelligence input from the front-line analysts, appear to pay more attention to the pattern of these indicators.

If substantiated by further work and analysis, such a finding can have an important impact on the relationships between these front-line and headquarters analysts since the first type of analyst acts as a perceptual filter for the information that is presented to the second type of analyst. The fact that these different types of analysts concentrate on different aspects of the available intelligence information (such as the number of active indicators or the pattern of indicators, for example) could introduce unexpected and unintentional biases in the interpretation of this information and lead to a misunderstanding of the nature of particular combat situations. The possibilities of such perceptual disconnects and their impact on I&W and command and control (C²) should be of concern to I&W analysts and others and further investigations with the aid of the IWCAT system would appear to be appropriate.

The following section of the report presents a description of the analysis of the I&W OMG threat data collected from five analysts who are members of Synectics staff and who have all had experience as intelligence analysts.

6.2.1 "ANALYST A"

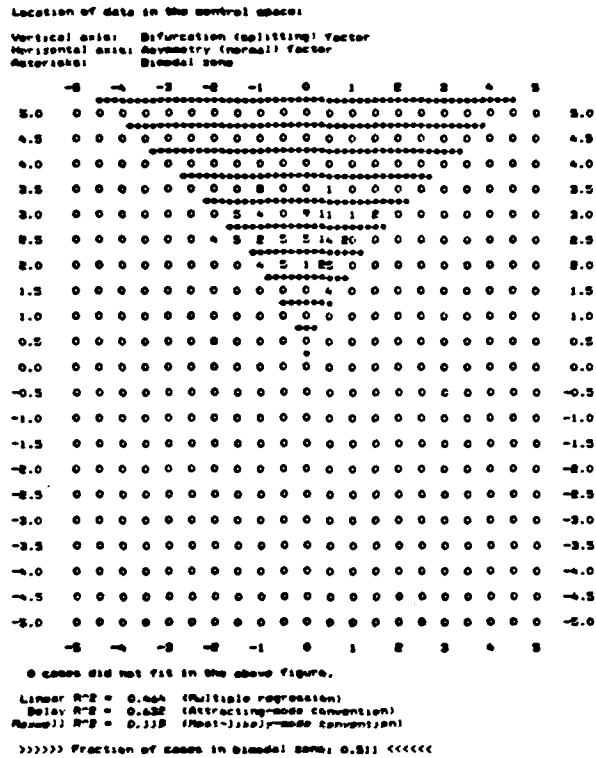
1. Impact of number of active primary indicators and level of confidence on OMG threat perception.

Exhibit 6-12 presents the analysis of the effect of number of primary indicators and level of confidence on OMG threat perception. The control plane plot (Exhibit 6-12a) shows that 51.1% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Statistical analysis shows that a catastrophe model

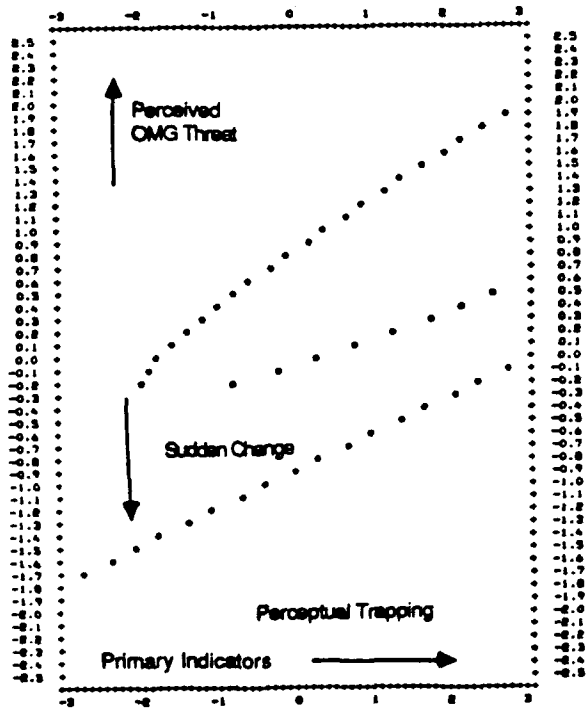
Exhibit 6-12

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot



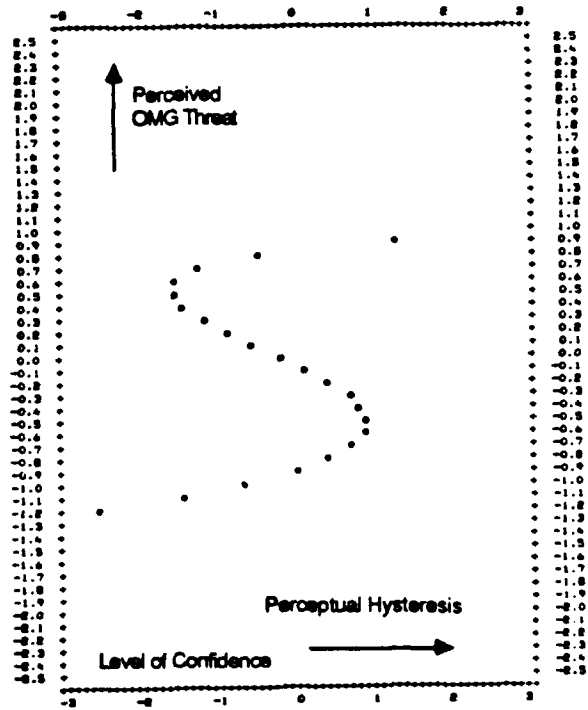
Effect of variable S, holding all others constant at their mean values.



• Mode symbol
• Antimode symbol

(b) Variable Primary Indicators, Fixed Level of Confidence

Effect of variable D, holding all others constant at their mean values.



• Mode symbol
• Antimode symbol

(c) Variable Level of Confidence, Fixed Primary Indicators

with a delay transition convention ($R^2 = 0.632$) appears to be a more suitable model for the data than a Maxwell convention-based model ($R^2 = 0.118$), or a linear model derived with the aid of multiple regression techniques ($R^2 = 0.464$).

A slice of the cusp model surface for a range of numbers of active primary indicators and with the level of confidence fixed at its mean value reveal situations in which partial perceptual trapping can occur (Exhibit 6-12b). Starting with a small number of active primary indicators and a low level of perceived OMG threat, an increase in the number of such indicators can lead to an approximately linear increase in the level of perceived OMG threat. By contrast, starting with a large number of active indicators and high level of OMG threat, a decrease in the number of active indicators can lead to an approximately linear decrease in perceived OMG threat level to a point at which a rapid decline in perceived threat can occur. Once this rapid decline has occurred, the analyst's perceptions appear to be "trapped" to the lower limb of the curve with no possibility of a return to the upper limb without changes in the level of confidence value, for example.

A slice of the cusp model surface for a range of values of the level of confidence and with the primary indicators fixed at their mean value reveal situations in which perceptual hysteresis can occur (Exhibit 6-12c) as the level of confidence is increased or decreased. Increasing the level of confidence of the indicators can lead to a rapid increase in perceived OMG threat while a decrease in level of confidence from a high to a low level can lead to a sudden decline in the level of perceived OMG threat. These sudden transitions in perceived threat level will generally take place at different levels of confidence.

2. Impact of number of active secondary indicators and level of confidence on OMG threat perception.

Exhibit 6-13 presents the analysis of the effect of number of secondary indicators and level of confidence on OMG threat perception. The control plane plot (Exhibit 6-13a) shows that 53.3% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Statistical analysis shows that a catastrophe model with a delay transition convention ($R^2 = 0.545$) appears to be a more suitable model for the data than a linear model derived with the aid of multiple regression techniques ($R^2 = 0.361$), or a model based on the Maxwell transition convention ($R^2 = 0.079$).

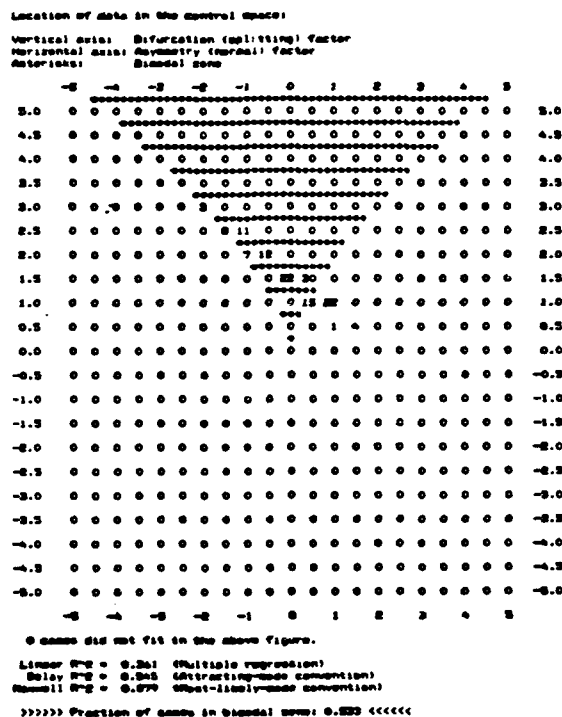
A slice of the cusp surface at a fixed level of confidence and variable number of active secondary indicators reveals situations in which perceptual trapping can occur. Increasing the number of these active indicators from a low to a higher level causes a gradual increase in perceived OMG threat to a condition at which a sudden change in these levels can take place (Exhibit 6-13b). Once at such a high perceived threat level, the analyst's perceptions are trapped at this level since no return path exists for a return to lower levels without changes in the values of the level of confidence factor. Under such circumstances, the analyst would not retract the OMG warning once issued, even with a marked decline in the number of active indicators.

A slice of the cusp model surface for a range of values of the level of confidence and with the secondary indicators fixed at their mean value reveal situations in which perceptual hysteresis can occur (Exhibit 6-13c) as the level of confidence is increased or decreased. Increasing the level of confidence of the indicators can lead to a rapid increase in perceived OMG threat while a decrease in level of confidence from a high to a low level can lead to a sudden decline in the level of perceived OMG threat. These sudden transitions in perceived threat level will generally take place at different level of confidence values.

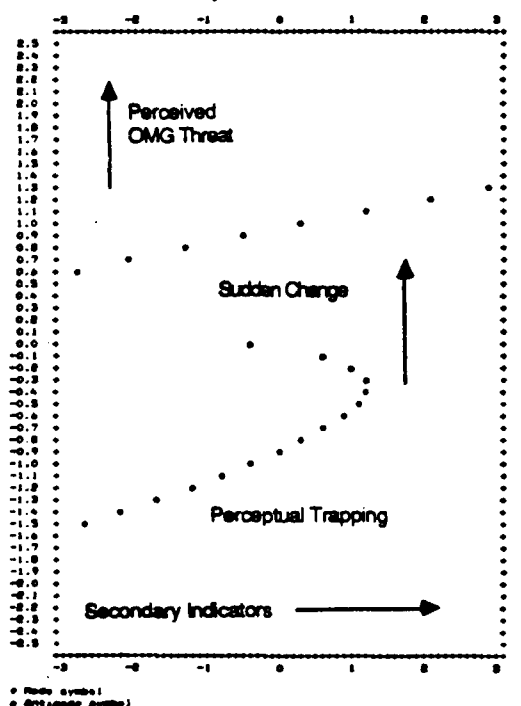
Exhibit 6-13

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot

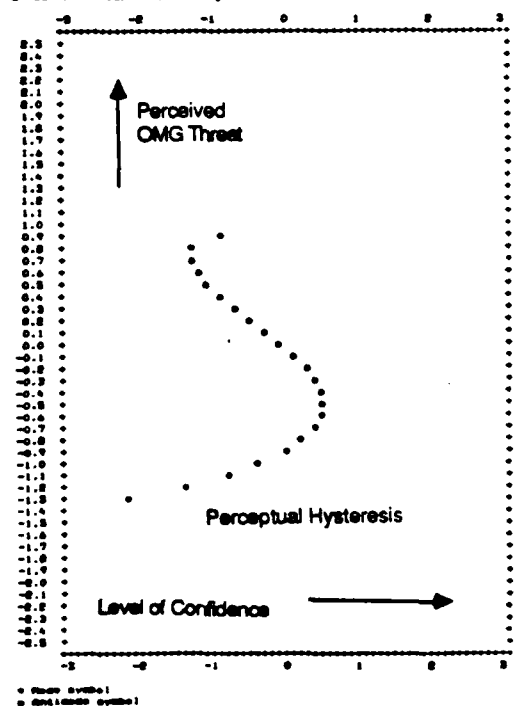


Effect of variable 9, holding all others constant at their mean values.



(b) Variable Secondary Indicators, Fixed Level of Confidence

Effect of variable 10, holding all others constant at their mean values.



(c) Variable Level of Confidence, Fixed Secondary Indicators

3. Impact of number of all active indicators and level of confidence on OMG threat perception.

Exhibit 6-14 presents the analysis of the effect of number of all active indicators and level of confidence on OMG threat perception. The control plane plot (Exhibit 6-14a) shows that none of the data are located within the bimodal zone. Statistical analysis shows that a linear model provides an adequate model for the data since the linear model derived with the aid of multiple regression techniques and catastrophe model with delay and Maxwell transition conventions have almost equal R^2 values (0.485, 0.487, and 0.487, respectively). This finding is also confirmed by the data presented in Exhibits 6-14b and 6-14c, which display the effect of variations of numbers of all active indicators and the level of confidence, respectively, on OMG threat perception.

A comparison of the material presented in Exhibits 6-12, 6-13, and 6-14 is revealing. While analysis of the data set in which impact of the primary or secondary indicators are examined provides graphs which suggest that sudden changes in perception can take place, analyzing the effect of these indicators as a whole reveals a linear response characteristic. Under such circumstances, the combination of the responses generated by the simultaneous consideration of the effect all the indicators can be described with the aid of a simple linear model. This demonstrates a major difficulty that might arise when data of different levels of importance to an analyst are combined for statistical or other purposes. Such observations should be a matter for further consideration.

4. Impact of sequence type and level of confidence on OMG threat perception.

Exhibit 6-15 presents the analysis of the effect of sequence type and level of confidence variables on OMG threat perception. The control plane plot (Exhibit 6-15a) shows that 41.5% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Statistical analysis shows that a catastrophe model with a delay transition convention ($R^2 = 0.472$) appears to be a more suitable model for the data than a Maxwell convention-based model ($R^2 = -0.299$), or a linear model derived with the aid of multiple regression techniques ($R^2 = 0.285$).

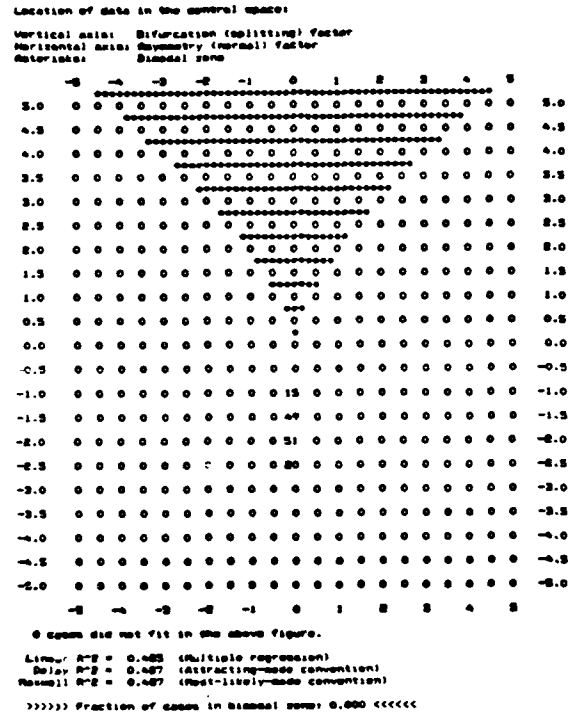
A slice of the cusp surface at a fixed level of confidence and variable sequence type reveals situations in which perceptual trapping can occur where the analyst's OMG threat perceptions are restricted either to a high value range or a low value range (Exhibit 6-15b).

A slice of the cusp model surface for a range of values of the level of confidence and with the sequence type fixed at its mean value reveals situations in which perceptual hysteresis can occur (Exhibit 6-15c) as the level of confidence is increased or decreased. Increasing the level of confidence of the indicators can lead to a rapid increase in perceived OMG threat while a decrease in level of confidence from a high to a low level can lead to a sudden decline in the level of perceived OMG threat. These sudden transitions in perceived threat level will generally take place at different level of confidence values.

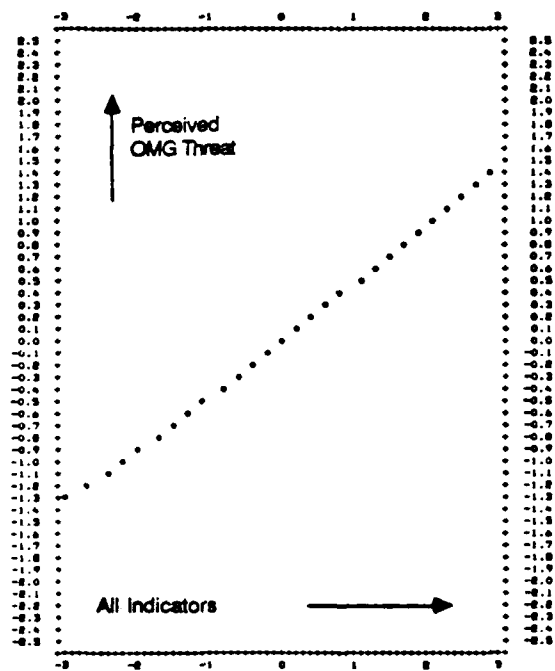
Exhibit 6-14

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot



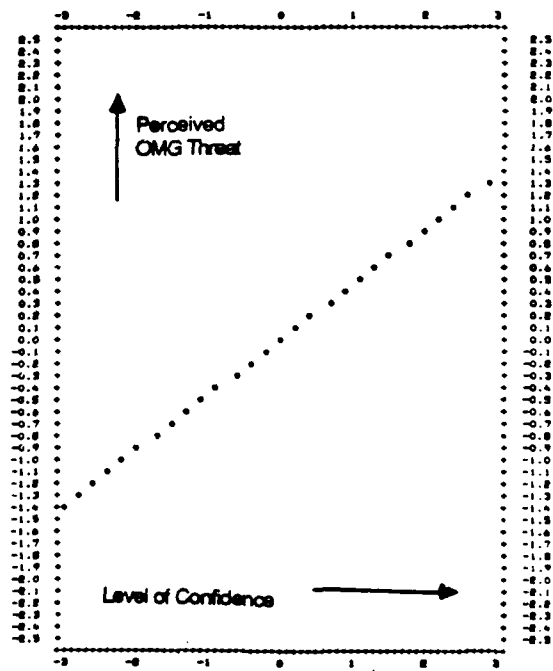
Effect of variable α , holding all others constant at their mean values.



o Mode symbol
o Antimode symbol

(b) Variable All Indicators, Fixed Level of Confidence

Effect of variable α , holding all others constant at their mean values.



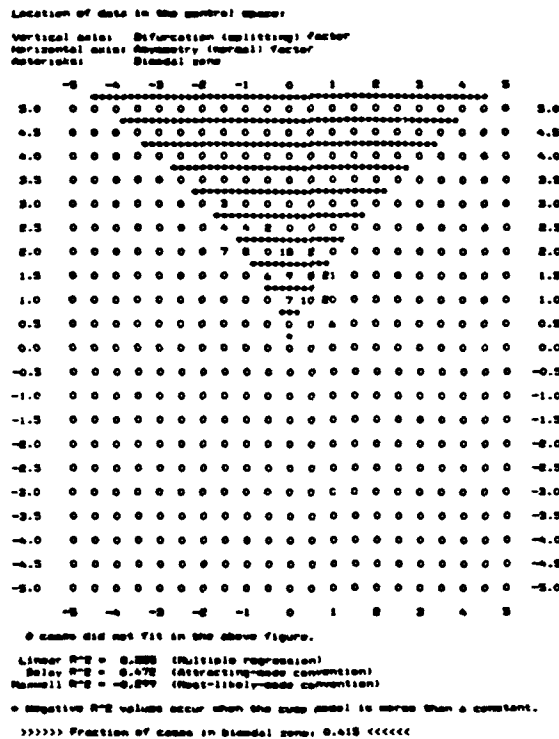
o Mode symbol
o Antimode symbol

(c) Variable Level of Confidence, Fixed All Indicators

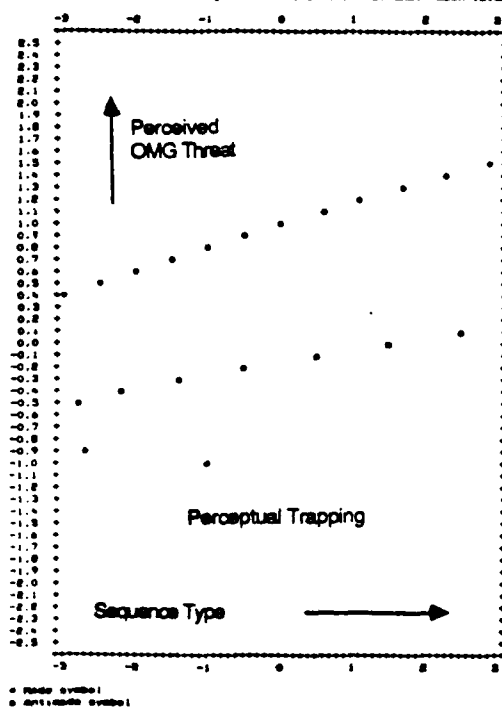
Exhibit 6-15

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot

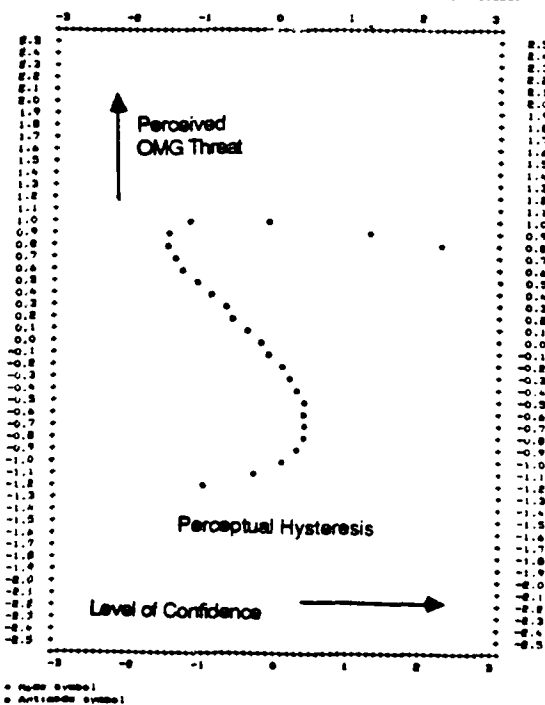


Effect of variable 5, holding all others constant at their mean values.



(b) Variable Sequence Type, Fixed Level of Confidence

Effect of variable 10, holding all others constant at their mean values.



(c) Variable Level of Confidence, Fixed Sequence Type

5. Impact of number of active primary indicators, sequence type, and level of confidence on OMG threat perception.

Exhibit 6-16 presents the analysis of the effect of number of active primary indicators, sequence type, and level of confidence variables on OMG threat perception. The control plane plot (Exhibit 6-16a) shows that 54.8% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Statistical analysis shows that a catastrophe model with a delay transition convention ($R^2 = 0.654$) appears to be a more suitable model for the data than a linear model derived with the aid of multiple regression techniques ($R^2 = 0.470$), or a Maxwell transition-based cusp model ($R^2 = 0.315$).

A slice of the cusp model surface for a range of values of the primary indicators and sequence type and level of confidence fixed at their mean values reveals situations in which partial perceptual trapping can occur (Exhibit 6-16b). Starting with a small number of active primary indicators and a low level of perceived OMG threat, an increase in the number of such indicators can lead to an approximately linear increase in the level of perceived OMG threat. By contrast, starting with a large number of active indicators and high level of OMG threat, a decrease in the number of active indicators can lead to an approximately linear decrease in perceived OMG threat level to a point at which a rapid decline in perceived threat can occur. Once this rapid decline has occurred, the analyst's perceptions appear to be "trapped" to the lower limb of the curve with no possibility of a return to the upper limb without changes in the level of confidence value, for example.

A slice of the cusp surface at a fixed level of confidence and number of active primary indicators and variable sequence type reveals situations in which perceptual trapping can occur with the analyst's OMG threat perceptions restricted either to a high value range or a low value range for all ranges of sequence type values (Exhibit 6-16c).

A slice of the cusp model surface for a range of values of the level of confidence parameter and with the numbers of primary indicators and sequence type fixed at their mean value reveals situations in which perceptual hysteresis can occur (Exhibit 6-16d) as the level of confidence is increased or decreased. Increasing the level of confidence of the indicators can lead to a rapid increase in perceived OMG threat while a decrease in level of confidence from a high to a low level can lead to a sudden decline in the level of perceived OMG threat. These sudden transitions in perceived threat level will generally take place at different levels of confidence.

6.2.2 "ANALYST B"

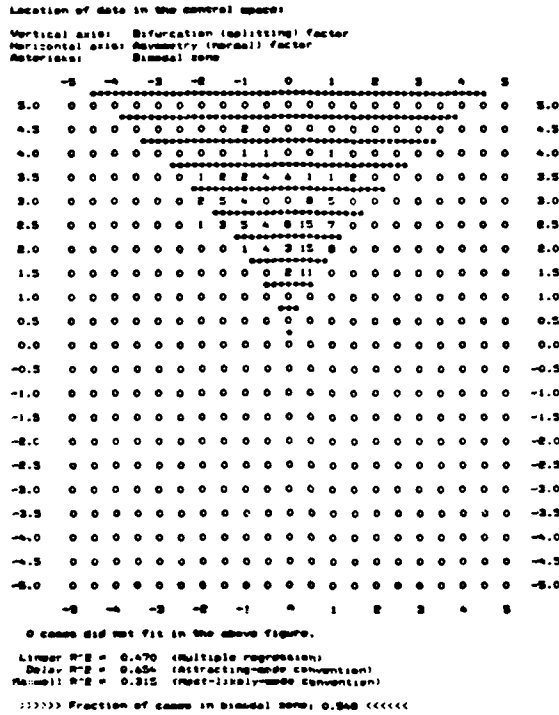
1. Impact of number of active primary indicators and level of confidence on OMG threat perception.

Exhibit 6-17 presents the analysis of the effect of number of active primary indicators and level of confidence on OMG threat perception. The control plane plot (Exhibit 6-17a) shows that 97.8% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Statistical analysis shows that a catastrophe model with a delay transition convention ($R^2 = 0.666$) appears to be a more suitable model for the data than a linear model derived with the aid of multiple regression techniques ($R^2 = 0.238$), or a Maxwell transition-based cusp model ($R^2 = -0.339$).

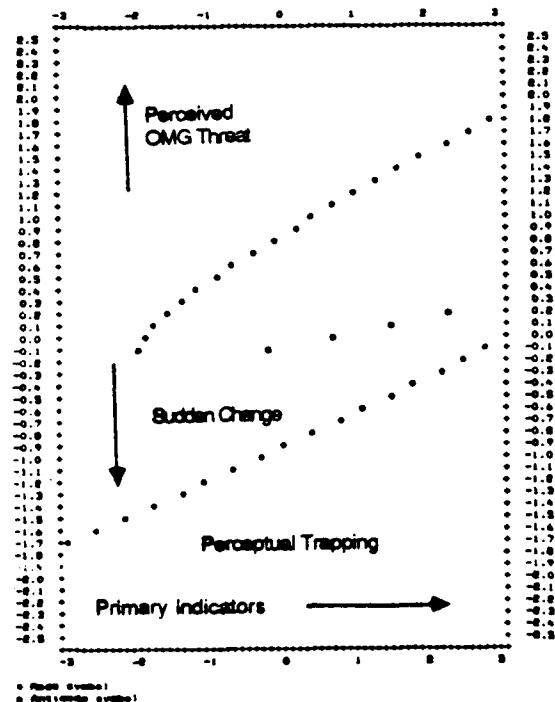
Exhibit 6-16

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot

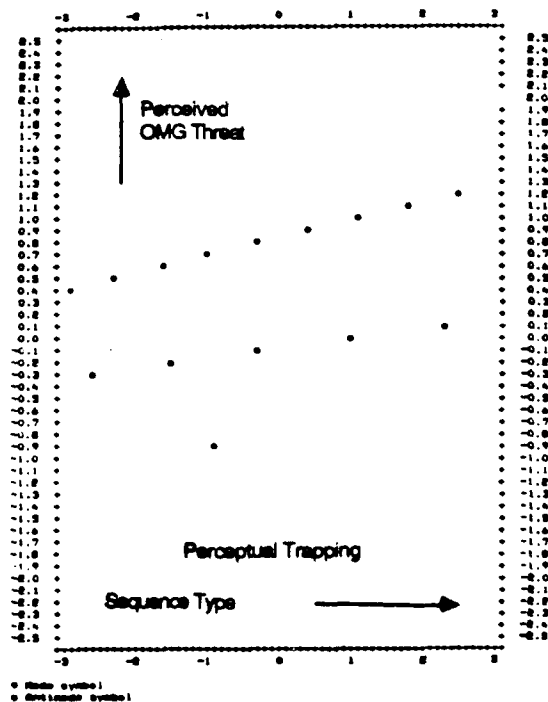


Effect of variable B, holding all others constant at their mean values.



(b) Variable Primary Indicators, Fixed Sequence Type and Level of Confidence

Effect of variable B, holding all others constant at their mean values.



(c) Variable Sequence Type, Fixed Primary Indicators and Level of Confidence

Exhibit 6-16 (Continued)

Analysis of OMG Threat Assessment Data

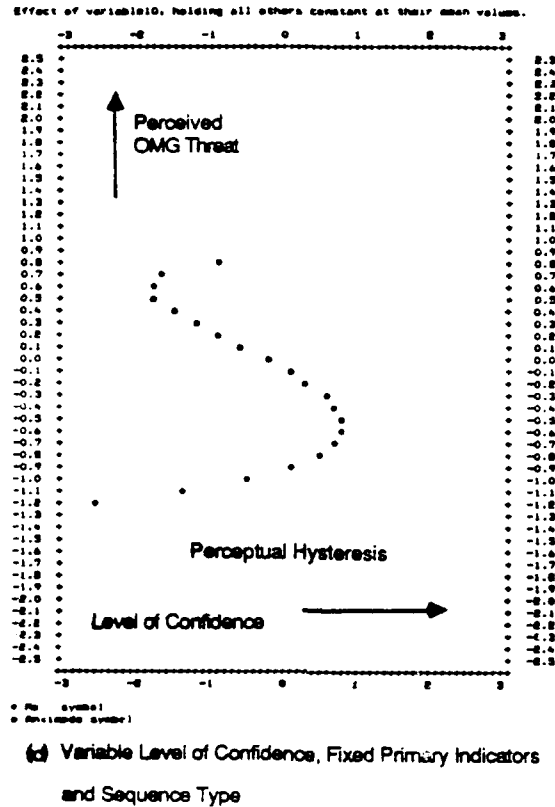
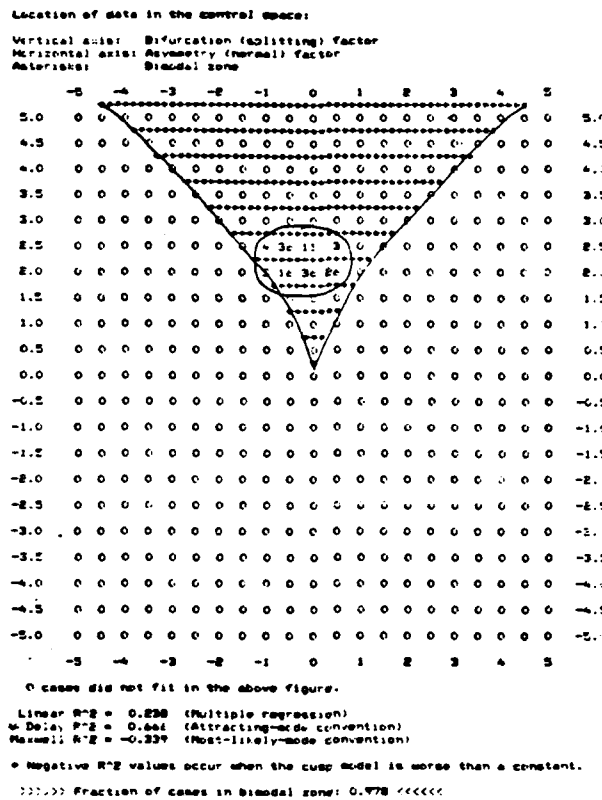


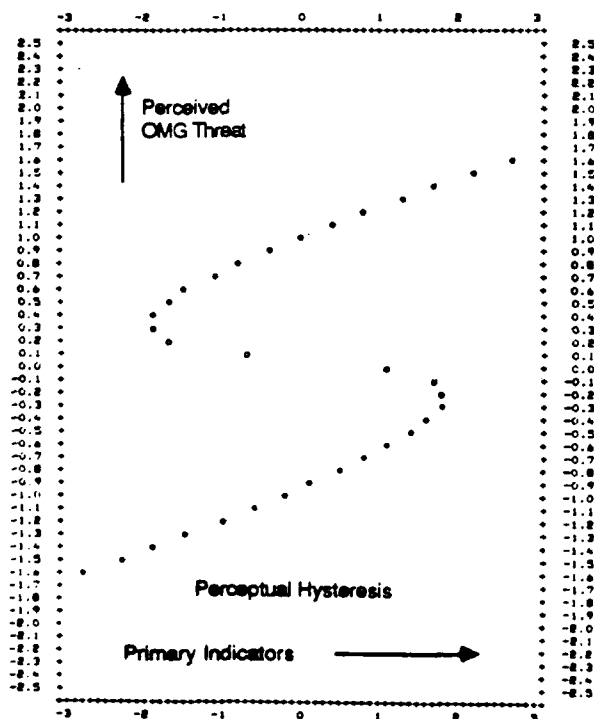
Exhibit 6-17

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot



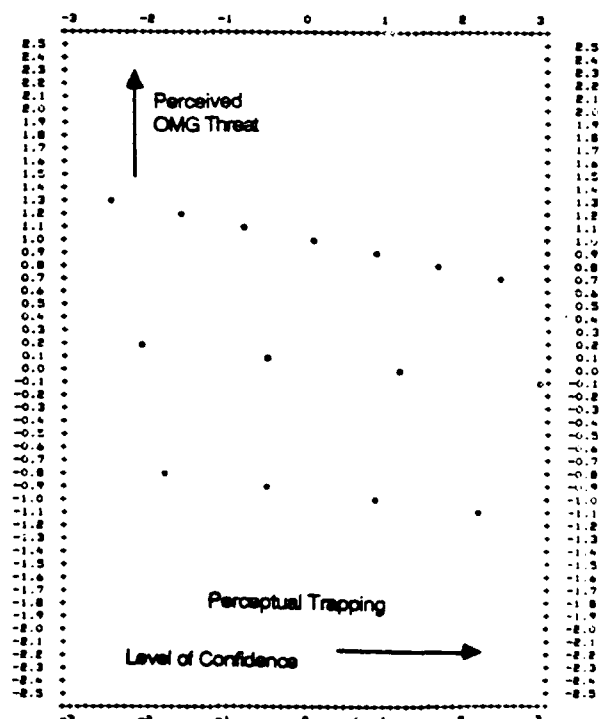
Effect of variable 2, holding all others constant at their mean values.



* Mode symbol
* Antimode symbol

(b) Variable Primary Indicators, Fixed Level of Confidence

Effect of variable 10, holding all others constant at their mean values.



(c) Variable Level of Confidence, Fixed Primary Indicators

A slice of the cusp model surface for a range of values of the primary indicators and with the level of confidence fixed at its mean value reveals situations in which perceptual hysteresis can occur (Exhibit 6-17b). Starting with a small number of active primary indicators and a low level of perceived OMG threat, an increase in the number of such indicators can lead to an approximately linear increase in the level of perceived OMG threat until a threshold is reached at which a sudden increase in threat occurs. By contrast, starting with a large number of active indicators and high level of OMG threat, a decrease in the number of active indicators can lead to an approximately linear decrease in perceived OMG threat level to a point at which a rapid decline in perceived threat can occur. The computed curve suggests that the analyst's perception would be subject to large and sudden changes as the number of active indicators for fixed level of confidence values. These sudden transitions in perceived threat level will generally take place at different levels of confidence.

A slice of the cusp surface at a fixed level of confidence and variable primary indicators reveals situations in which perceptual trapping can occur with the analyst's OMG threat perceptions restricted either to a high value range or a low value range for all ranges of sequence type values (Exhibit 6-17c).

2. Impact of number of active secondary indicators and level of confidence on OMG threat perception.

Exhibit 6-18 presents the analysis of the effect of number of active secondary indicators and level of confidence on OMG threat perception. The control plane plot (Exhibit 6-18a) shows that 72.6% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Statistical analysis shows that a catastrophe model with a delay transition convention ($R^2 = 0.653$) appears to be a more suitable model for the data than a Maxwell transition model ($R^2 = -0.203$), or a linear model derived with the aid of multiple regression techniques ($R^2 = 0.326$).

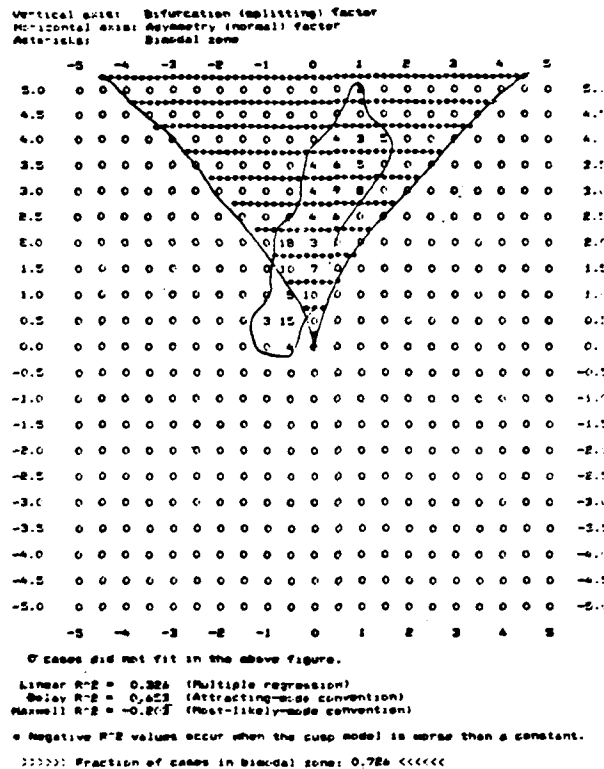
A slice of the cusp surface at a fixed level of confidence and variable number of active secondary indicators reveals situations in which perceptual trapping can occur. Starting with a low level of active indicators and low perceived threat level, the analyst's perceptions would remain trapped on the low OMG portion of the curve. By contrast, starting with a high level of OMG threat and number of secondary indicators and decreasing the number of these active indicators from a high to a lower level causes a gradual decrease in perceived OMG threat to a condition at which a sudden decline in these levels can take place (Exhibit 6-18b). Once having achieved such a low perceived threat level, the analyst's perceptions are trapped at this level since no return path exists for a return to higher levels without changes in the overall level of confidence in the indicators, for example.

A slice of the cusp surface at a fixed level of confidence and variable number of active secondary indicators reveals situations in which perceptual trapping can occur. Increasing the number of these active indicators from a low to a higher level causes a gradual increase in perceived OMG threat to a condition at which a sudden change in these levels can take place (Exhibit 6-18c). Once at such a high perceived threat level, the analyst's perceptions are trapped at this level since no return path exists for a return to lower levels without changes in the values of the level of confidence factor. Under such circumstances, the analyst would not retract the OMG warning once issued, even with a marked decline in the level of confidence.

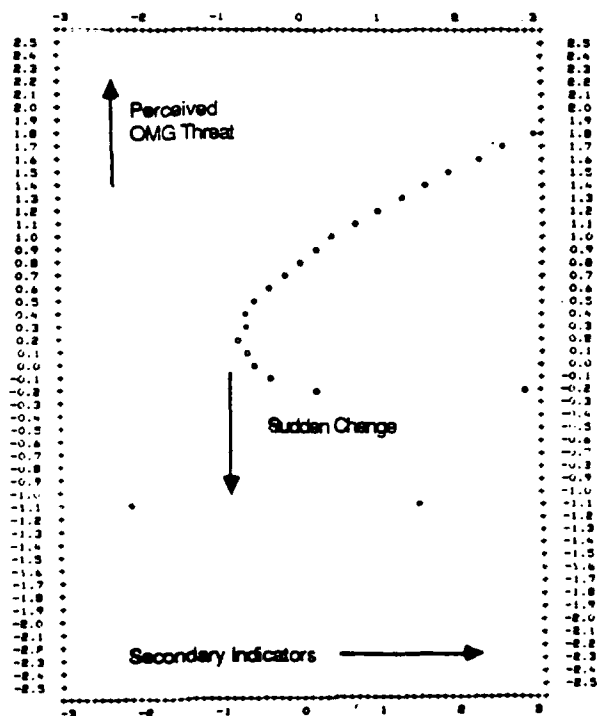
Exhibit 6-18

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot

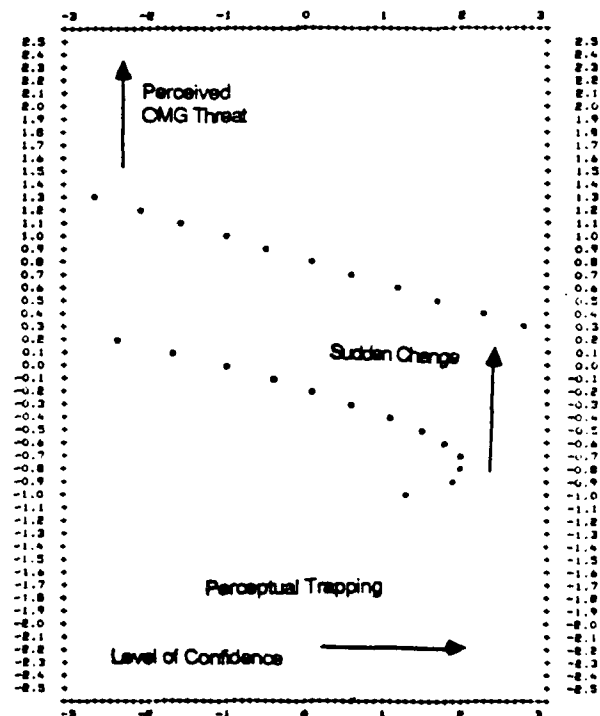


Effect of variable 3, holding all others constant at their mean values.



(b) Variable Secondary Indicators, Fixed Level of Confidence

Effect of variable 10, holding all others constant at their mean values.



• Mode symbol
• Anti-mode symbol

(c) Variable Level of Confidence, Fixed Secondary Indicators

3. Impact of sequence type and level of confidence on OMG threat perception.

Data collected from this analyst for variable sequence type could not be formed into a cusp model because convergence of the computational process to a particular model structure did not take place. This result suggests that the analyst was not considering information about sequence type in the analysis of the OMG threat data and forming this information into a model based on the cusp catastrophe. The possibility that a linear or more complicated nonlinear model would be a more appropriate model of these data remains open.

6.2.3 "ANALYST C"

1. Impact of number of active primary indicators and level of confidence on OMG threat perception.

Exhibit 6-19 presents the analysis of the effect of number of active primary indicators and level of confidence on OMG threat perception. The control plane plot (Exhibit 6-19a) shows that 23.0% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Statistical analysis shows that a catastrophe model with a delay transition convention ($R^2 = 0.348$) appears to be a somewhat more suitable model for the data than a linear model derived with the aid of multiple regression techniques ($R^2 = 0.295$), and certainly more suitable than a Maxwell convention model ($R^2 = 0.145$).

A slice of the cusp model surface for a range of number of active primary indicators and with the level of confidence fixed at its mean values reveals situations in which partial perceptual trapping can occur (Exhibit 6-19b). Starting with a small number of active primary indicators and a low level of perceived OMG threat, an increase in the number of such indicators can lead to an increase in the level of perceived OMG threat without sudden changes taking place. By contrast, starting with a large number of active indicators and high level of OMG threat, a decrease in the number of active indicators can lead to an approximately linear decrease in perceived OMG threat level to a point at which a rapid decline in perceived threat can occur. Once this rapid decline has occurred, the analyst's perceptions appear to be "trapped" on the lower limb of the curve with no possibility of a return to the upper limb without changes in the overall level of confidence in the OMG indicators, for example.

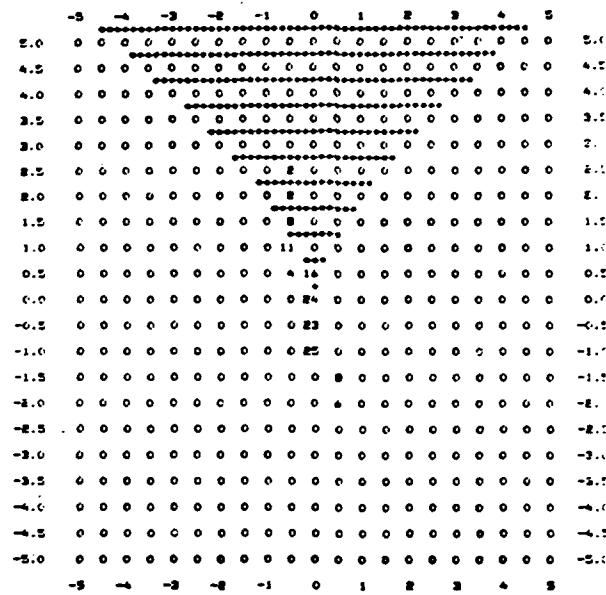
A slice of the cusp model surface for a range of values of level of confidence and the number of primary indicators fixed at its mean value reveals situations in which partial perceptual trapping and paradoxical reversal in behavior can occur (Exhibit 6-19c). Starting with a small number of active primary indicators and a low level of perceived OMG threat, an increase in the number of such indicators can lead to an increase in the level of perceived OMG threat which does not exhibit sudden transitions. By contrast, starting with a low level of confidence level and high level of OMG threat, an increase in the number of active indicators can lead to an approximately linear decrease in perceived OMG threat level to a point at which a rapid, paradoxical, decline in perceived threat can occur. Once this rapid decline has occurred, the analyst's perceptions would also appear to be "trapped" to the lower limb of the curve with no possibility of a return to the upper limb without changes in the overall level of number of active primary indicators, for example.

Exhibit 6-19

Analysis of OMG Threat Assessment Data

Location of data in the control space:

Vertical axis: Bifurcation (splitting) factor
Horizontal axis: Asymmetry (normal) factor
Materials: Bimodal zone

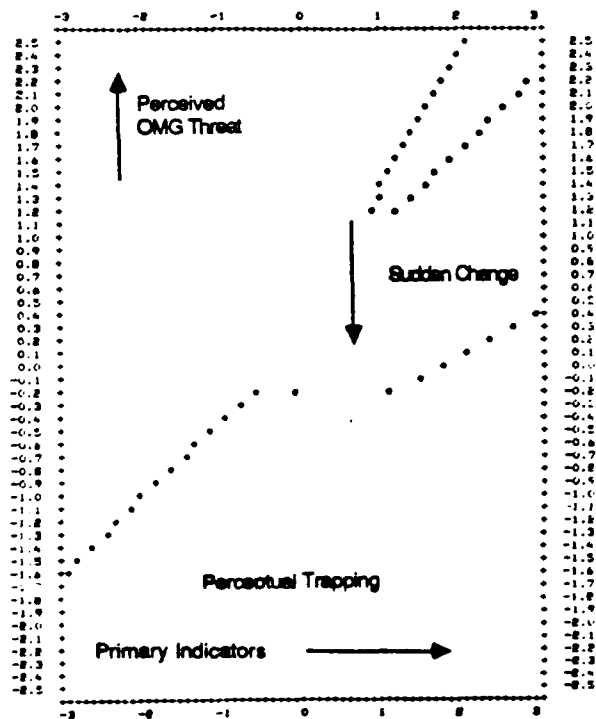


0 cases did not fit in the above figure.

Linear R² = 0.295 (Multiple regression)
Delay R² = 0.348 (Attracting-mode convention)
Hawthill R² = 0.145 (Rest-like/lymode convention)

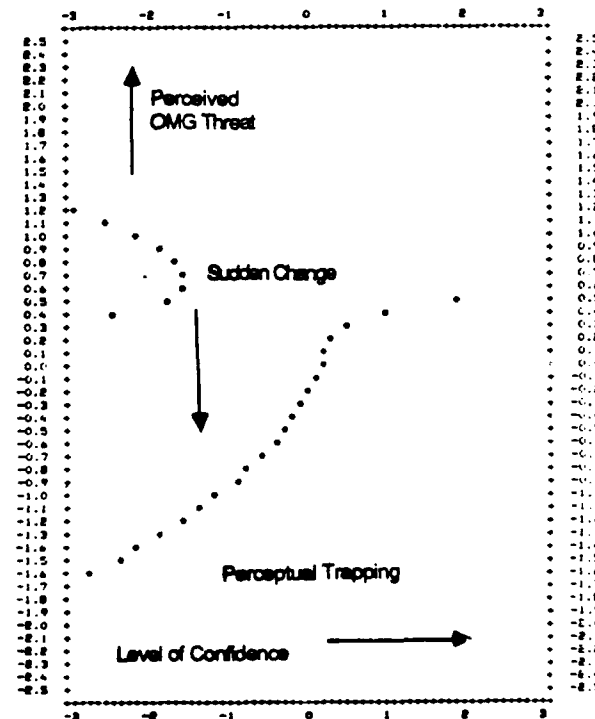
>>>> Fraction of cases in Bimodal zone: 0.220 <<<<<<

Effect of variable E, holding all others constant at their mean values.



(b) Variable Primary Indicators, Fixed Level of Confidence

Effect of variable S, holding all others constant at their mean values.



• Mode symbol
• Attracted symbol

(c) Variable Level of Confidence, Fixed Primary Indicators

2. Impact of number of active secondary indicators and level of confidence on OMG threat perception.

Exhibit 6-20 presents the analysis of the effect of number of active secondary indicators and level of confidence on OMG threat perception. The control plane plot (Exhibit 6-20a) shows that 26.7% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Statistical analysis shows that a catastrophe model with a delay transition convention ($R^2 = 0.489$) appears to be a more suitable model for the data than a linear model derived with the aid of multiple regression techniques ($R^2 = 0.392$), and certainly more suitable than a Maxwell convention model ($R^2 = 0.099$).

A slice of the cusp surface at a fixed level of confidence and variable number of active secondary indicators reveals situations in which perceptual trapping can occur. Increasing the number of these active indicators from a low to a higher level causes a gradual and approximately linear increase in perceived OMG threat to high level (Exhibit 6-20b). By contrast, starting with a large number of secondary indicators and consequential moderate level of perceived OMG threat, reducing the number of these indicators can lead to conditions under which a rapid increase in the level of perceived threat can occur.

A slice of the cusp model surface for a range of values of the level of confidence and with the secondary indicators fixed at their mean value reveals situations in which perceptual hysteresis can occur (Exhibit 6-20c) as the level of confidence is increased or decreased. Increasing the level of confidence of the indicators can lead to a rapid increase in perceived OMG threat while a decrease in level of confidence from a high to a low level can lead to a sudden decline in the level of perceived OMG threat. These sudden transitions in perceived threat level will occur at different level of confidence values.

3. Impact of number of all active indicators and level of confidence on OMG threat perception.

Exhibit 6-21 presents the analysis of the effect of the number of all active indicators and level of confidence on OMG threat perception. This actual data was the first collected from any analyst during the IWCAT pre-testing and was performed before the concept of primary and secondary indications was established. This development took place during the "Analyst C" test session and the complete set of indicators was subsequently broken down into these components after discussions with the analyst concerned. The control plane plot (Exhibit 6-21a) shows that 31.1% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Statistical analysis shows that a catastrophe model with a delay transition convention ($R^2 = 0.570$) appears to be a more suitable model for the data than a linear model derived with the aid of multiple regression techniques ($R^2 = 0.414$), and certainly more suitable than a Maxwell convention model ($R^2 = 0.125$).

A slice of the cusp surface at a fixed level of confidence and variable number of active indicators reveals situations in which perceptual trapping can occur. Increasing the number of these active indicators from a low to a higher level causes a gradual and approximately linear increase in perceived OMG threat to high level (Exhibit 6-21b). By contrast, starting with a large number of active indicators and consequential moderate level of perceived OMG threat

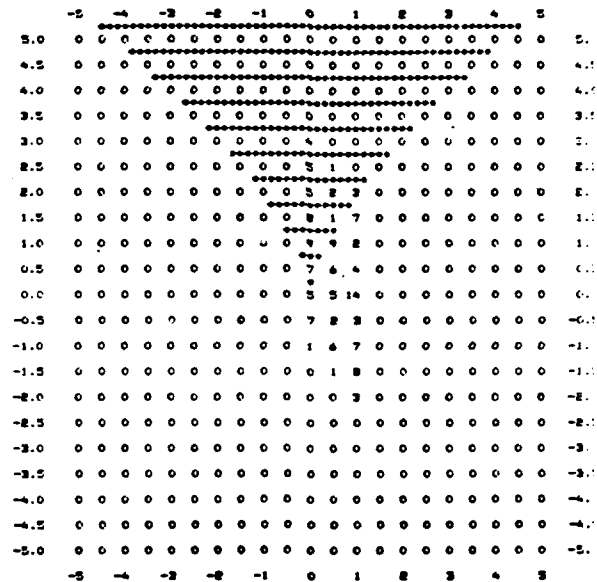
Exhibit 6-20

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot

Location of data in the control space:

Vertical axis: Bifurcation (splitting) factor
Horizontal axis: Asymmetry (normal) factor
Antisymmetry: Bimodal zone

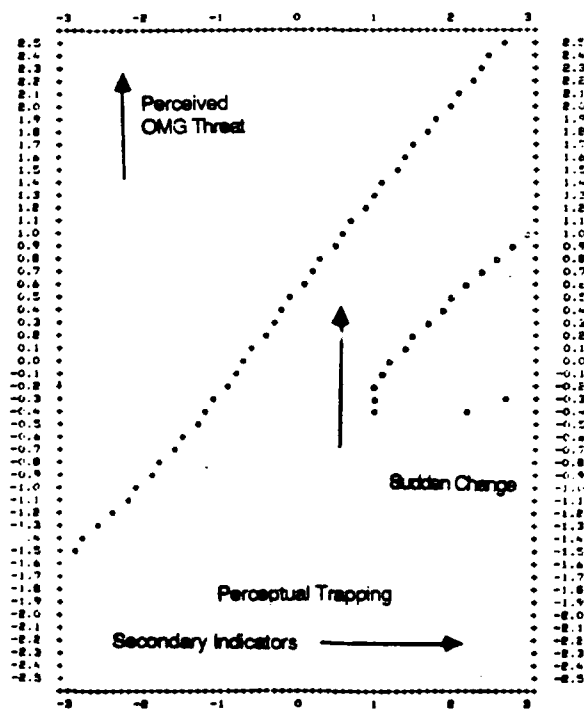


0 cases did not fit in the above figure.

Linear R² = 0.992 (Multiple regression)
Delay R² = 0.489 (Attracting-mode convention)
Maxwell R² = 0.099 (Most-likely-mode convention)

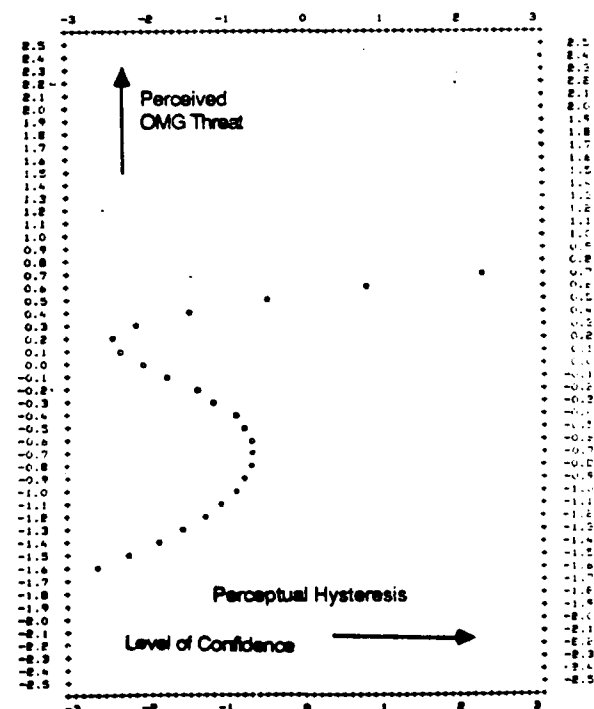
0.000: Fraction of cases in bimodal zone: 0.847 <<<<<

Effect of variable 3, holding all others constant at their mean values.



(b) Variable Secondary Indicators, Fixed Level of Confidence

Effect of variable 6, holding all others constant at their mean values.

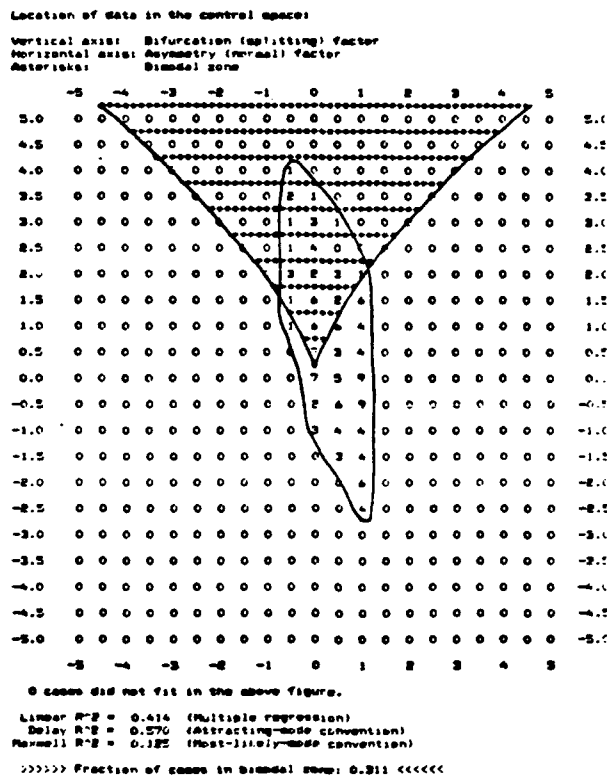


(c) Variable Level of Confidence, Fixed Secondary Indicators

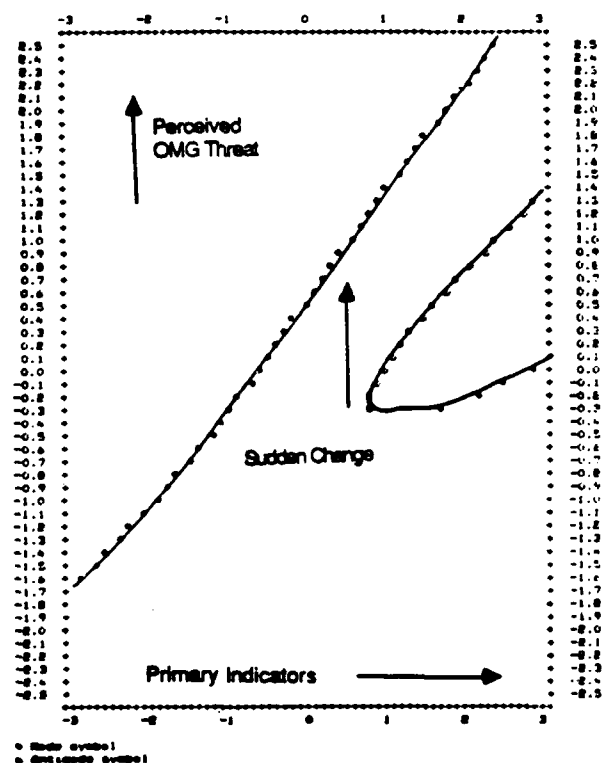
Exhibit 6-21

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot

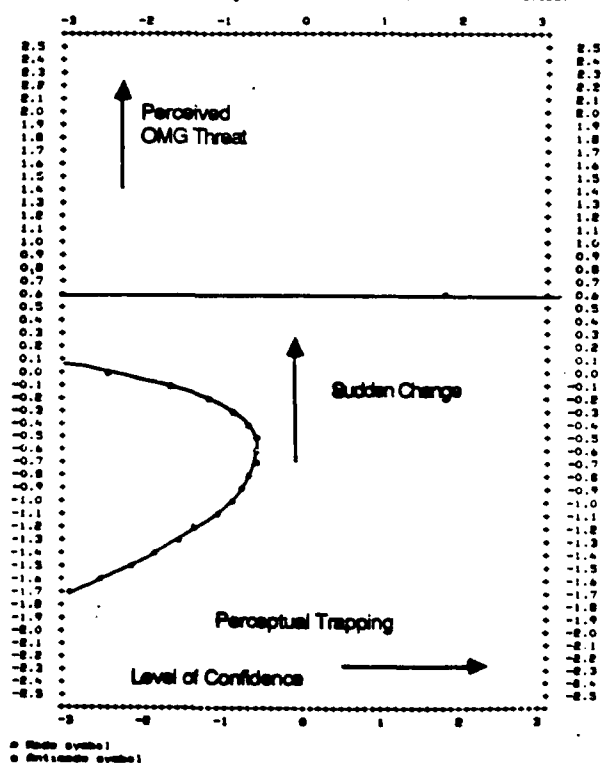


Effect of variable 2, holding all others constant at their mean values.



(b) Variable All Indicators, Fixed Level of Confidence

Effect of variable 5, holding all others constant at their mean values.



(c) Variable Level of Confidence, Fixed All Indicators

and reducing the number of these indicators can lead to conditions under which a rapid increase in the level of perceived threat can occur.

A slice of the cusp surface at a fixed level of confidence and variable number of active indicators also reveals situations in which perceptual trapping can occur. Increasing the number of these active indicators from a low to a higher level causes a gradual increase in perceived OMG threat to a condition at which a sudden change in these levels can take place (Exhibit 6-21c). Once at such a high perceived threat level, the analyst's perceptions are trapped at this level since no return path exists for a return to lower levels without changes in the overall number of active indicators. Under such circumstances, the analyst would not retract the OMG warning once issued, even with a marked decline in the level of confidence in the active indicators, for example.

4. Impact of sequence type and level of confidence on OMG threat perception.

Data collected from this analyst for sequence type variation could not be formed into a cusp model because convergence of the computational process to a particular model structure did not take place. This result suggests that the analyst was not considering information about sequence type in the analysis of the OMG threat data and forming this information into a model based on the cusp catastrophe. The possibility that a linear or more complicated nonlinear model would be a more appropriate model of these data remains open.

6.2.4 "ANALYST D"

1. Impact of number of primary, secondary, and all active indicators and level of confidence on OMG threat perception.

Data collected from this analyst for primary, secondary, and all active indicators and analyzed separately could not be formed into a cusp model because convergence of the computational process to a particular model structure did not take place. However, a model could be formed from sequence type and level of confidence data produced by this analyst (see below). This result suggests that the analyst was not considering information about the number of active indicators, per se, in the analysis of the OMG threat data, or that another type of model (such as some form of linear model, or much more complicated nonlinear model) of these data was more appropriate.

2. Impact of sequence type and level of confidence on OMG threat perception.

Exhibit 6-22 presents the analysis of the effect of sequence type and level of confidence on OMG threat perception. The control plane plot (Exhibit 6-22a) shows that 74.1% of the data are located within the bimodal zone and represent analyst assessment conditions which are subject to ambiguity. Statistical analysis shows that a catastrophe model with a delay transition convention ($R^2 = 0.606$) appears to be a more suitable model for the data than a linear model derived with the aid of multiple regression techniques ($R^2 = 0.097$), or a Maxwell transition-based cusp model ($R^2 = -0.740$).

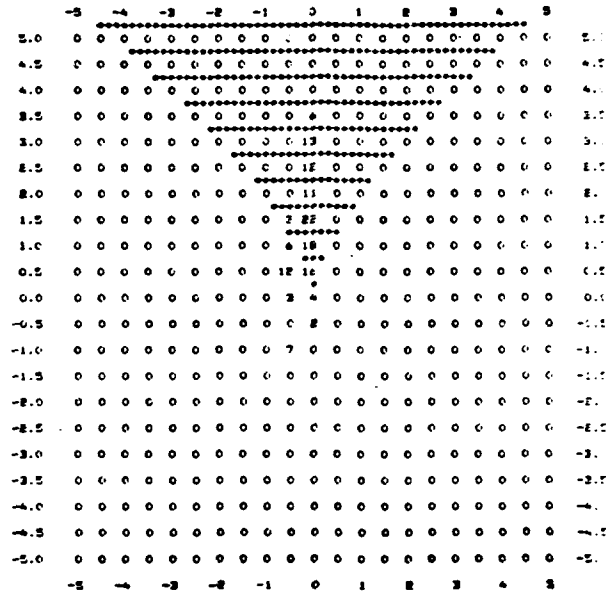
Exhibit 6-22

Analysis of OMG Threat Assessment Data

(a) Control Plane Plot

Location of data in the control space:

Vertical axis: Bifurcation (splitting) factor
Horizontal axis: Asymmetry (normal) factor
Asterisk: Bimodal zone



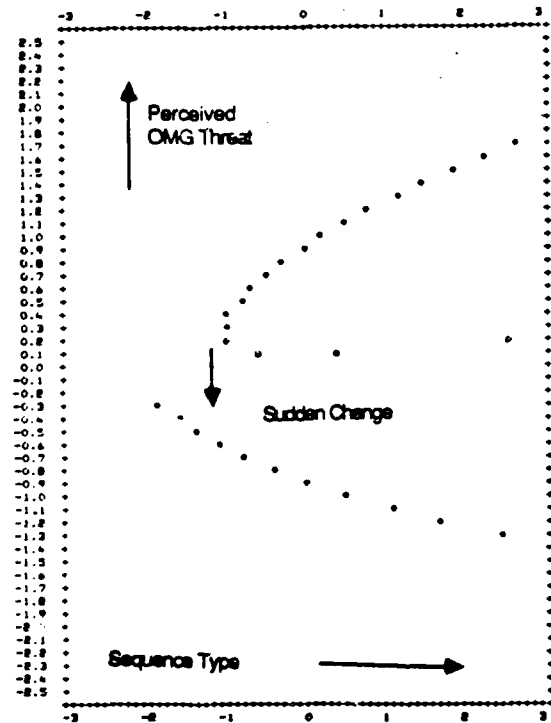
0 cases did not fit in the above figure.

Linear R² = 0.097 (Multiple regression)
Delay P² = 0.606 (Attracting-mode convention)
Maxwell R² = -0.740 (Most-likely-mode convention)

* Negative R² values occur when the cube model is worse than a constant.

>>>>> Fraction of cases in bimodal zone: 0.741 <<<<<

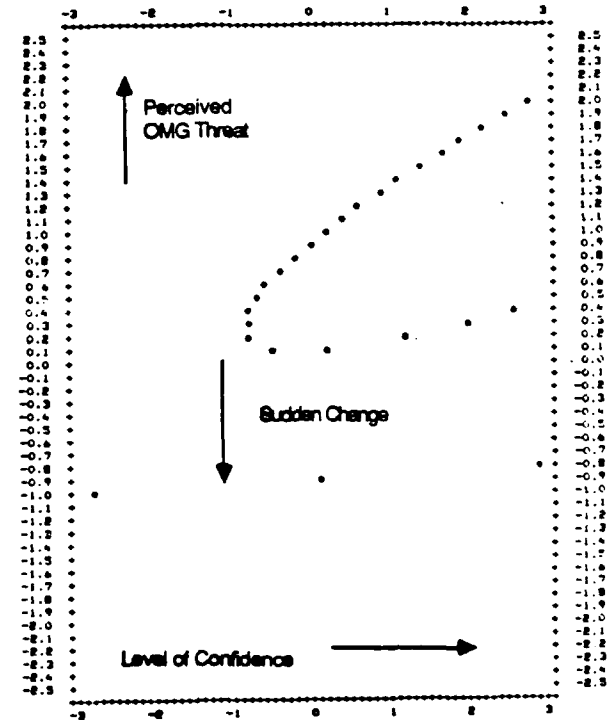
Effect of variable 5, holding all others constant at their mean values.



• Mode symbol
• Asymmetry symbol

(b) Variable Sequence Type, Fixed Level of Confidence

Effect of variable 10, holding all others constant at their mean values.



• Mode symbol
• Asymmetry symbol

(c) Variable Level of Confidence, Fixed Sequence Type

A slice of the cusp surface at a fixed level of confidence and variable sequence type reveals situations in which partial perceptual trapping can occur. Increasing the sequence type parameter from a low to a higher level causes a gradual and approximately linear decrease in perceived OMG threat (Exhibit 6-22b). By contrast, starting with a large sequence parameter value and a high level of perceived OMG threat and reducing this parameter can lead to conditions under which a rapid decrease in the level of perceived threat can occur and where perceptual trapping can take place.

A slice of the cusp surface at a variable level of confidence and fixed sequence parameter reveals situations in which partial perceptual trapping can occur. Increasing the level of confidence from a low to a higher level causes almost no change in perceived OMG threat to high level (Exhibit 6-22c). By contrast, starting with a large level of confidence and high level of perceived OMG threat and reducing the level of confidence of the I&W input data can lead to conditions under which a rapid increase in the level of perceived threat and perceptual trapping can occur.

6.2.5 "ANALYST E"

Despite collecting a complete data set from this analyst, computer analysis of the data revealed that these data would be best fitted by a linear model. Indeed, the statistical analysis was terminated because of the low level of the cubic term in the analysis or the nonlinear nature of the data itself.

APPENDIX A: AN OVERVIEW OF CATASTROPHE THEORY

Catastrophe theory and related mathematical modeling techniques can illustrate the impact of command and control on military behavior and be used to support the activities of I&W analysts. They can also provide new methods to support the military decision-making process that can exploit intelligence and other types of data. Models developed by Woodcock (1986) and based on the theory describe the impact of command and control, firepower, and force strength on military force survivability. Other mathematical models developed by Woodcock (1986) describe spatial and temporal oscillations in military force strength and the emergence of chaotic behavior under appropriate conditions.

Catastrophe theory provides a rigorous mathematical framework to support the "top-down" analysis of complicated military systems in general, and the high level management policy- and decision-making activities of military planners and others in the command and control and indications and warnings arena, in particular. The theory can be used to analyze those systems in which gradually changing forces can give rise to either gradual or sudden changes in behavior in the same system under different conditions. This behavior can be described with the aid of specialized catastrophe diagrams that capture the essence of system behavior by expressing the causal relationships between the input variables (known as control or conflicting variables) and output variables (known as behavior variables).

Catastrophe theory can be used to analyze those systems which exhibit at least some of the properties of hysteresis, bimodality, and divergence. The theory is particularly useful under those circumstances in which gradually changing forces can give rise to either gradual or sudden changes in behavior in the same system under different conditions. It is also useful under conditions involving ambiguity and uncertainty such as those faced by I&W analysts.

A.1 CATASTROPHE THEORY - A PROVEN FRAMEWORK

Invented by Thom in the 1960's (Thom, 1969, 1975; Broucker and Lander, 1975; Poston and Stewart, 1978; Zeeman, 1977; Zeeman and Trotman, 1977; Woodcock and Poston, 1974), the theory has been used in many applications in the mathematical, physical, life, and social sciences (see Arnold, 1984; Berry, 1976; Cobb, 1978; Gilmore, 1981; Hilton, 1978; Janich, 1974; Lu, 1980; Stewart, 1981; Stewart and Peregoy, 1982; Stewart and Woodcock, 1981, 1984; Wilson, 1981; Woodcock, 1978, 1979; and Woodcock and Davis, 1978; for example). Catastrophe theory has also been used in a series of military applications by Woodcock, 1986; Woodcock and Dockery, 1984 a, b; 1986 a, b; Dockery and Chiatti, 1986; Dockery and Woodcock, 1986 a, b; Isnard and Zeeman, 1976; and Holt, Job, and Marcus, 1978; and to model the phenomenon of bistable and multistable perception by Poston and Stewart (1978b) and Stewart and Peregoy (1982).

Thom called sudden changes in behavior "catastrophes" and developed a theory (subsequently referred to as "catastrophe theory" by Zeeman) as a new method for analyzing and classifying these changes. The elementary catastrophes are the simplest distinct patterns of changes that can occur in many types of systems. There are seven distinct elementary catastrophe manifolds. These manifolds are geometric surfaces that are generated from combinations of the stationary state points of systems with up to four input variables (referred to as control factors) and two output variables (referred to as behavior variables).

The elementary catastrophe manifolds have a complexity ranging from the two-dimensional fold catastrophe (with one control factor and one behavior variable) to the six-dimensional parabolic umbilic catastrophe (with four control factors and two behavior variables). Mathematical details of these catastrophes are summarized in Exhibit A-1. The popular names presented in Exhibit A-1 are descriptive of the geometry of their catastrophe manifolds as shown in Woodcock and Poston (1974) for example.

Exhibit A-1
The Elementary Catastrophes

Popular Name of Catastrophe	Control Factors or Input Variables	Behavior or Output Variables	Potential Function Equation
Fold	1	1	$x^3/3 + ax$
Cusp	2	1	$x^4/4 + ax^2/2 + bx$
Swallowtail	3	1	$x^5/5 + ax^3/3 + bx^2/2$
Butterfly	4	1	$x^6/6 + ax^4/4 + bx^3/3 + cx^2/2 + dx$
Hyperbolic Umbilic	3	2	$x^3 + y^3 + wxy + ux + vy$
Elliptic Umbilic	3	2	$x^3 - 3xy^2 + w(x^2 + y^2) + ux + vy$
Parabolic Umbilic	4	2	$x^2y + y^4 + ty^2 + wx^2 + ux + vy$

where (a, b, c, d, u, v, w, and t are control factors and x and y are behavior variables).

In applications in which the elementary catastrophes are used, an attempt is made to devise the simplest possible model (that is a model which uses as few control factors and behavior variables as possible) that can capture the essence of system behavior. Once the number of dependent and independent variables at work in a particular system has been identified, elementary catastrophe theory provides an indication of which of the catastrophe manifold diagrams is appropriate for expressing the causal relationships between these variables. Such diagrams can provide a basis for undertaking a series of "thought-experiments" in order to determine the response of the system to changes in the magnitude of the controls to which it is subjected.

Thom's theorem claims that the stationary state behavior of all systems (including physical, chemical, biological, and societal systems) with up to four control factors (or input variables), two behavior variables (or outputs), and which possess an associated potential function can be described with the aid of one of the elementary catastrophes. Subsequent mathematical analysis by Zeeman and Trotman (1977), for example, has proved Thom's theorem.

The use of catastrophe theory to model a particular system, such as that associated with military combat, will require the identification of a suitable set of control factors and behavior variables. Two and four control factor models of military behavior, which are based on the cusp and butterfly catastrophes respectively, have been developed by Woodcock and Dockery (1984 a, b). The development of these models was motivated, in part, by the need to provide models with relatively few control factors that resemble the properties or "axes" around which a battle commander organizes his perceptions.

Following this work, Dockery and Chiatti (1986) have used a statistical program developed by Cobb (1983) to fit simulated combat data to the surface of the cusp catastrophe manifold. Such models can provide a useful type of interface between the military commander and elaborate computer-based combat simulations which provide large amounts of data. These models can provide a series of diagrams, which can be referred to as "problem-solving landscapes," that can aid the commander in tracking events during combat and support his tactical and strategic decision-making processes by enabling him to perform a series of situation assessments and impact analyses on an essentially real time basis.

While elaborate simulations can provide considerable insight into the nature of the combat process, the complexity and magnitude of the data that they produce serves to reduce their utility as a guide for the military commander. By contrast, the catastrophe models provide a method for presenting information in a method which can aid comprehension and "turn old facts into new knowledge" (Thompson, 1917).

A.2 THE PROPERTIES OF THE CUSP CATASTROPHE

The properties of the elementary catastrophes can be illustrated by the cusp catastrophe. The cusp catastrophe potential function ($V_{CC}(x)$) is the sum of the germ ($g_{CC}(x)$) and unfolding ($u_{CC}(x)$) components of the catastrophe, that is:

$$V_{CC}(x) = g_{CC}(x) + u_{CC}(x) = x^4/4 + ax^2/2 + bx \quad (A1)$$

where a and b are the control factors, x is the behavior variable, and $g_{CC}(x)$ and $u_{CC}(x)$ are the germ and unfolding of the catastrophe, respectively. The properties of equation (A1) are illustrated in Exhibit A-2 for various values of the a and b control factors.

Conditions in which both a and b in equation (A1) are zero describe the germ of the catastrophe (Exhibit A-2). Nonzero control factor values represent conditions in which the germ is perturbed or unfolded and the effect of such control factor changes reveals the underlying structure of the germ of the catastrophe.

Stationary states of the cusp catastrophe potential function (equation (A1)) occur when its differential with respect to the behavioral variable (x), vanishes, that is, when:

$$dV_{CC}(x)/dx = x^3 + ax + b = 0 \quad (A2)$$

This equation describes a three-dimensional (x, a, b) curved surface known as the catastrophe manifold which represents the stationary states of the function (A2) as shown in Exhibit A-3. The catastrophe manifold equation has either one or three real solutions depending on the values of the a and b control factors. The single solution is a minimum while the triple solution represents two separate minima separated by a maximum. Equation (A3), representing the bifurcation set (that is, the set of control factor values at which one or other of the minima is

Exhibit A-2

The Cusp Catastrophe Function $V_{CC}(x)$

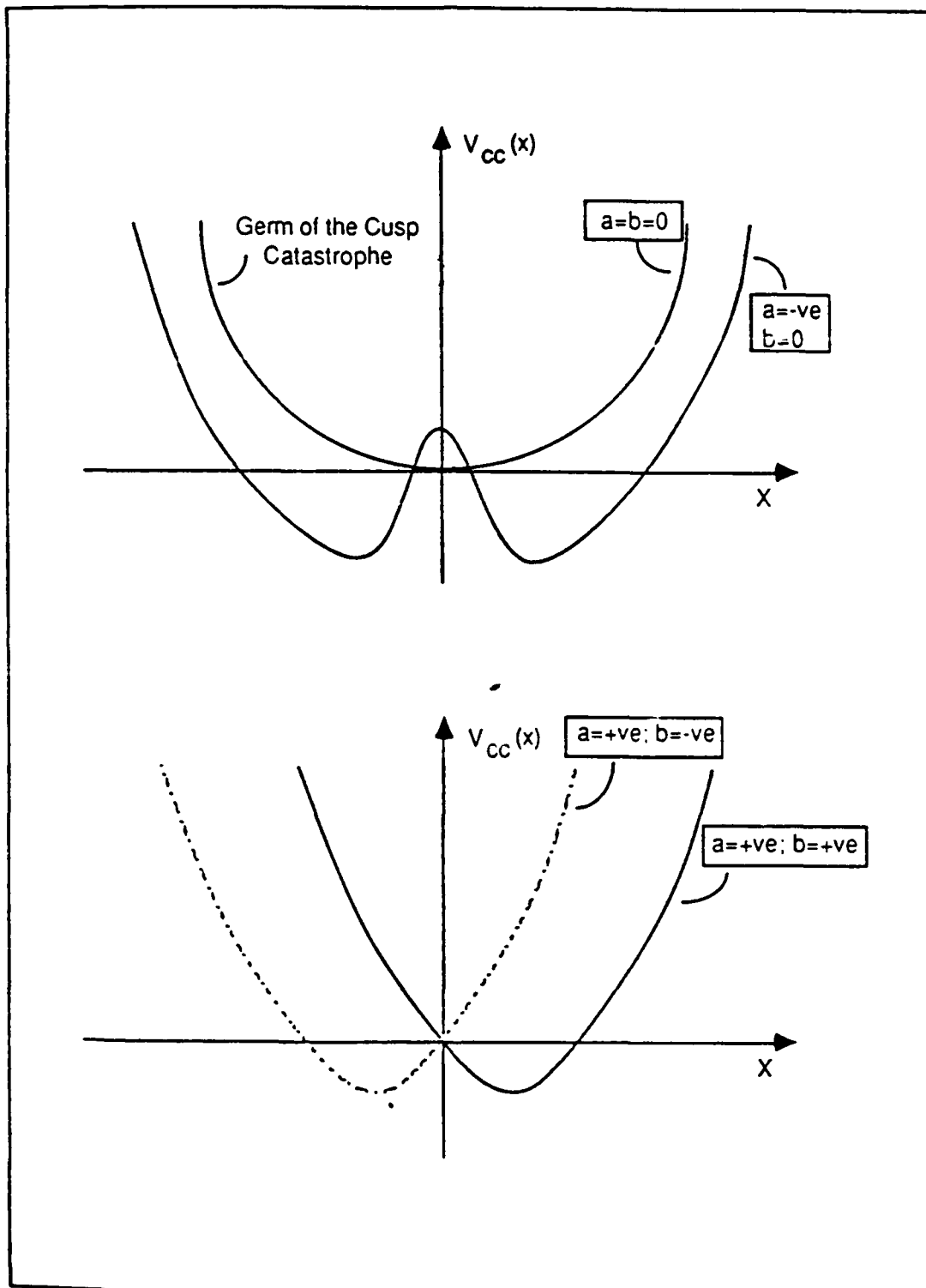
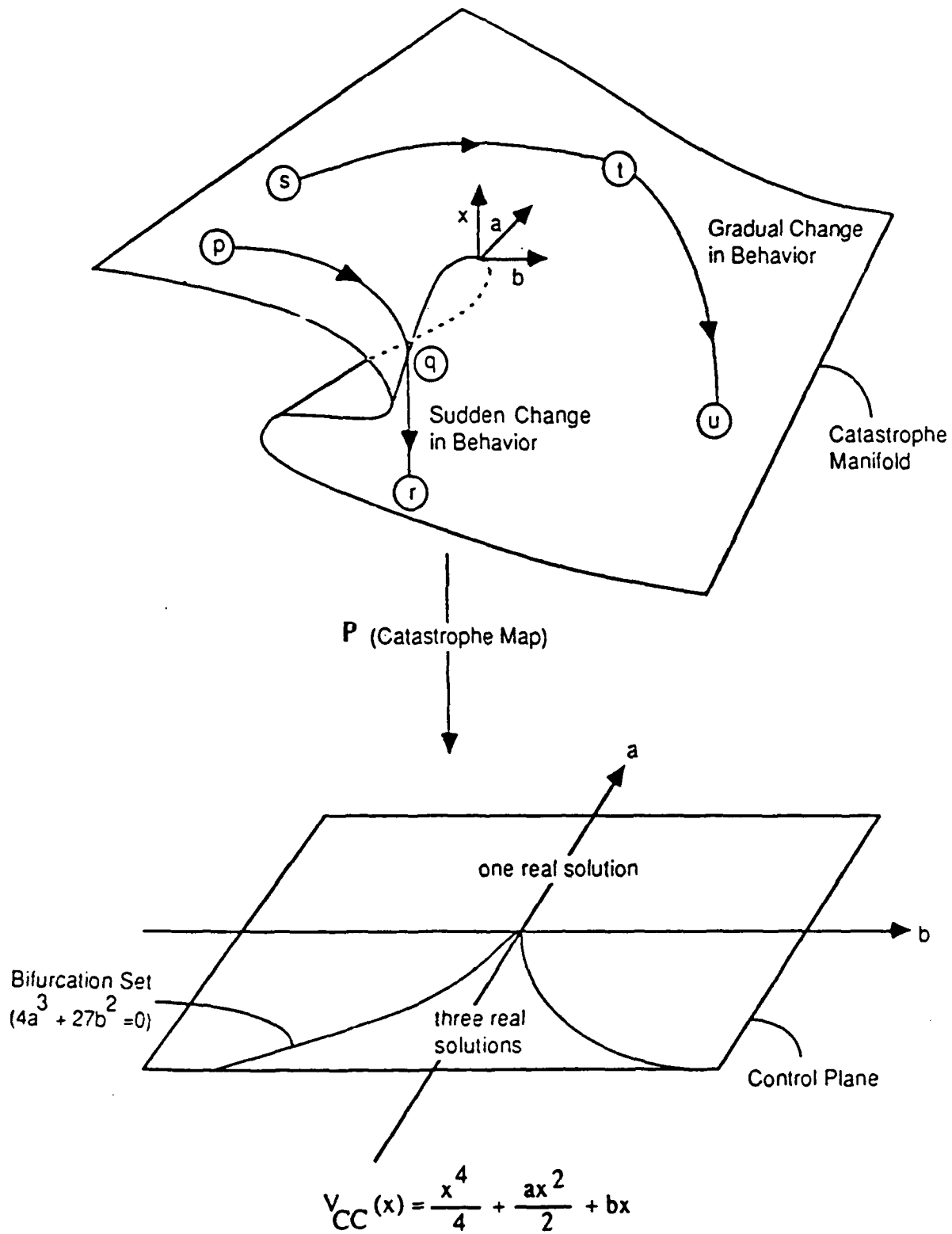


Exhibit A-3

The Cusp Catastrophe Manifold and Control Plane



destroyed in response to system changes), can be obtained from the cusp function (1) by differentiation and rearrangement:

$$4a^3 + 27b^2 = 0 \quad (A3)$$

Sudden or catastrophic transitions of state can occur in response to changes in control factor values at the bifurcation set as one of the two minima of the catastrophe function (A2) disappears and becomes a point of inflection. Non-catastrophic changes in behavior can occur when changes in control factor values do not cause a change in the number of stationary states of the catastrophe function.

Both types of changes in system behavior can be illustrated by the pattern of movement of a point, known as the state point, on the catastrophe manifold surface in response to changes in control factor values. Thus, path (p, q, r) (Exhibit A-3) represents a sudden or catastrophic change in system behavior while path (s, t, u) represents a gradual change in system behavior.

A.3 THE PROPERTIES OF THE BUTTERFLY CATASTROPHE

The butterfly catastrophe function ($V_{BC}(x)$) has the form:

$$V_{BC}(x) = x^6/6 + ax^4/4 + bx^3/3 + cx^2/2 + dx \quad (A4)$$

which represents a potential function with at most three stable stationary states separated by two unstable stationary states for particular values of the control factors (Exhibit A-4a). The third stable stationary state can be considered as an intermediate or compromise state between two polarized extreme states. It is represented by an additional, intermediate layer of the butterfly catastrophe manifold surface (Exhibit A-4b). This exhibit is a three-dimensional projection of the five-dimensional manifold drawn with the a and b control dimensions fixed in value. Other projections look similar to the cusp manifold (see Exhibit A-3, for example) except that the folded region of the butterfly manifold can move back and forth. In the four factor model of military behavior described below, this intermediate layer represents a state of intermediate Blue force survivability which is supported by appropriate levels of Blue force firepower and command and control capabilities.

A.4 CATASTROPHE THEORY-BASED MODELS OF MILITARY BEHAVIOR

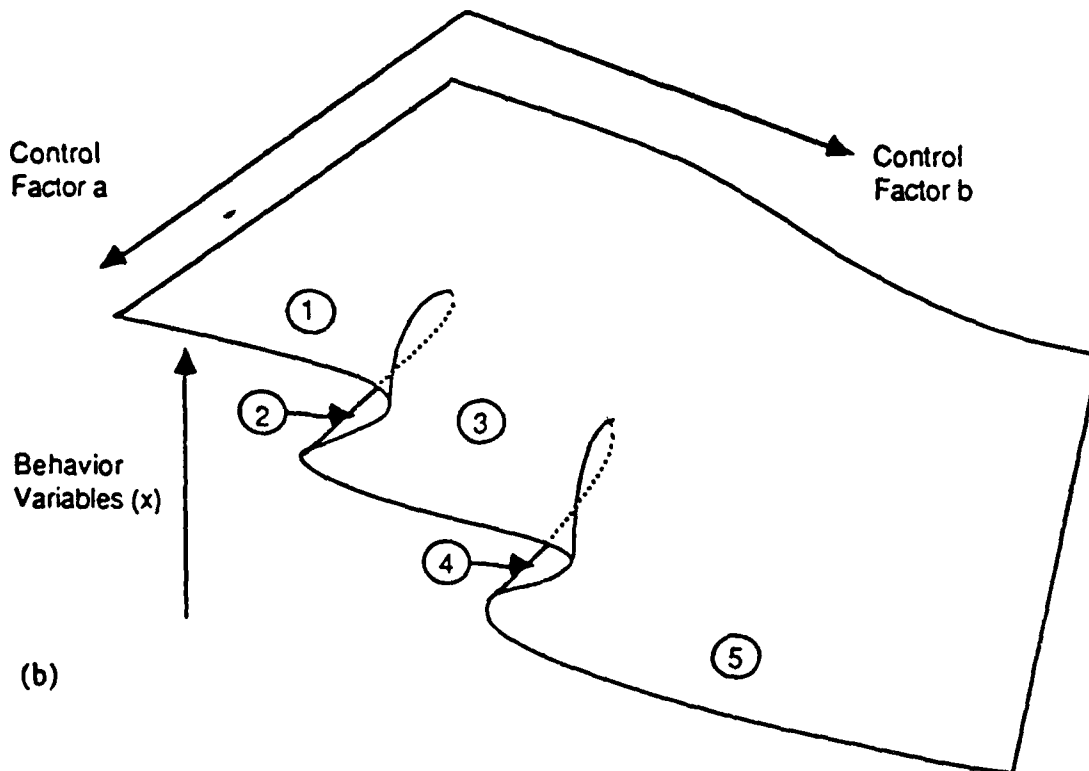
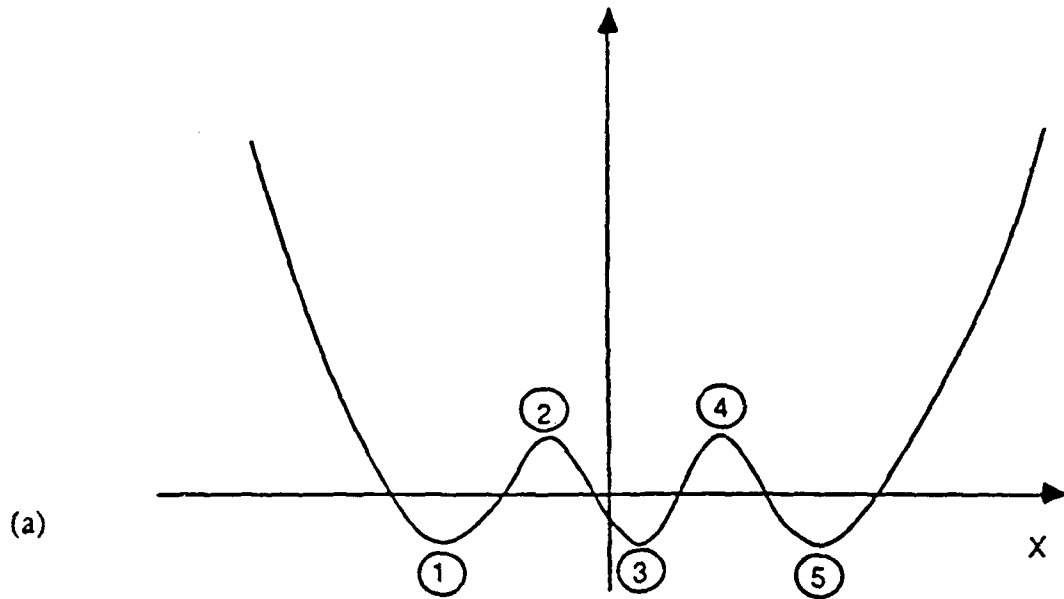
The following is a description of two models of military behavior based on catastrophe theory that has been developed by Woodcock and Dockery (1984 a, b; 1986 a).

A.4.1 A CUSP-CATASTROPHE-BASED MODEL

The first catastrophe-theory model of the combat process developed by Woodcock and Dockery (1984 a, b) describes the impact of opposing Red and Blue forces on the survivability of the Blue forces. This model involves the impact of two influences on system behavior and, as a consequence of Thom's theorem, is based on the cusp catastrophe.

Exhibit A-4

The Butterfly Catastrophe Potential Function $V_{BC}(x)$ and Manifold



The Butterfly Catastrophe can represent systems with up to three stable stationary states and two unstable stationary states and whose behavior is determined by the impact of four key influences (or control factors).

$$V_{BC}(x) = x^6/6 + ax^4/4 + bx^3/3 + cx^2/2 + dx$$

The concept of using conflicting factors (which is due to Zeeman (Isnard and Zeeman, 1976)) in place of control factors was employed by Woodcock and Dockery (1984 a, b; 1986 a) in both the two factor and four factor models in order to capture the inherently conflictual nature of the military combat process (Exhibit A-5). Two areas of the cusp catastrophe manifold, which represent two qualitatively different types of system behavior, can be identified. A condition of high Blue and Red Force strengths is represented by a point at position (a) on the catastrophe manifold surface. As the combat proceeds, a relatively large degradation of the Blue Force strength compared to that of the Red Forces can lead to a rapid decrease in Blue Force survivability path (a-b-c).

This two factor model has illustrated the impact of Blue and Red force strength on the survivability of the Blue forces. Together with the fitting of simulated data with a program developed by Cobb (1983), and by Dockery and Chiatti (1986), one result of which is illustrated in Exhibit A-6, this model has provided an anchor for further investigations. The line of reasoning takes quite literally the concept that lower dimensional manifolds are embedded in those of higher dimensions.

A.4.2 A BUTTERFLY-CATASTROPHE-BASED MODEL

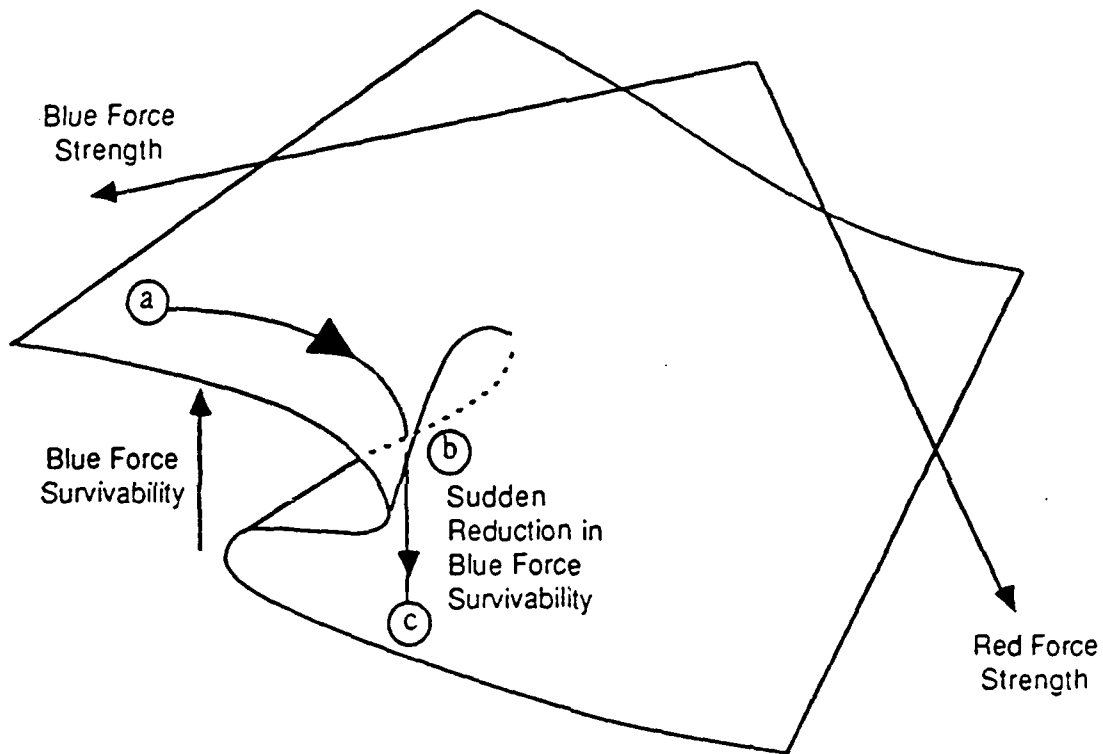
The second model describes the impact of Red and Blue force strength, firepower, and command and control capabilities on the survivability of the Blue forces and is based on the butterfly catastrophe (Exhibit A-7). The relative firepower and command and control capabilities available to the two forces will be represented by scales which will be placed below the drawing of the three-dimensional section of the five-dimensional butterfly catastrophe manifold. As the position of the indicator arrow moves on the scale, the shape of the manifold changes. We track these changes by comparing the changes in the shape of the "footprint" of the folded region of the manifold.

The conditions in which the Blue force has a significant advantage in firepower and command and control capabilities compared to the Red force, is illustrated in the model by a distortion of the manifold surface to produce a relatively large region. This is identified with conditions of high Blue force survivability. These additional influences can thus be seen to offset the effect of a relatively low intrinsic Blue Force strength.

A reduction in the Blue force command and control capability advantage during combat will lead to a reduction in Blue force survivability. Under these circumstances, it would perhaps appear to the Blue Force Commander that the "ground" was falling away from under his feet as the folded region of the surface moves in response to changes in the level of command and control capabilities (path a-b, Exhibit A-7) in response to a decrease in the level of Blue force command and control capabilities. The hatched area drawn on the plane in this Exhibit represents those sets of factor values for which such a reduction in force survivability can occur.

Exhibit A-5

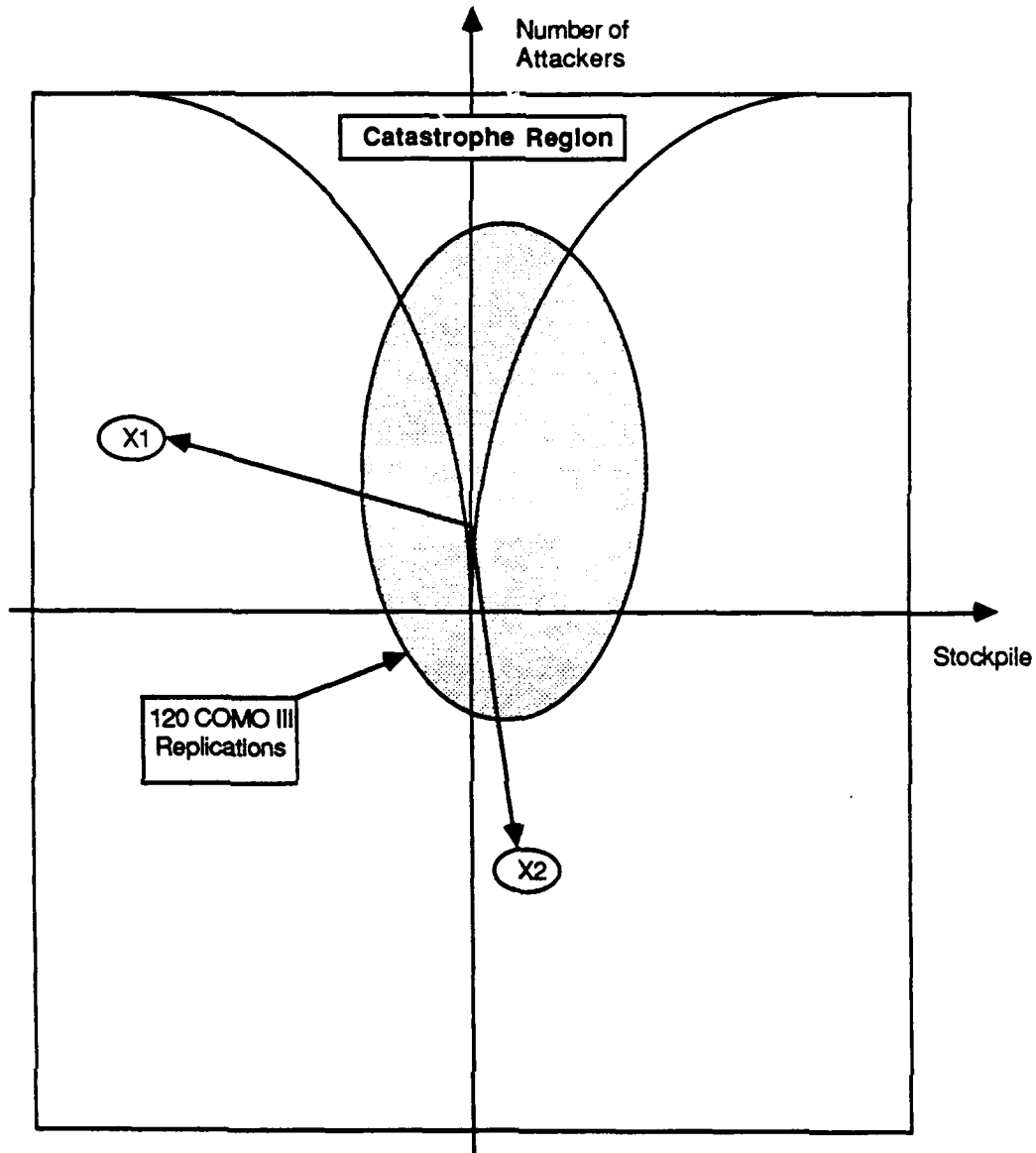
A Military Analysis and Problem-Solving Landscape



- A two factor catastrophe model
- force balance determines force survivability

Exhibit A-6

Application of Catastrophe Theory to Military Analysis



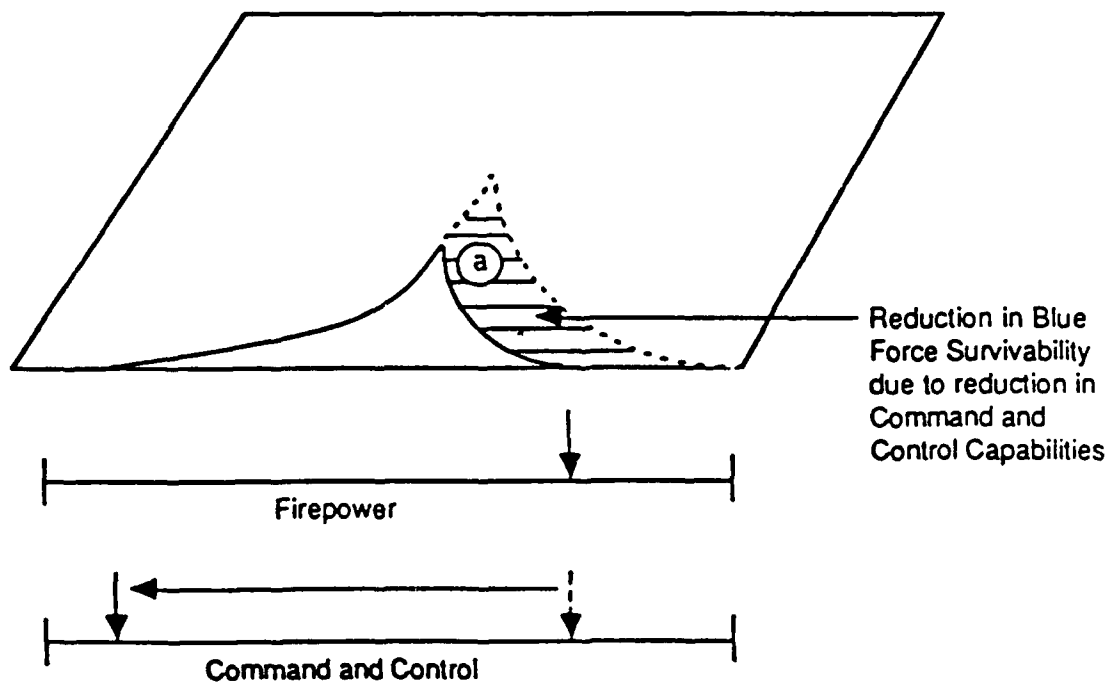
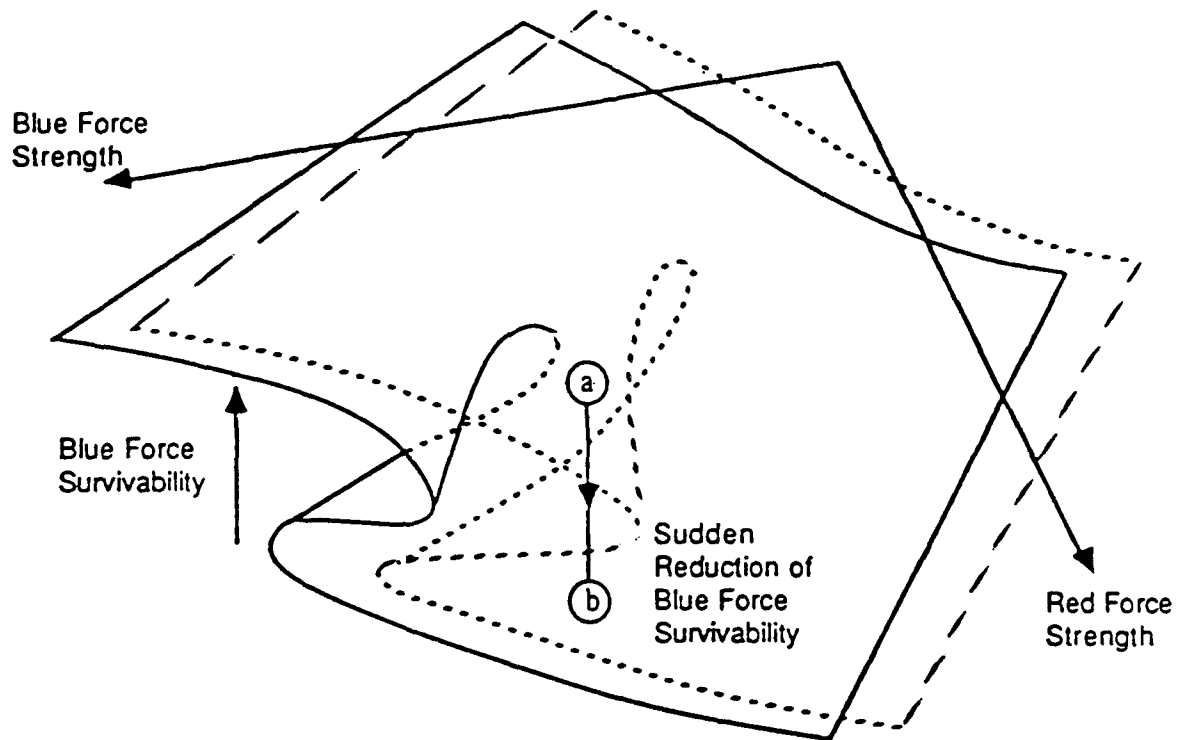
(Re-drawn and modified, after Dockery and Chiatti, 1986)

Control plane plot of 120 replications of the second scenario taken from the event driven simulation. The independent variables are stockpile (X_1) and number of attackers (X_2).

Values of X_1 , X_2 used were total over several time periods.

Exhibit A-7

A Military Analysis and Problem-Solving Landscape



Force balance, firepower, and command and control capabilities determine force survivability.

BIBLIOGRAPHY

- Arnold, V. I., Catastrophe Theory (Translated by R. K. Thomas). Berlin and New York: Springer-Verlag, 1984.
- Berry, M. V., "Waves and Thom's Theorem." Adv. Phys., 1976, 25, 1-26.
- Bonder, S. and J. G. Honig. "An Analytical Model of Ground Combat: Design and Application." Proceedings of the Tenth Annual U.S. Army Operations Research Symposium. Durham, North Carolina, 1971.
- Brocker, T. and L. Lander. "Differential Germs and Catastrophes." London Mathematical Society Lecture Notes, 17, 1975, Cambridge: Cambridge University Press.
- Cobb, L. "Stochastic Catastrophe Models and Multimodal Distributions." Behavioral Science, 1978, 23, 360-374.
- Cobb, L. "Estimation Theory for the Cusp Catastrophe Theory Model." Proceedings of the Section on Survey Research. Washington, DC: American Statistical Association, 1980.
- Cobb, L. A Maximum Likelihood Computer Program to Fit a Statistical Cusp Hypothesis. The Hague, The Netherlands: SHAPE Technical Centre, 1983.
- Cobb, L. and G. Harrison. A Computer Program to Solve Stochastic Lanchester Equations. Washington, DC: Organization of the Joint Chiefs of Staff, 1985.
- Dick, C. J. "Soviet Operational Maneuver Groups: A Closer Look." International Defense Review. 769-772, 1983.
- Dockery, J. T., and L. Chiatti. "Application of Catastrophe Theory to the Problems of Military Analysis." European Journal of Operations Research, 1986, 24, 46-53.
- Dockery, J. T., and A. E. R. Woodcock. "Models of Combat II: Catastrophe Theory and Chaotic Behavior." 1986a, preprint.
- Dockery, J. T., and A. E. R. Woodcock. "Models of Combat III: Combat Rheology." 1986b (in preparation).
- Donnelly, C. N. "The Soviet Operational Maneuver Group: A New Challenge for NATO." International Defense Review. 1177-1186, 1982.
- Gilmore, R. Catastrophe Theory for Scientists and Engineers. New York: Wiley Interscience, 1981.
- Guckenheimer, J. and P. Holmes. Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. New York: Springer-Verlag, 1983.
- Hilton, P. "Structural Stability, the Theory of Catastrophes and Applications." Lecture Notes in Mathematics, 525. Berlin and New York: Springer-Verlag, 1978.

BIBLIOGRAPHY (Continued)

- Holt, R. T., B. Job and L. Marcus. "Catastrophe Theory and the Study of War." Journal of Conflict Resolution. 1978, 22, 171-208.
- Isnard, C. A. and E. C. Zeeman. "Some Models from Catastrophe Theory in the Social Sciences." The Use of Models in the Social Sciences. London: Tavistock Publications, 1976. Collins, L. (ed.) 44-100.
- Janich, K. "Caustics and Catastrophes." Math. Ann. 1974, 209, 161-180.
- Janus, I. L. and L. Mann. Decision Making. New York: The Free Press, 1977.
- Lu, Y. C. Singularity Theory and an Introduction to Catastrophe Theory. Berlin and New York: Springer-Verlag, 1980.
- Petersen, R. "On the Logarithmic Law of Attrition and its Application to Tank Combat." Operations Research. 1967, 15, 557-558.
- Poston, T. and I. Stewart. Catastrophe Theory and its Applications. London: Pitman, 1978.
- Stewart, I. "Applications of Catastrophe Theory in the Physical Sciences." Physica. 1981, 2D, 245-305.
- Stewart, I. N. and P. L. Peregoy. "Catastrophe Theory Modelling in Psychology." Preprint. Coventry, Warwickshire: University of Warwick Mathematics Institute, 1982.
- Stewart, I. N. and A. E. R. Woodcock. "On Zeeman's Equations for the Nerve Impulse." Bulletin Mathematical Biology. 1981, 43, 279-325.
- Stewart, I. and A. E. R. Woodcock. "Bifurcation and Hysteresis Varieties for the Thermal Chainbranching Model II: Positive Modal Parameter." Math. Proc. Camb. Phil. Soc. 1984, 96, 331-349.
- Thom, R. "Topological Models in Biology." Topology. 1969, 8, 313-335.
- Thom, R. Structural Stability and Morphogenesis. Reading, Massachusetts: W. A. Benjamin, 1975.
- Thompson, D. A. W. On Growth and Form. Cambridge: Cambridge University Press, 1917.
- Wassermann, G. "Stability of Unfoldings in Space and Time." Acta Mathematica. 1975, 135, 57-128.
- Woodcock, A. E. R. An Investigation of Catastrophe Theory as a Command and Control Device. Rome, New York: Syntectics Corporation, 1986.

BIBLIOGRAPHY (Continued)

Woodcock, A. E. R. "Catastrophe Theory and Cellular Determination, Transdetermination, and Differentiation." Bulletin of Mathematical Biology, 1979, 41, 101-117.

Woodcock, A. E. R. "On the Geometry of Space- and Time-Equivalent Catastrophes." Bulletin of Mathematical Biology, 1978, 40, 1-25.

Woodcock, A. E. R. and M. Davis. Catastrophe Theory. New York: E. P. Dutton, 1978.

Woodcock, A. E. R. and J. T. Dockery. Application of Catastrophe Theory to the Analysis of Military Behavior. The Hague, The Netherlands: SHAPE Technical Centre Consultants Report, STC CR-56, 1984a.

Woodcock, A. E. R. and J. T. Dockery. "Artificial Intelligence and Catastrophe Theory." The Hague, The Netherlands: SHAPE Technical Centre, Technical Report, TM749, The Use of Artificial Intelligence in the Analysis of Command and Control, J. T. Dockery and J. van den Driessche.

Woodcock, A. E. R. and J. T. Dockery. Models of Combat I: Catastrophe Theory and the Lanchester Equations (Appendix A). Rome, New York: Synectics Corporation, 1986a.

Woodcock, A. E. R. and J. T. Dockery. Models of Combat IV: Population-dynamics Models of Military Behavior (Appendix B). Rome, New York: Synectics Corporation, 1986b.

Woodcock, A. E. R. and J. T. Dockery. Models of Combat V: The Tactical Control of Insurgents (Appendix C). Rome, New York: Synectics Corporation, 1986.

Woodcock, A. E. R. and T. Poston. "A Geometrical Study of the Elementary Catastrophes." Lecture Notes in Mathematics, 373. Berlin and New York: Springer-Verlag, 1974.

Zeeman, E. C. "Catastrophe Theory, Selected Papers, 1972-1977." Reading, Massachusetts: Addison Wesley, 1977.

Zeeman, E. C. and D. Trotman. "The Classification of Elementary Catastrophes ≤ 5 . Structural Stability, The Theory of Catastrophes and Applications." Lecture Notes in Mathematics, 525, 263-327. Berlin and New York: Springer-Verlag, 1976.



MISSION of Rome Air Development Center

RADC plans and executes research, development, test and selected acquisition programs in support of Command, Control, Communications and Intelligence (C³I) activities. Technical and engineering support within areas of competence is provided to ESD Program Offices (POs) and other ESD elements to perform effective acquisition of C³I systems. The areas of technical competence include communications, command and control, battle management, information processing, surveillance sensors, intelligence data collection and handling, solid state sciences, electromagnetics, and propagation, and electronic, maintainability, and compatibility.